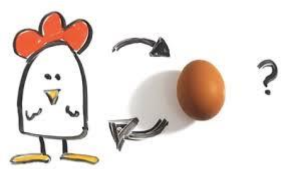
Department of National Defense Longitudinal Analysis Training Seminar

Presented by: Simon Houle

In collaboration with Dr. Alexandre Morin and the Substantive Methodological Synergy Research Laboratory

Why Longitudinal Data?

- Directionality



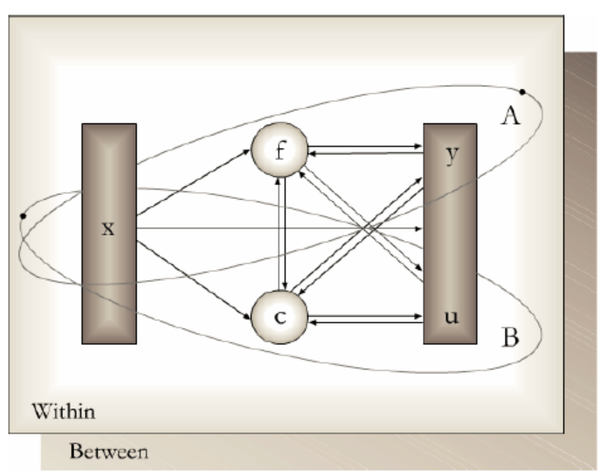
- Change

- Shape of change

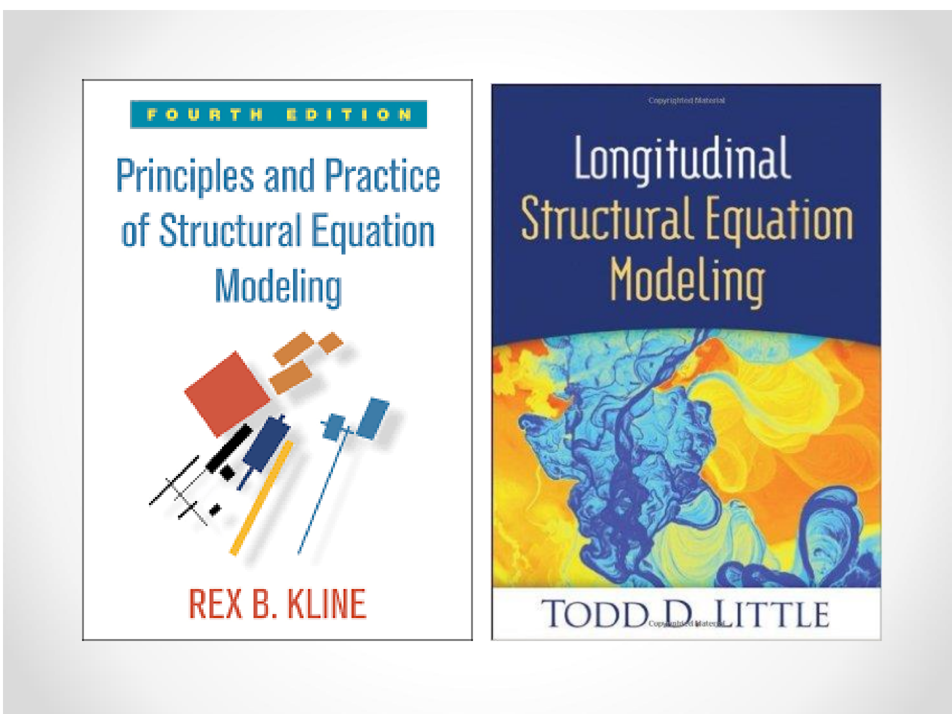
Overview

- Brief introduction to M*plus*
- Setting the scale for latent variables
- The meaning of time
- Longitudinal measurement invariance in M*plus*
- Basic longitudinal models
- Auto-regressive crossed-lagged models
- State-trait-error models
- Latent change models
- Cross sectional application for latent change models
- Latent curve models

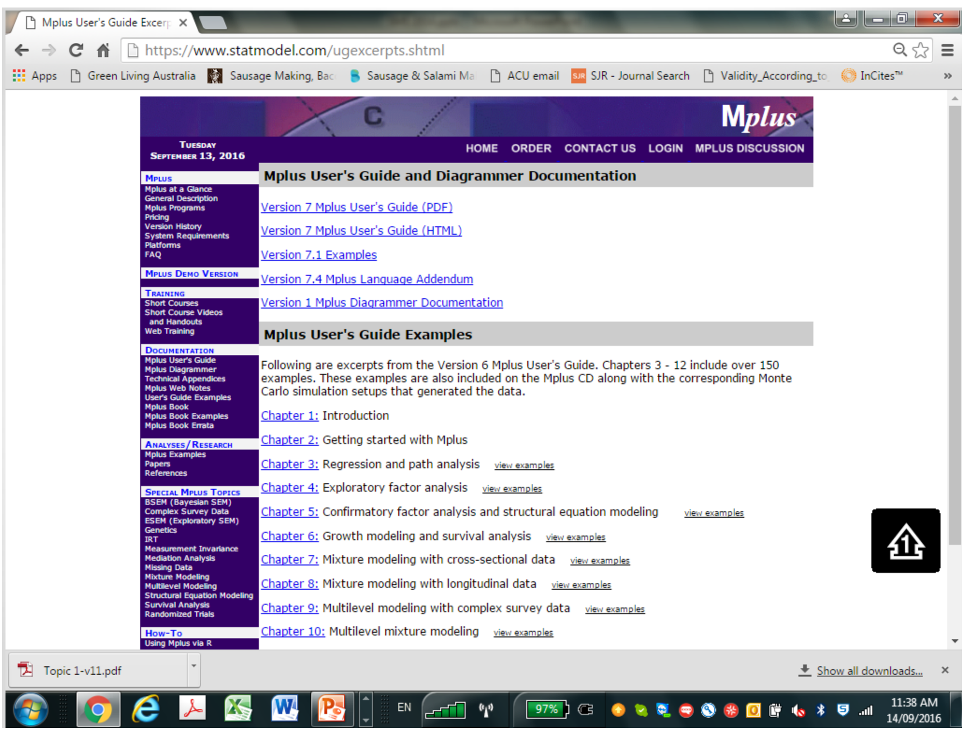
M*plus*

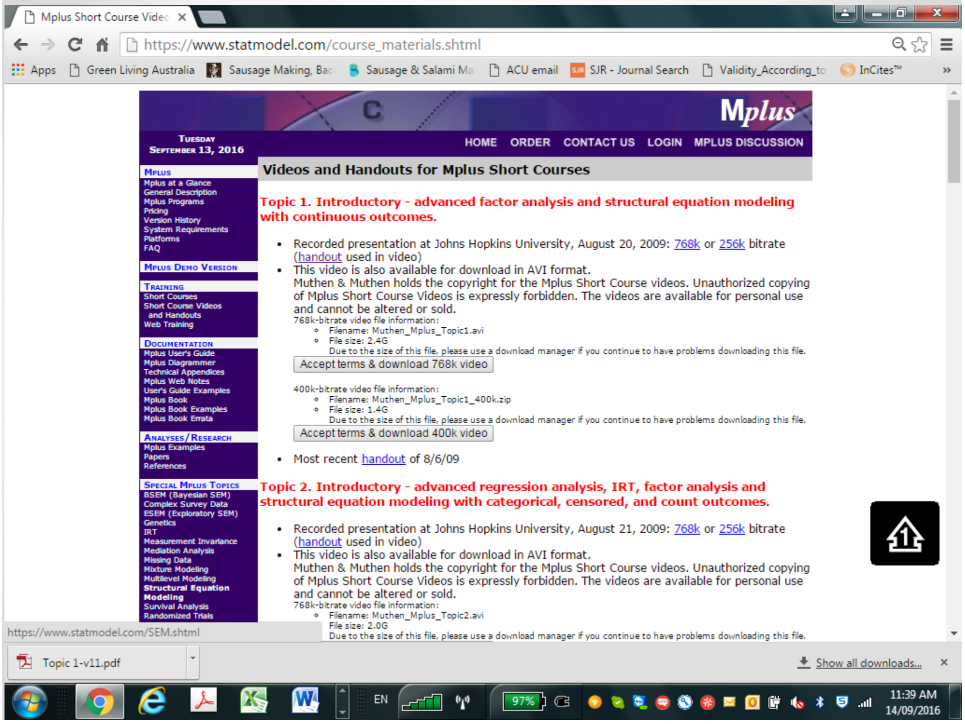



Structural Equation Modeling With Mplus
Basic Concepts, Applications, and Programming
Barbara M. Byrne
DATA ANALYSIS WITH MPLUS
Christian Geiser
SECOND EDITION
Using Mplus for Structural Equation Modeling
A Researcher's Guide
E. Kevin Kelloway
WILEY
Structural Equation Modeling
Applications Using Mplus
Jichuan Wang · Xiaoqian Wang
WILEY SERIES IN PROBABILITY AND STATISTICS


FOURTH EDITION
Principles and Practice of Structural Equation Modeling
REX B. KLINE
Longitudinal Structural Equation Modeling
TODD D. LITTLE


Muthén & Muthén, Mplus
https://www.statmodel.com
Mplus
HOME ORDER CONTACT US LOGIN MPLUS DISCUSSION
Tuesday September 13, 2016
Mplus Papers
Using Special Mplus Features
Latest News
Announcing our new book Regression And Mediation Analysis Using Mplus. 140 Mplus inputs, outputs, and data sets where available are posted for the book examples. The book can be ordered from the online store.
Mplus favored for Bayesian analysis.
Mplus Version 7.4 is now available. The Version 7.4 Mplus Language Addendum is found on the website along with the revised Mplus Version 7 User's Guide. Registered users who purchased Mplus within the last year and those with a current Mplus Upgrade and Support Contract can download Version 7.4 using our online system at no cost.
Recent Postings
Recent papers
Recent web notes
Recent presentations
Mplus Demo Version
Student Pricing for Mplus Version 7.4
Mplus Version 7 User's Guide and Examples
Mplus Web Training and Handouts
Papers Using Special Mplus Features
Topic 1-v11.pdf
Show all downloads...
11:37 AM
14/09/2016


Mplus User's Guide Excerpts
https://www.statmodel.com/ugexcerpts.shtml
Mplus
HOME ORDER CONTACT US LOGIN MPLUS DISCUSSION
Tuesday September 13, 2016
Mplus User's Guide and Diagrammer Documentation
Version 7 Mplus User's Guide (PDF)
Version 7 Mplus User's Guide (HTML)
Version 7.1 Examples
Version 7.4 Mplus Language Addendum
Version 1 Mplus Diagrammer Documentation
Mplus User's Guide Examples
Following are excerpts from the Version 6 Mplus User's Guide. Chapters 3 - 12 include over 150 examples. These examples are also included on the Mplus CD along with the corresponding Monte Carlo simulation setups that generated the data.
Chapter 1: Introduction
Chapter 2: Getting started with Mplus
Chapter 3: Regression and path analysis view examples
Chapter 4: Exploratory factor analysis view examples
Chapter 5: Confirmatory factor analysis and structural equation modeling view examples
Chapter 6: Growth modeling and survival analysis view examples
Chapter 7: Mixture modeling with cross-sectional data view examples
Chapter 8: Mixture modeling with longitudinal data view examples
Chapter 9: Multilevel modeling with complex survey data view examples
Chapter 10: Multilevel mixture modeling view examples
Topic 1-v11.pdf
Show all downloads...
11:38 AM
14/09/2016



Mplus Short Course Video

https://www.statmodel.com/course_materials.shtml

Apps | Green Living Australia | Sausage Making, Bac | Sausage & Salami Ma | ACU email | SJR - Journal Search | Validity_According_to | InCites™

Mplus

Tuesday September 13, 2016

HOME ORDER CONTACT US LOGIN MPLUS DISCUSSION

Mplus: Mplus at a Glance, General Description, Mplus Programs, Pricing, Version History, System Requirements, Platforms, FAQ

Mplus Demo Version

Training: Short Courses, Short Course Videos and Handouts, Web Training

Documentation: Mplus User's Guide, Mplus Diagrammer, Technical Appendices, Mplus Web Notes, User's Guide Examples, Mplus Book, Mplus Book Examples, Mplus Book Errata

Analyses/Research: Mplus Examples, Papers, References

Special Mplus Topics: BSEM (Bayesian SEM), Complex Survey Data, ESEM (Exploratory SEM), Genetics, IRT, Measurement Invariance, Mediation Analysis, Missing Data, Mixture Modeling, Multilevel Modeling, Structural Equation Modeling, Survival Analysis, Randomized Trials

Videos and Handouts for Mplus Short Courses

Topic 1. Introductory - advanced factor analysis and structural equation modeling with continuous outcomes.

- Recorded presentation at Johns Hopkins University, August 20, 2009: 768k or 256k bitrate (handout used in video)
- This video is also available for download in AVI format.
  Muthen & Muthen holds the copyright for the Mplus Short Course videos. Unauthorized copying of Mplus Short Course Videos is expressly forbidden. The videos are available for personal use and cannot be altered or sold.
  768k-bitrate video file information:
  - Filename: Muthen_Mplus_Topic1.avi
  - File size: 2.4G
  Due to the size of this file, please use a download manager if you continue to have problems downloading this file.
  Accept terms & download 768k video
  400k-bitrate video file information:
  - Filename: Muthen_Mplus_Topic1_400k.zip
  - File size: 1.4G
  Due to the size of this file, please use a download manager if you continue to have problems downloading this file.
  Accept terms & download 400k video
- Most recent handout of 8/6/09

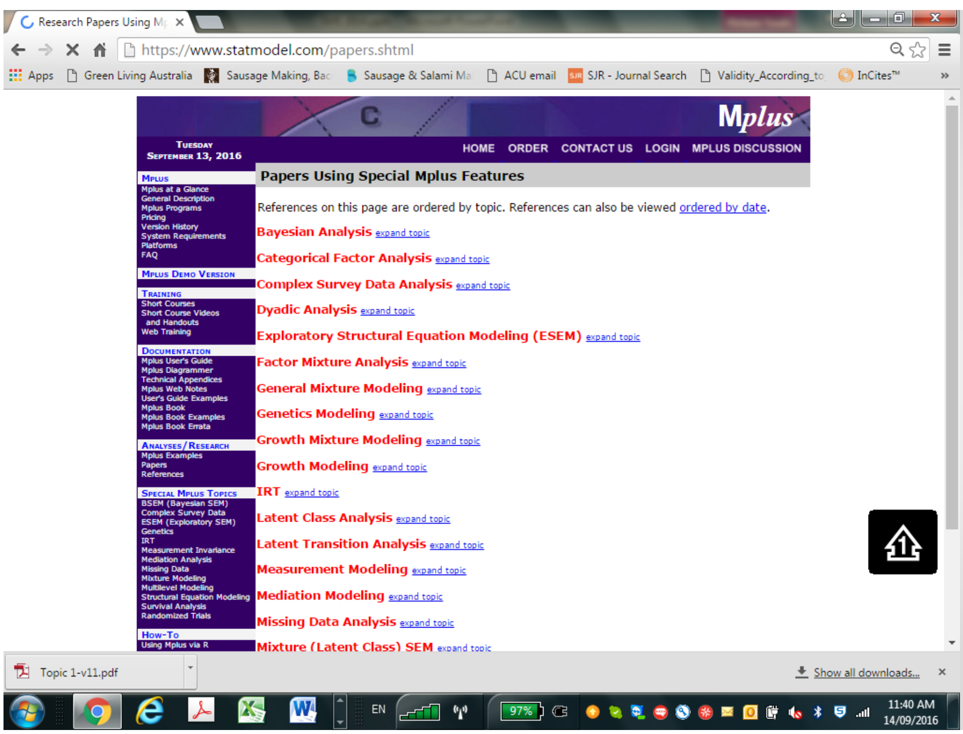
Topic 2. Introductory - advanced regression analysis, IRT, factor analysis and structural equation modeling with categorical, censored, and count outcomes.

- Recorded presentation at Johns Hopkins University, August 21, 2009: 768k or 256k bitrate (handout used in video)
- This video is also available for download in AVI format.
  Muthen & Muthen holds the copyright for the Mplus Short Course videos. Unauthorized copying of Mplus Short Course Videos is expressly forbidden. The videos are available for personal use and cannot be altered or sold.
  768k-bitrate video file information:
  - Filename: Muthen_Mplus_Topic2.avi
  File size: 2.0G
  Due to the size of this file, please use a download manager if you continue to have problems downloading this file.

https://www.statmodel.com/SEM.shtml

Topic 1-v11.pdf | Show all downloads...

EN 97% 11:39 AM 14/09/2016



Research Papers Using M

https://www.statmodel.com/papers.shtml

Apps | Green Living Australia | Sausage Making, Bac | Sausage & Salami Ma | ACU email | SJR - Journal Search | Validity_According_to | InCites™

Mplus

Tuesday September 13, 2016

HOME ORDER CONTACT US LOGIN MPLUS DISCUSSION

Mplus: Mplus at a Glance, General Description, Mplus Programs, Pricing, Version History, System Requirements, Platforms, FAQ

Mplus Demo Version

Training: Short Courses, Short Course Videos and Handouts, Web Training

Documentation: Mplus User's Guide, Mplus Diagrammer, Technical Appendices, Mplus Web Notes, User's Guide Examples, Mplus Book, Mplus Book Examples, Mplus Book Errata

Analyses/Research: Mplus Examples, Papers, References

Special Mplus Topics: BSEM (Bayesian SEM), Complex Survey Data, ESEM (Exploratory SEM), Genetics, IRT, Measurement Invariance, Mediation Analysis, Missing Data, Mixture Modeling, Multilevel Modeling, Structural Equation Modeling, Survival Analysis, Randomized Trials

How-To: Using Mplus via R

Papers Using Special Mplus Features

References on this page are ordered by topic. References can also be viewed ordered by date.

Bayesian Analysis expand topic

Categorical Factor Analysis expand topic

Complex Survey Data Analysis expand topic

Dyadic Analysis expand topic

Exploratory Structural Equation Modeling (ESEM) expand topic

Factor Mixture Analysis expand topic

General Mixture Modeling expand topic

Genetics Modeling expand topic

Growth Mixture Modeling expand topic

Growth Modeling expand topic

IRT expand topic

Latent Class Analysis expand topic

Latent Transition Analysis expand topic

Measurement Modeling expand topic

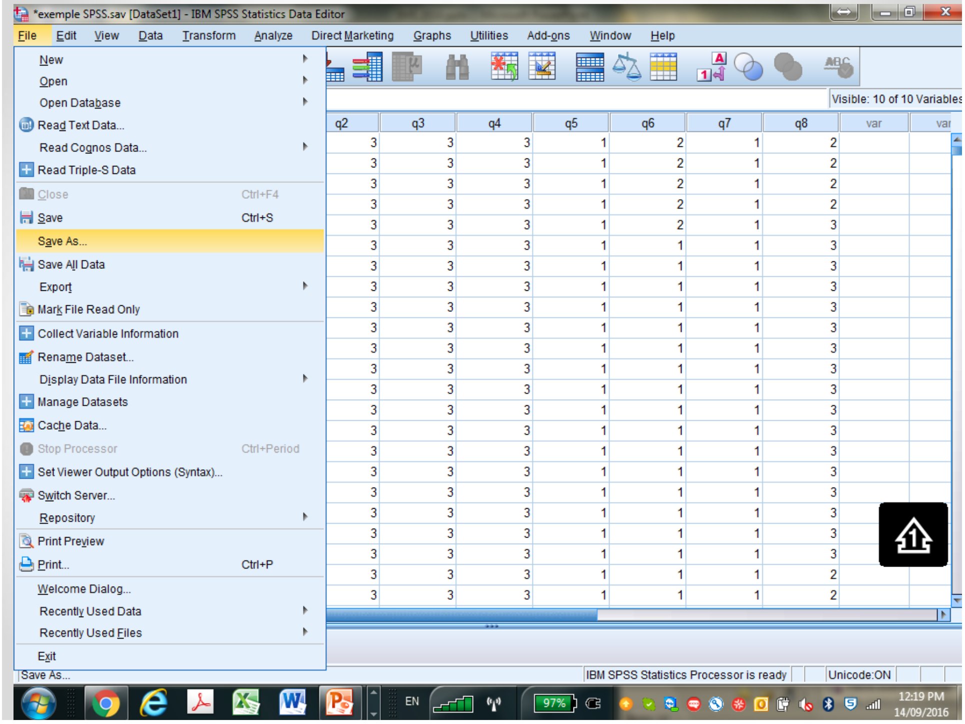
Mediation Modeling expand topic

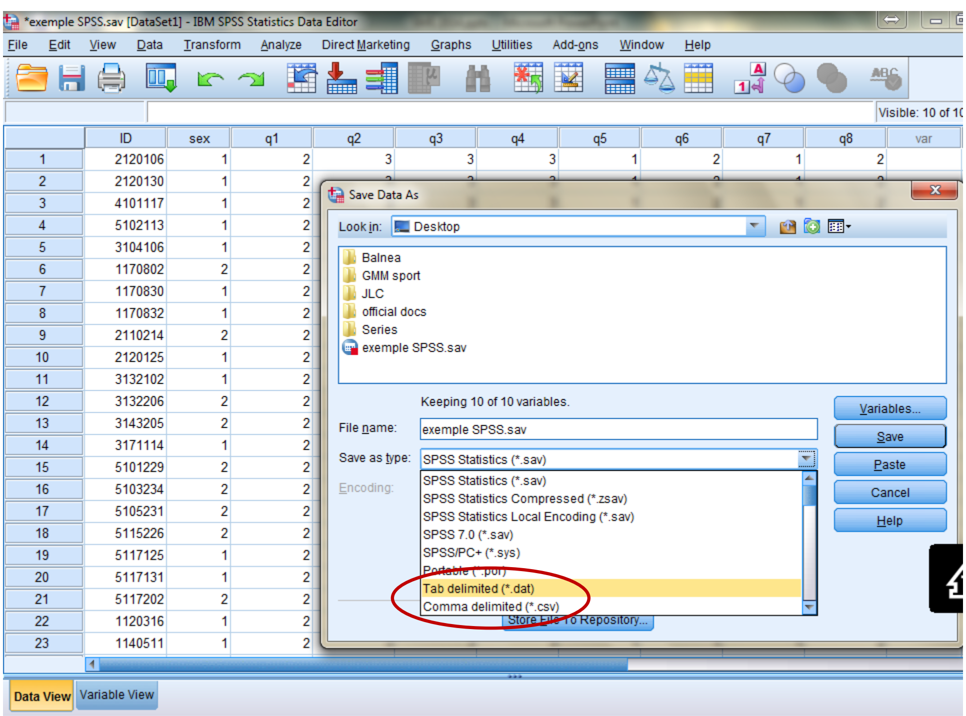
Missing Data Analysis expand topic

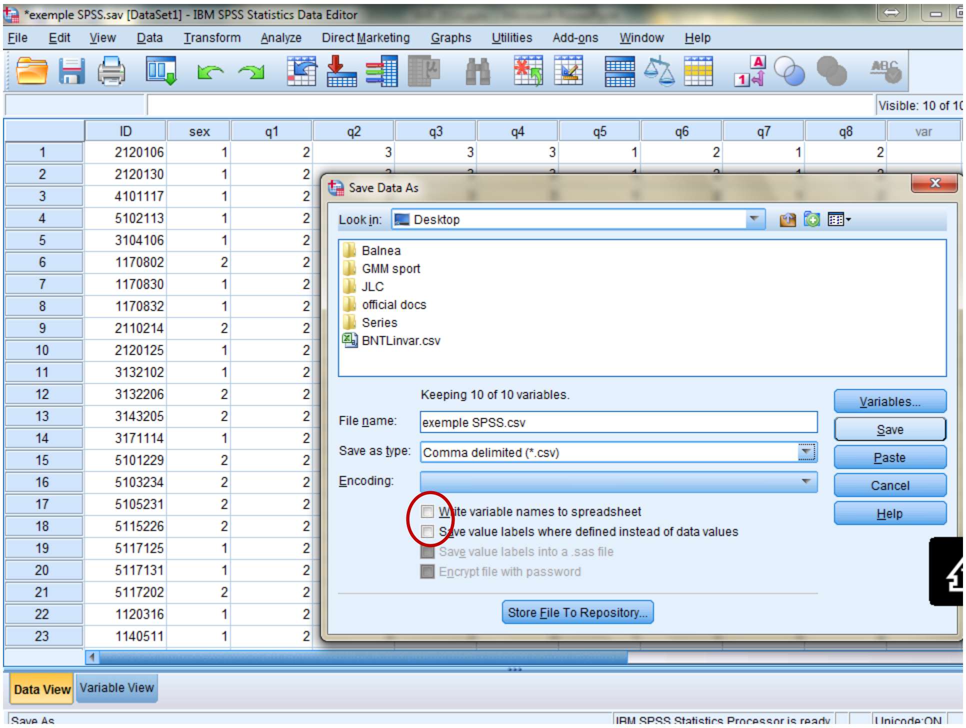
Mixture (Latent Class) SEM expand topic

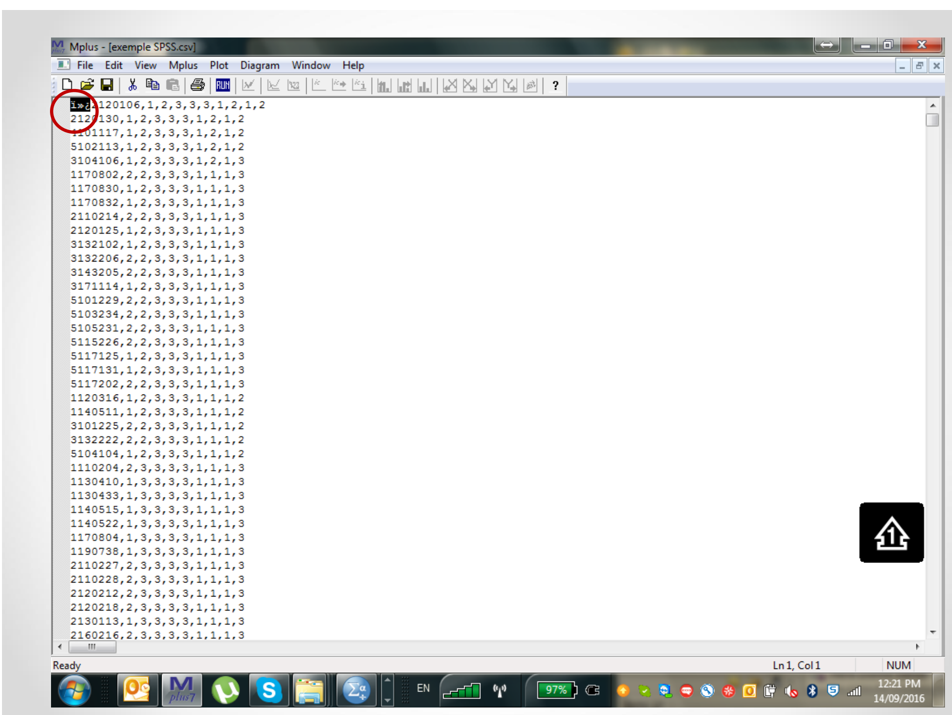
Topic 1-v11.pdf | Show all downloads...

EN 97% 11:40 AM 14/09/2016


*exemple SPSS.sav [DataSet1] - IBM SPSS Statistics Data Editor
File Edit View Data Transform Analyze Direct Marketing Graphs Utilities Add-ons Window Help
New
Open
Open Database
Read Text Data...
Read Cognos Data...
Read Triple-S Data
Close Ctrl+F4
Save Ctrl+S
Save As...
Save All Data
Export
Mark File Read Only
Collect Variable Information
Rename Dataset...
Display Data File Information
Manage Datasets
Cache Data...
Stop Processor Ctrl+Period
Set Viewer Output Options (Syntax)...
Switch Server...
Repository
Print Preview
Print... Ctrl+P
Welcome Dialog...
Recently Used Data
Recently Used Files
Exit
Visible: 10 of 10 Variables
Save As...
IBM SPSS Statistics Processor is ready
Unicode:ON
12:19 PM
14/09/2016


*exemple SPSS.sav [DataSet1] - IBM SPSS Statistics Data Editor
File Edit View Data Transform Analyze Direct Marketing Graphs Utilities Add-ons Window Help
Save Data As
Look in: Desktop
Balnea
GMM sport
JLC
official docs
Series
exemple SPSS.sav
Keeping 10 of 10 variables.
File name: exemple SPSS.sav
Save as type: SPSS Statistics (*.sav)
Encoding:
SPSS Statistics (*.sav)
SPSS Statistics Compressed (*.zsav)
SPSS Statistics Local Encoding (*.sav)
SPSS 7.0 (*.sav)
SPSS/PC+ (*.sys)
Portable (*.por)
Tab delimited (*.dat)
Comma delimited (*.csv)
Store File To Repository...
Variables...
Save
Paste
Cancel
Help
Data View
Variable View
Save As
IBM SPSS Statistics Processor is ready
Unicode:ON


*exemple SPSS.sav [DataSet1] - IBM SPSS Statistics Data Editor
File Edit View Data Transform Analyze Direct Marketing Graphs Utilities Add-ons Window Help
Visible: 10 of 10
ID sex q1 q2 q3 q4 q5 q6 q7 q8 var
Save Data As
Look in: Desktop
Balnea
GMM sport
JLC
official docs
Series
BNTLinvar.csv
Keeping 10 of 10 variables.
File name: exemple SPSS.csv
Save as type: Comma delimited (*.csv)
Encoding:
Write variable names to spreadsheet
Save value labels where defined instead of data values
Save value labels into a .sas file
Encrypt file with password
Store File To Repository...
Variables...
Save
Paste
Cancel
Help
Data View
Variable View
Save As
IBM SPSS Statistics Processor is ready
Unicode:ON


Mplus - [exemple SPSS.csv]
File Edit View Mplus Plot Diagram Window Help
ï»¿2120106,1,2,3,3,3,1,2,1,2
2120130,1,2,3,3,3,1,2,1,2
4101117,1,2,3,3,3,1,2,1,2
5102113,1,2,3,3,3,1,2,1,2
3104106,1,2,3,3,3,1,2,1,3
1170802,2,2,3,3,3,1,1,1,3
1170830,1,2,3,3,3,1,1,1,3
1170832,1,2,3,3,3,1,1,1,3
2110214,2,2,3,3,3,1,1,1,3
2120125,1,2,3,3,3,1,1,1,3
3132102,1,2,3,3,3,1,1,1,3
3132206,2,2,3,3,3,1,1,1,3
3143205,2,2,3,3,3,1,1,1,3
3171114,1,2,3,3,3,1,1,1,3
5101229,2,2,3,3,3,1,1,1,3
5103234,2,2,3,3,3,1,1,1,3
5105231,2,2,3,3,3,1,1,1,3
5115226,2,2,3,3,3,1,1,1,3
5117125,1,2,3,3,3,1,1,1,3
5117131,1,2,3,3,3,1,1,1,3
5117202,2,2,3,3,3,1,1,1,3
1120316,1,2,3,3,3,1,1,1,2
1140511,1,2,3,3,3,1,1,1,2
3101225,2,2,3,3,3,1,1,1,2
3132222,2,2,3,3,3,1,1,1,2
5104104,1,2,3,3,3,1,1,1,2
1110204,2,3,3,3,3,1,1,1,3
1130410,1,3,3,3,3,1,1,1,3
1130433,1,3,3,3,3,1,1,1,3
1140515,1,3,3,3,3,1,1,1,3
1140522,1,3,3,3,3,1,1,1,3
1170804,1,3,3,3,3,1,1,1,3
1190738,1,3,3,3,3,1,1,1,3
2110227,2,3,3,3,3,1,1,1,3
2110228,2,3,3,3,3,1,1,1,3
2120212,2,3,3,3,3,1,1,1,3
2120218,2,3,3,3,3,1,1,1,3
2130113,1,3,3,3,3,1,1,1,3
2160216,2,3,3,3,3,1,1,1,3
Ready
Ln 1, Col 1
NUM
EN
97%
12:21 PM
14/09/2016

Title:

Title of the model to be estimated;

! Annotations following "!" are discarded by the program

! The TITLE function is not mandatory.

! All commands end with ";"

! All section titles end with ":"

Data:

File is longitudinaldata.csv;

! The FILE function of the DATA section is used to identify your

! data set. If the data set is in the same folder, then this is fine.

! If the data set is in another folder, then the full link is indicated.

File is D:\DOCUMENTS\Cours\LATENT VARIABLE MODELING\longitudinaldata.csv;

VARIABLE:

Names are ID sex q1 q2 q3 q4 q5 q6 q7 q8;

! The NAMES function lists, in order, all variables in the data set.

Usevariables are q1 q2 q3 q4 q5 q6 q7 q8;

! The USEVARIABLES function lists those used in the analysis.

Missing are all (-999);

! The MISSING function identifies the missing data code.

Idvariable = ID;

! The IDVARIABLE identifies the unique identifier.

Auxiliary = sex;

Auxiliary = sex (m);

! Sometimes, one wants to save the results from an analysis to an

! external data file (e.g., scores on the factors). This external data

! file will include all variables included in the analyses + those

! listed as auxiliary. The (m) indicators allows auxiliary variables

! to be taken into account in the missing data process.

ANALYSIS:
TYPE = General;
ESTIMATOR = MLR; ! Or ML, etc.
! MLR is robust to multivariate non-normality
! MLR can be made to be robust to nesting.

MODEL:
!!! This is where everything happens !

A simple correction for nesting:

VARIABLE:
NAMES = ID sex q1 q2 q3 q4 q5 q6 q7 q8;
USEVARIABLES = q1 q2 q3 q4 q5 q6 q7 q8;
Missing are all (-999);
Idvariable = ID;
CLUSTER = Unit;
Analysis:
TYPE = **COMPLEX**;
ESTIMATOR = MLR;

MODEL:
!!! This is where everything happens !
OUTPUT:
SAMPSTAT STANDARDIZED MODINDICES CINTERVAL RESIDUAL SVALUES TECH1 TECH3 TECH4 ;

SAMPSTAT: sample descriptive.
STANDARDIZED: Standardized parameter estimates.
CINTERVAL: Confidence intervals for parameter estimates.
RESIDUAL: Residuals for parameter estimates.
MODINDICES: Modification indices.
SVALUES: Starts Values.
TECH1: Parameter specifications and starts values (not for EFA).
TECH3: Correlations and covariances for parameter estimates.
TECH4: Means, Correlations and covariances for the latent variables.

MODEL:

ON: Defines a regression e.g., Y ON X;
WITH: Defines a correlation e.g., X WITH Y;
BY: Defines a factor loading e.g., F1 BY X1 X2 X3;
[ ]: Variable names within brackets define intercepts and means e.g., [X1]; or [F1];
Variable names: By themselves, variable names define variances, uniquenesses and disturbances e.g., X1; or F1;
***:** Is used to request the free estimation of a parameter that would otherwise be constrained e.g., F1 BY X1* X2 X3; or to provide a start value for a parameter e.g., F1 BY X1*.900 X2*.850 X3*800;
@: Is used to constrain a parameter to a specific value e.g., F1 BY X1@1 X2 X3;
(): alphanumeric codes in parentheses following a parameter can be used to constrain parameters to equality, e.g. F1 BY X1* (l1) X2 (l2);

*** WARNING
Data set contains cases with missing on x-variables.
These cases were not included in the analysis.
Number of cases with missing on x-variables: 61
*** WARNING
Data set contains cases with missing on all variables except
x-variables. These cases were not included in the analysis.
Number of cases with missing on all variables except x-variables: 30

By explicitly requesting the free
Estimation of the variance of the
Exogenous variables in the MODEL:
section, FIML will be activated.
X1 X2 X3;

THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION

Number of Free Parameters 13

Loglikelihood

- H0 Value -7221.664
- H0 Scaling Correction Factor for MLR 1.6859
- H1 Value -7221.604
- H1 Scaling Correction Factor for MLR 1.6256

Information Criteria

- Akaike (AIC) 14469.328
- Bayesian (BIC) 14537.107
- Sample-Size Adjusted BIC 14495.811 (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

Value 97.470*

Degrees of Freedom 19

P-Value 0.0000

Scaling Correction Factor 1.4070 for MLR

* The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used for chi-square difference testing in the regular way. MLM, MLR and WLSM chi-square difference testing is described on the Mplus website. MLMV, WLSMV, and ULSMV difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)

Estimate 0.055

90 Percent C.I. 0.045 0.066

Probability RMSEA <= .05 0.203

CFI/TLI

CFI 0.947

TLI 0.922

SRMR (Standardized Root Mean Square Residual)

Value 0.040

$\Delta\chi^2$

The difference in the χ^2 of two nested models is distributed as a χ^2 with degrees of freedom corresponding to the difference in degrees of freedom between the two models.

BUT

* The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used for chi-square difference testing in the regular way. MLM, MLR and WLSM chi-square difference testing is described on the Mplus website. MLMV, WLSMV, and ULSMV difference testing is done using the DIFFTEST option.

For MLR, one needs to rely on the Satorra-Bentler correction.

https://www.statmodel.com/chidiff.shtml

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
POSF **BY**				
Q1	1.000	0.000	999.000	999.000
Q2	1.175	0.061	19.371	0.000
POSF **ON**				
ZSELFEST	0.225	0.027	8.272	0.000
ZDEPRESS **ON**				
ZSELFEST	-0.346	0.037	-9.418	0.000
ZSELFEST **WITH**				
ZLONELY	-0.246	0.032	-7.688	0.000
Means				
Intercepts				
Variances				
Residual Variances				
POSF	0.936	0.050	18.751	0.000
Q1	0.883	0.049	17.982	[illegible]

Unstandardized loading

Unstandardized regression (b)

Covariance

Means, Intercept, Variances

Residuals (Disturbances, uniquenesses)

STANDARDIZED MODEL RESULTS
STDYX Standardization

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
POSF **BY**				
Q1	0.583	0.028	21.101	0.000
Q2	0.727	0.023	31.835	0.000
POSF **ON**				
ZSELFEST	0.225	0.027	8.472	[illegible]
ZDEPRESS **ON**				
ZSELFEST	-0.346	0.034	-10.305	0.000
ZSELFEST **WITH**				
ZLONELY	-0.246	0.028	-8.875	0.000
Means				
Intercepts				
Variances				
Residual Variances				
POSF	0.936	0.013	72.868	0.000
Q1	0.882	0.022	39.564	0.000

Standardized loading

Standardized regression (β)

Correlation (r)

NA

Standardized Residuals

R-SQUARE

% Explained variance

Communality (h^2)

Observed Variable	Estimate	S.E.	Est./S.E.	
ZGPA	0.064	0.013	4.948	
ZDEPRESS	0.118	0.022	5.297	
Q1	0.340	0.032	10.550	
Q2	0.529	0.033	15.917	0.000
Q3	0.430	0.035	12.212	0.000
Q4	0.551	0.035	15.578	0.000
Q5	0.753	0.071	10.544	0.000

CONFIDENCE INTERVALS OF MODEL RESULTS

	Lower .5%	Lower 2.5%	Lower 5%	Estimate	Upper 5%	Upper 2.5%	Upper .5%
ZGPA ON							
ZSELFEST	0.155	0.172	0.180	0.225	0.270	0.278	0.295
ZDEPRESS	-0.141	-0.122	-0.112	-0.060	-0.009	0.001	0.021
ZDEPRESS ON							
ZSELFEST	-0.441	-0.418	-0.407	-0.346	-0.286	-0.274	-0.252
ZLONELY	-0.087	-0.068	-0.059	-0.009	0.041	0.050	0.069

[…]

CONFIDENCE INTERVALS OF STANDARDIZED MODEL RESULTS

STDYX Standardization

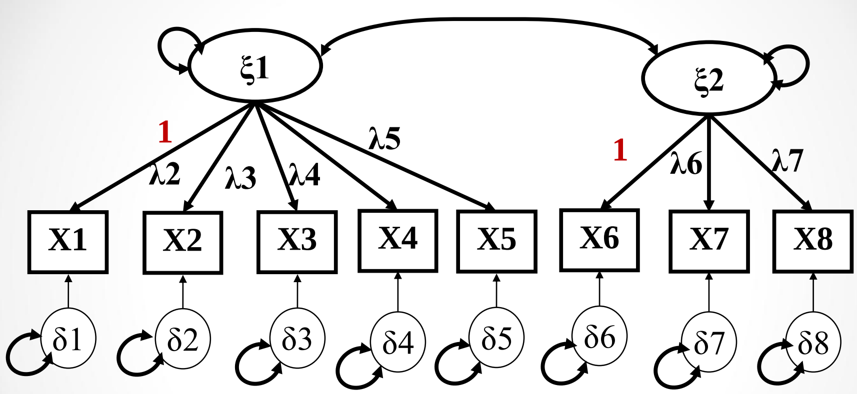
	Lower .5%	Lower 2.5%	Lower 5%	Estimate	Upper 5%	Upper 2.5%	Upper .5%
ZGPA ON							
ZSELFEST	0.157	0.173	0.181	0.225	0.269	0.277	0.293
ZDEPRESS	-0.142	-0.122	-0.112	-0.060	-0.008	0.001	0.021

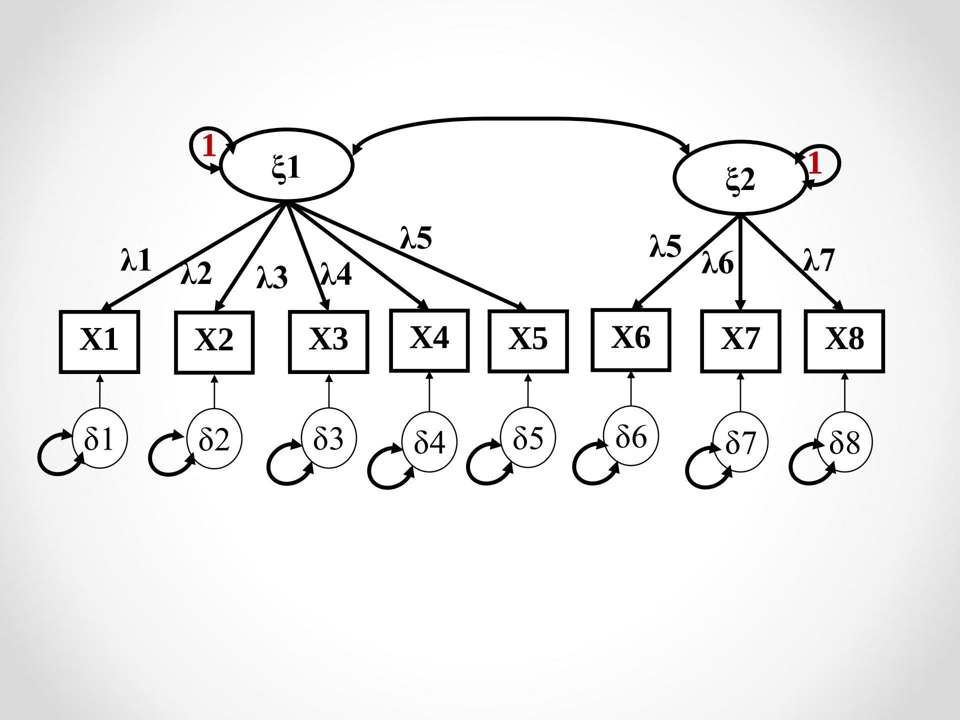
Preliminary Measurement Models

Setting the Scale?

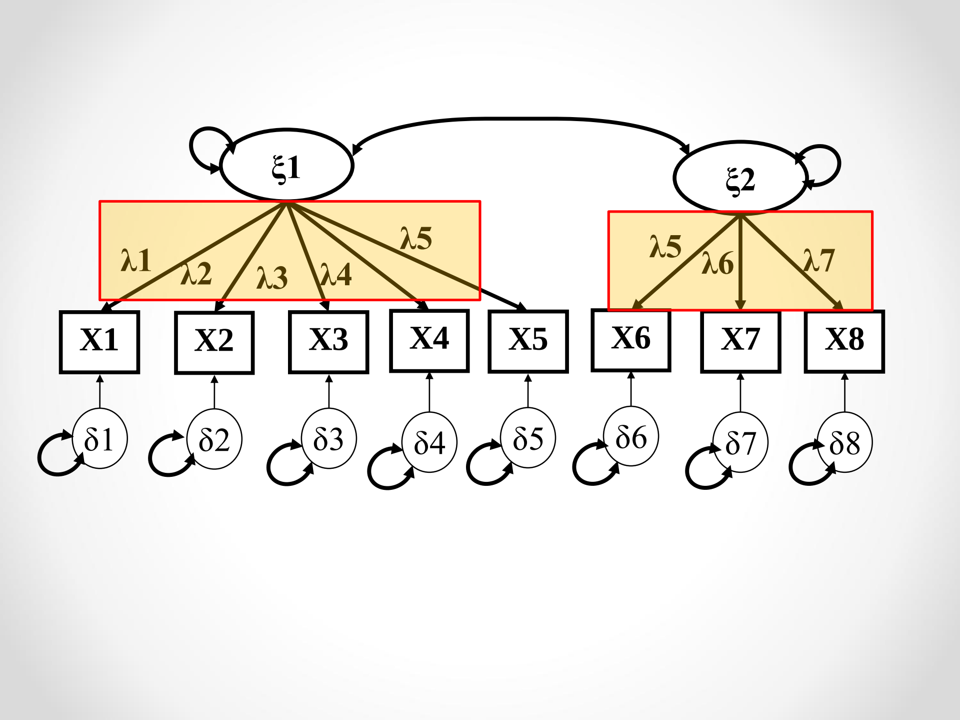
Unless the units are meaningful, all methods are "almost" equivalent.

- Referent Variable: This scale is arbitrary and can change depending on the selected referent variable.
- Latent Standardization: The "natural" scale is lost, and the mean of the intercept factor becomes 0. This method also means that it is not the mean/variance of the first order factor that is constrained, but their intercepts/disturbances (as they are themselves explained by the second order intercept and slope factors). Another possibility is to fix the loading of a referent indicator to its value in the longitudinal CFA model, in order to freely estimate a variance that will be close to 1.
- Effects Coding: Ideal, but more prone to convergence problems.

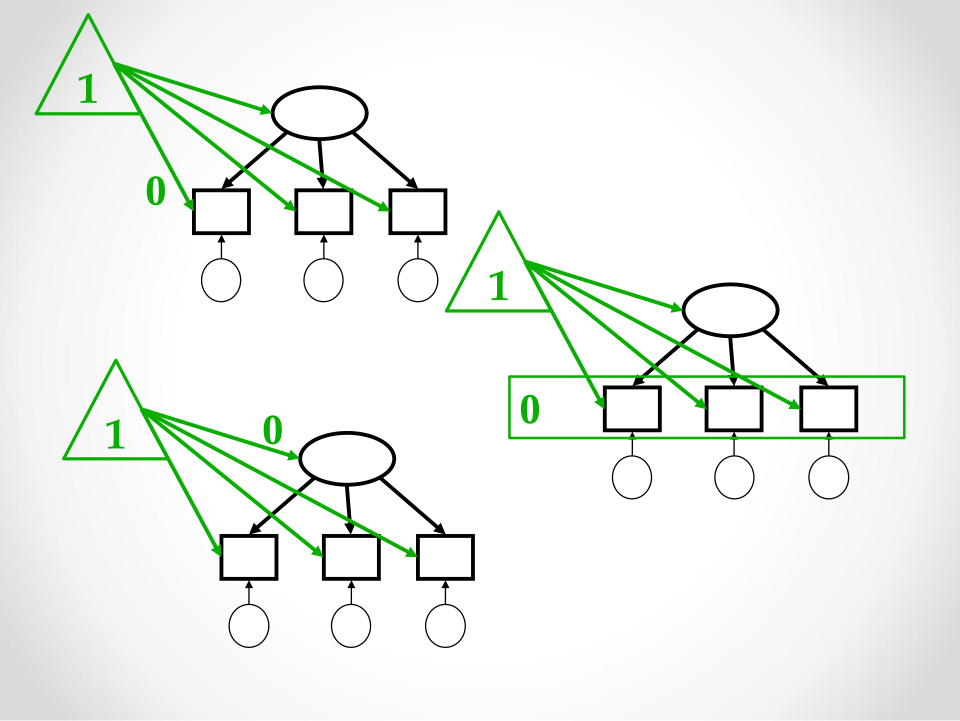




1
ξ1
ξ2
1
λ1
λ2
λ3
λ4
λ5
λ5
λ6
λ7
X1
X2
X3
X4
X5
X6
X7
X8
δ1
δ2
δ3
δ4
δ5
δ6
δ7
δ8



ξ1
ξ2
λ1
λ2
λ3
λ4
λ5
λ5
λ6
λ7
X1
X2
X3
X4
X5
X6
X7
X8
δ1
δ2
δ3
δ4
δ5
δ6
δ7
δ8



Referent Indicator

Model:
POSF BY q1 q2 q3 q4 q8;
NEGF BY q5 q6 q7;
[q1@0];
[q5@0];
[POSF*];
[NEGF*];

Means and Intercepts are identified with brackets []

Explicit

Model:
POSF BY q1@1 q2 q3 q4 q8;
NEGF BY q5@1 q6 q7;
POSF*; NEGF*;
[q1@0];
[q5@0];
[POSF*];
[NEGF*];

Standardized factors

Model:
POSF BY q1* q2 q3 q4 q8;
NEGF BY q5* q6 q7;
POSF@1; NEGF@1;

By default, Mplus freely estimates all intercepts and constrain factors means to be 0.

Explicit

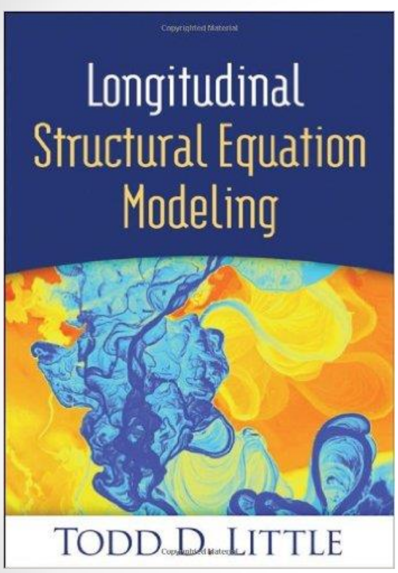
Model:
POSF BY q1* q2 q3 q4 q8;
NEGF BY q5* q6 q7;
POSF@1; NEGF@1;
[POSF@0];
[NEGF@0];

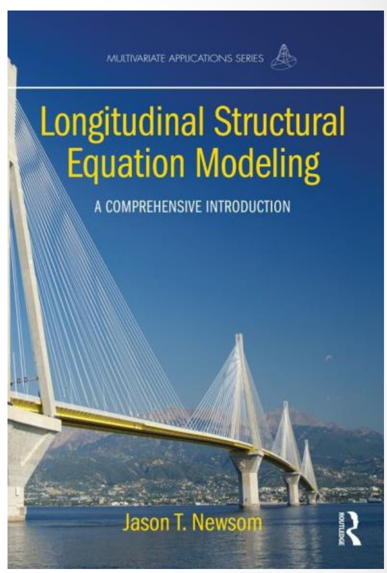
Effects Coding

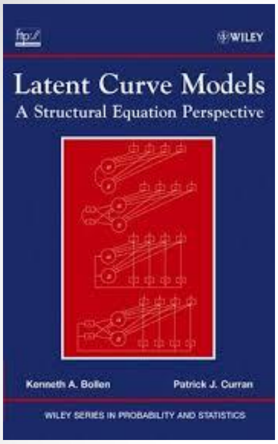
Model:
POSF BY q1* (l1)
q2 q3 q4 q8 (l2-l5);
NEGF BY q5* (l6)
q6 q7 (l7-l8);
POSF*; NEGF*;
[q1-q8] (i1-i8);
[POSF*]; [NEGF*];

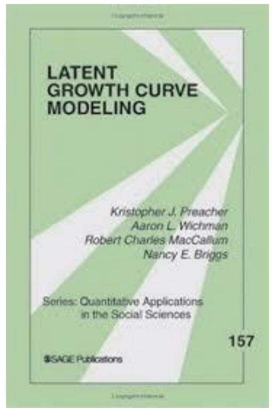
MODEL CONSTRAINT:
l1 = 5 - l2 - l3 - l4 – l5 ;
l4 = 3 – l7 – l8;
i1 = 0-i2-i3-i4-i8;
i5 = 0-i6-i7;

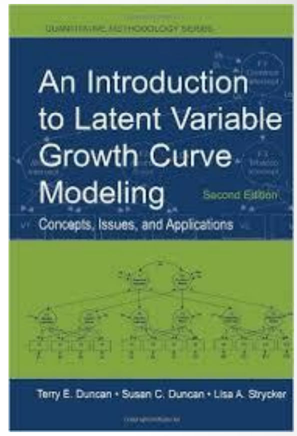
Longitudinal Structural Equation Modeling (continuous latent variables)

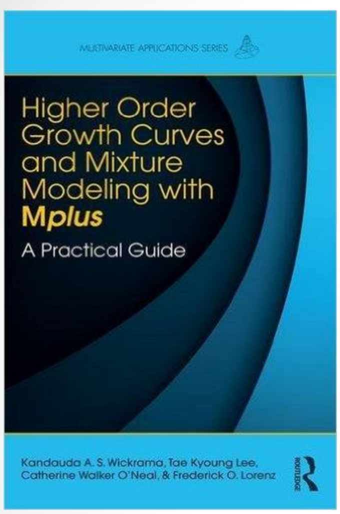


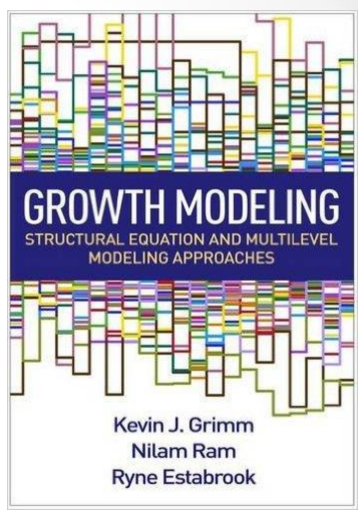



WILEY
Latent Curve Models
A Structural Equation Perspective
Kenneth A. Bollen
Patrick J. Curran
WILEY SERIES IN PROBABILITY AND STATISTICS


LATENT GROWTH CURVE MODELING
Kristopher J. Preacher
Aaron L. Wichman
Robert Charles MacCallum
Nancy E. Briggs
Series: Quantitative Applications in the Social Sciences
157
SAGE Publications

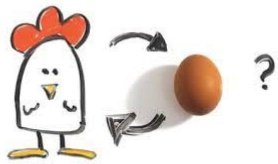

An Introduction to Latent Variable Growth Curve Modeling
Second Edition
Concepts, Issues, and Applications
Terry E. Duncan • Susan C. Duncan • Lisa A. Strycker


MULTIVARIATE APPLICATIONS SERIES
Higher Order Growth Curves and Mixture Modeling with Mplus
A Practical Guide
Kandauda A. S. Wickrama, Tae Kyoung Lee, Catherine Walker O'Neal, & Frederick O. Lorenz
ROUTLEDGE


GROWTH MODELING
STRUCTURAL EQUATION AND MULTILEVEL MODELING APPROACHES
Kevin J. Grimm
Nilam Ram
Ryne Estabrook

Why Longitudinal Data?

- Directionality



- Change

- Shape of change

Assumptions

(1) The Meaning of Time.
(2) Measurement Invariance.
(3) Changes in Means and Variances.
(4) Wording Effects (with Latent Models).
(5) Predictive Equilibrium.

The Meaning(s) of Time

- **Temporal Ordering**: In some longitudinal studies, time simply serves to map temporal ordering: Year 1 of data collection, year 2, year 3, among a sample of participants of different age groups.
- **Meaningful Variable**: In some other studies, time is a meaningful variable and reflects the effect of age, tenure, etc.
- **Change**: Yet, in some other studies, time will reflect the impact of some natural (puberty, life transition) or experimental (intervention, experimentation, etc.) change that is also important.
- In a autoregressive cross-lagged models, time typically simply serves to map the temporal ordering of the relations. However, these analyses can also serve to test the effects of changes in a "pre-post" manner.
- When participants are really far apart in age, it might be useful to control for age in the estimation of the models.

The Meaning(s) of Time

- Over and above these considerations, many have noted the importance to think about the specific time interval that is used in a specific study, showing that it did importantly change the results.
- Not always possible.
- What do we know about the stability of the various constructs?
- Over which length of time can we expect change, both empirically and theoretically?
- How does that fit the research question.

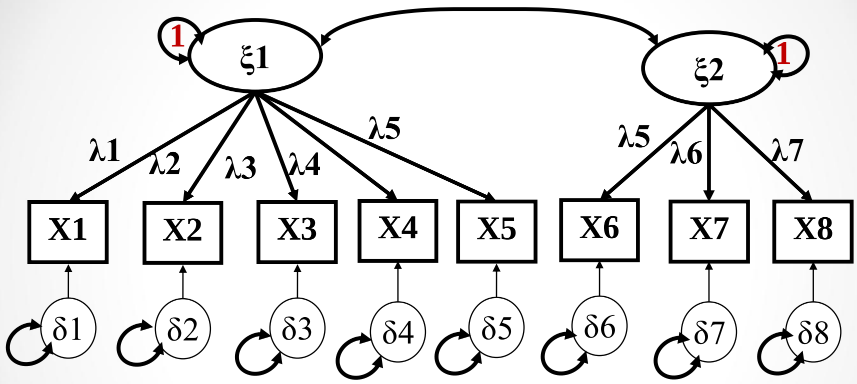
Cole, D.A., & Maxwell, S. E. (2003). Testing meditational models with longitudinal data: Questions and tips in the use of structural equation modeling. *Journal of Abnormal Psychology*, *112*, 558–577.

Maxwell, S. E., & Cole, D.A. (2007). Bias in cross-sectional analyses of longitudinal mediation. *Psychological Methods*, *12*, 23–44.

Selig, J.P. & Preacher, K.J. (2009). Mediation models for longitudinal data in developmental research. *Research in Human Development*, *6*, 144-164.

Measurement invariance

- It is important to ascertain that the meaning of the constructs of interest has not changed over **time**.
- When **latent** constructs are used, and the only objective is to test for the **relations** among constructs, only weak (or partial weak) invariance (i.e. factor loadings) is required with continuous indicators.
- When **latent** constructs are used, and one also wants to test for latent **mean** differences, then strong (or partial strong) invariance (i.e. factors loadings and intercepts) is required.
- When **manifest scores** are used, then strict invariance (or partial strict (i.e., factors loadings, intercepts, and uniquenesses) is required.



Changes in Means and Variances

- A longitudinal measurement model can also be used to test for latent variances, latent covariances, and latent means differences as a function of time.
- This is the latent variable approach to repeated measures ANOVAs.
- When moving from a measurement model to a predictive model, even though only weak/strong invariance is required in a fully latent model, it remains useful to start from a model of strict invariance, and even from a model in which the latent variances and means are set to invariance when this is possible, as this induces parsimony in the model (maximising power).
- It also makes the variances and means easier to interpret (in SD units). Otherwise, they will still be interpretable in SD units of the first time point (if relying on the standardized factor approach).
- Not the covariances, as these do not have the same meaning in measurement versus predictive (residual covariances) models.

Changes in Means and Variances

- Sometimes it is not possible to rely on a fully latent predictive model: The complexity of the model becomes too great, and the model crashes. However, sometimes, it is still possible to estimate the fully latent measurement model.
- In these situations, invariance can, and should be tested.
- When this does not work, it should still be tested across pairs of time points.
- When invariance can be tested, it can be useful to rely on factor scores, rather than scale scores, as these provide a partial control for measurement errors, and can be saved from a model of strict invariance, or even of latent variances, covariances, and means invariance.

Wording Effects (Latent Models)

- In a longitudinal study, the same set of factor indicators are utilized at the different time points.
- This creates a methodological artefact given that what is "unique" to each item (uniqueness) becomes shared with what is unique to the same item measured at the other time points.
- Latent longitudinal models thus need to include correlated uniquenesses among matching indicators.
- Not doing so would risk inflating the autoregressive (stability) parameters.

Fully Latent Models

- In fully latent model, the decision to rely on the referent indicator, latent standardisation, or effects coding method will have a direct effect on the estimates of the average intercept ([i]), but also on the estimate of the means and variances of the other growth factors ([s] will be expressed as deviation from [i] in SD units if using the standardized factor approach).

- Preliminary: Longitudinal invariance of the repeated measures:
  - Requirement: Strong invariance.
  - Better: Strict invariance (makes the model more parsimonious)
  - Latent variance-covariances: not necessary, and results in invariant time-specific residuals in the LCM, not variances.
  - Latent means: invariance suggest that, on the average, there is no growth.
  - The fully latent model should start from a model of strong or strict invariance – maximum.

Longitudinal Invariance

```
FX_T1 BY X1_t1* X2_t1 X3_t1;
FY_T1 BY Y1_t1* Y2_t1 Y3_t1;

[X1_t1 X2_t1 X3_t1];
[Y1_t1 Y2_t1 Y3_t1];

X1_t1 X2_t1 X3_t1;
Y1_t1 Y2_t1 Y3_t1;

FX_T1@1; FY_T1@1;
[FX_T1@0]; [FY_T1@0];
```

```
FX_T2 BY X1_t2* X2_t2 X3_t2;
FY_T2 BY Y1_t2* Y2_t2 Y3_t2;

[X1_t2 X2_t2 X3_t2];
[Y1_t2 Y2_t2 Y3_t2];

X1_t2 X2_t2 X3_t2;
Y1_t2 Y2_t2 Y3_t2;

FX_T2@1; FY_T2@1;
[FX_T2@0]; [FY_T2@0];
```

```
X1_t1 X2_t1 X3_t1 pwith X1_t2 X2_t2 X3_t2;
! Equivalent to
! X1_t1 WITH X1_t2; X2_t1 WITH X2_t2; X3_t1 WITH X3_t2;
```

Weak Invariance

```
FX_T1 BY X1_t1* (l1)
X2_t1 X3_t1 (l2-l3);
FY_T1 BY Y1_t1*  (l4)
Y2_t1  Y3_t1 (l5-l6);

[X1_t1 X2_t1 X3_t1];
[Y1_t1  Y2_t1  Y3_t1];

X1_t1 X2_t1 X3_t1;
Y1_t1  Y2_t1  Y3_t1;

FX_T1@1;  FY_T1@1;
[FX_T1@0];  [FY_T1@0];
```

```
FX_T2 BY X1_t2* (l1)
X2_t2 X3_t2 (l2-l3);
FY_T2 BY  Y1_t2*  (l4)
Y2_t2  Y3_t2 (l5-l6);

[X1_t2 X2_t2 X3_t2];
[Y1_t2  Y2_t2  Y3_t2];

X1_t2 X2_t2 X3_t2;
Y1_t2  Y2_t2  Y3_t2;

FX_T2*;  FY_T2*;
[FX_T2@0];  [FY_T2@0];
```

```
X1_t1 X2_t1 X3_t1 pwith X1_t2 X2_t2 X3_t2;
```

Strong Invariance

```
FX_T1 BY X1_t1* (l1)
X2_t1 X3_t1 (l2-l3);
FY_T1 BY  Y1_t1*  (l4)
Y2_t1  Y3_t1 (l5-l6);

[X1_t1 X2_t1 X3_t1] (i1-i3);
[Y1_t1  Y2_t1  Y3_t1] (i4-i6);

X1_t1 X2_t1 X3_t1;
Y1_t1  Y2_t1  Y3_t1;

FX_T1@1;  FY_T1@1;
[FX_T1@0];  [FY_T1@0];
```

```
FX_T2 BY X1_t2* (l1)
X2_t2 X3_t2 (l2-l3);
FY_T2 BY  Y1_t2*  (l4)
Y2_t2  Y3_t2 (l5-l6);

[X1_t2 X2_t2 X3_t2] (i1-i3);
[Y1_t2  Y2_t2  Y3_t2] (i4-i6);

X1_t2 X2_t2 X3_t2;
Y1_t2  Y2_t2  Y3_t2;

FX_T2*;  FY_T2*;
[FX_T2*];  [FY_T2*];
```

```
X1_t1 X2_t1 X3_t1 pwith X1_t2 X2_t2 X3_t2;
```

Strict Invariance

```
FX_T1 BY X1_t1* (l1)
X2_t1 X3_t1 (l2-l3);
FY_T1 BY Y1_t1*  (l4)
Y2_t1  Y3_t1 (l5-l6);

[X1_t1 X2_t1 X3_t1] (i1-i3);
[Y1_t1  Y2_t1  Y3_t1] (i4-i6);

X1_t1 X2_t1 X3_t1 (u1-u3);
Y1_t1  Y2_t1  Y3_t1 (u4-u6);

FX_T1@1;  FY_T1@1;
[FX_T1@0];  [FY_T1@0];
```

```
FX_T2 BY X1_t2* (l1)
X2_t2 X3_t2 (l2-l3);
FY_T2 BY  Y1_t2*  (l4)
Y2_t2  Y3_t2 (l5-l6);

[X1_t2 X2_t2 X3_t2] (i1-i3);
[Y1_t2  Y2_t2  Y3_t2] (i4-i6);

X1_t2 X2_t2 X3_t2 (u1-u3);
Y1_t2  Y2_t2  Y3_t2 (u4-u6);

FX_T2*;  FY_T2*;
[FX_T2*];  [FY_T2*];
```

```
X1_t1 X2_t1 X3_t1 pwith X1_t2 X2_t2 X3_t2;
```

Var-Covar Invariance

```
FX_T1 BY X1_t1* (l1)
X2_t1 X3_t1 (l2-l3);
FY_T1 BY  Y1_t1*  (l4)
Y2_t1  Y3_t1 (l5-l6);
[X1_t1 X2_t1 X3_t1] (i1-i3);
[Y1_t1  Y2_t1  Y3_t1] (i4-i6);
X1_t1 X2_t1 X3_t1 (u1-u3);
Y1_t1  Y2_t1  Y3_t1 (u4-u6);
FX_T1@1;  FY_T1@1;
FX_T1 WITH FY_T1 (c1);
[FX_T1@0];  [FY_T1@0];
```

```
FX_T2 BY X1_t2* (l1)
X2_t2 X3_t2 (l2-l3);
FY_T2 BY  Y1_t2*  (l4)
Y2_t2  Y3_t2 (l5-l6);
[X1_t2 X2_t2 X3_t2] (i1-i3);
[Y1_t2  Y2_t2  Y3_t2] (i4-i6);
X1_t2 X2_t2 X3_t2 (u1-u3);
Y1_t2  Y2_t2  Y3_t2 (u4-u6);
FX_T2@1;  FY_T2@1;
FX_T2 WITH FY_T2 (c1);
[FX_T2*];  [FY_T2*];
```

```
X1_t1 X2_t1 X3_t1 pwith X1_t2 X2_t2 X3_t2;
```

Latent Means Invariance

```
FX_T1 BY X1_t1* (l1)
X2_t1 X3_t1 (l2-l3);
FY_T1 BY Y1_t1* (l4)
Y2_t1 Y3_t1 (l5-l6);
[X1_t1 X2_t1 X3_t1] (i1-i3);
[Y1_t1 Y2_t1 Y3_t1] (i4-i6);
X1_t1 X2_t1 X3_t1 (u1-u3);
Y1_t1 Y2_t1 Y3_t1 (u4-u6);
FX_T1@1; FY_T1@1;
FX_T1 WITH FY_T1 (c1);
[FX_T1@0]; [FY_T1@0];
```

```
FX_T2 BY X1_t2* (l1)
X2_t2 X3_t2 (l2-l3);
FY_T2 BY Y1_t2* (l4)
Y2_t2 Y3_t2 (l5-l6);
[X1_t2 X2_t2 X3_t2] (i1-i3);
[Y1_t2 Y2_t2 Y3_t2] (i4-i6);
X1_t2 X2_t2 X3_t2 (u1-u3);
Y1_t2 Y2_t2 Y3_t2 (u4-u6);
FX_T2@1; FY_T2@1;
FX_T2 WITH FY_T2 (c1);
[FX_T2@0]; [FY_T2@0];
```

```
X1_t1 X2_t1 X3_t1 pwith X1_t2 X2_t2 X3_t2;
```

Basic Longitudinal Models

Longitudinal Models.

When different variables are measured at each time point.

- Can be analyzed as cross-sectional data, with no added source of complexity, except in the treatment of missing data.
- This provides a way to analyse data while taking into account the temporal ordering of the data, but precluding clear tests of the directionality of the associations.
- **Basic rule:** Use all available cases, unless there is a specific reason not to (i.e., a key variable is completely missing, ethics/consents, etc.).

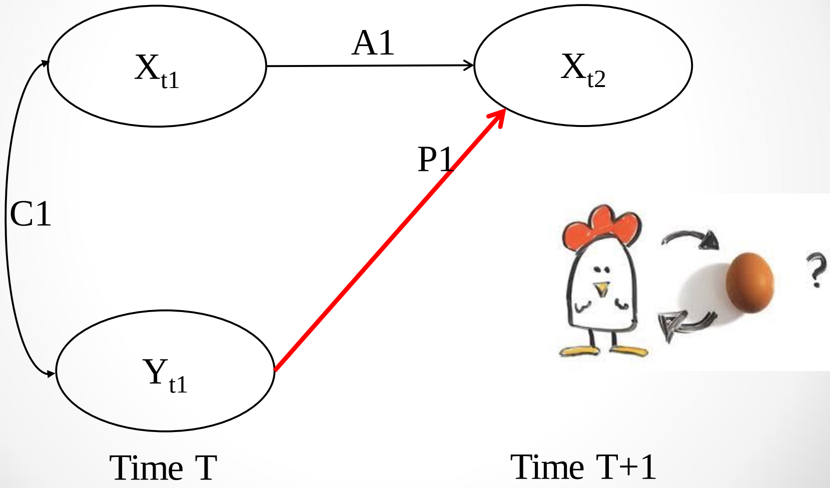
Longitudinal Models.

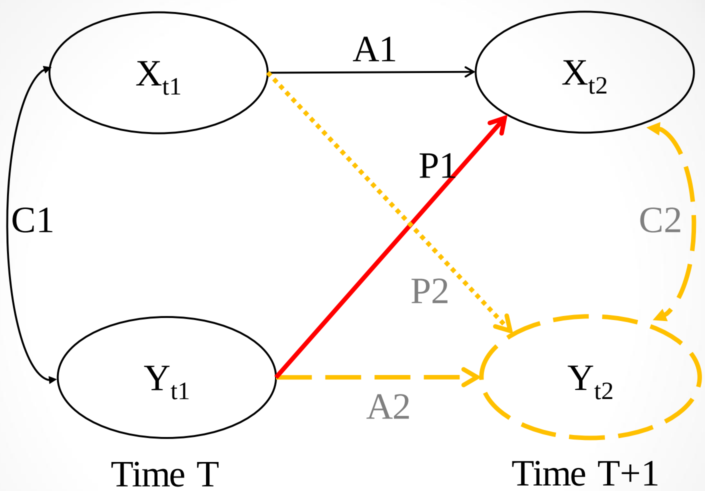
When the variables are measured at all time points.

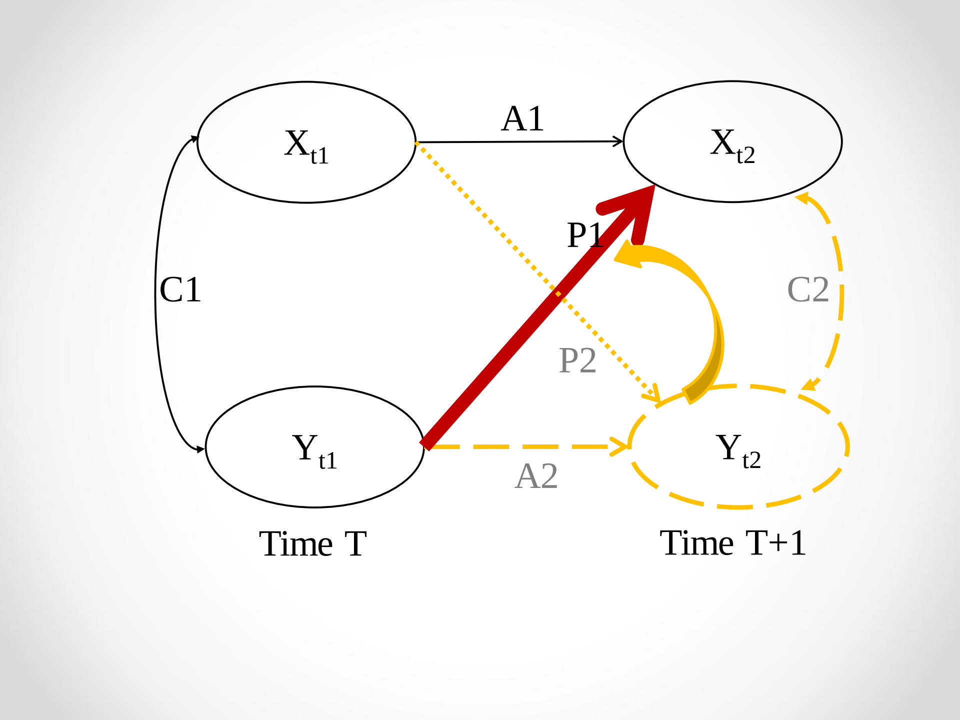
Provides a way to control for baseline levels of the key variables of interest, and thus to model the "impact" of (or relation between) one variable and "changes" over time in the levels of an outcome variable (i.e., the influence of one variable on the part of the outcome that remains unexplained by baseline levels of that same outcome).

i.e., effects "over and above" the stability of the outcome.

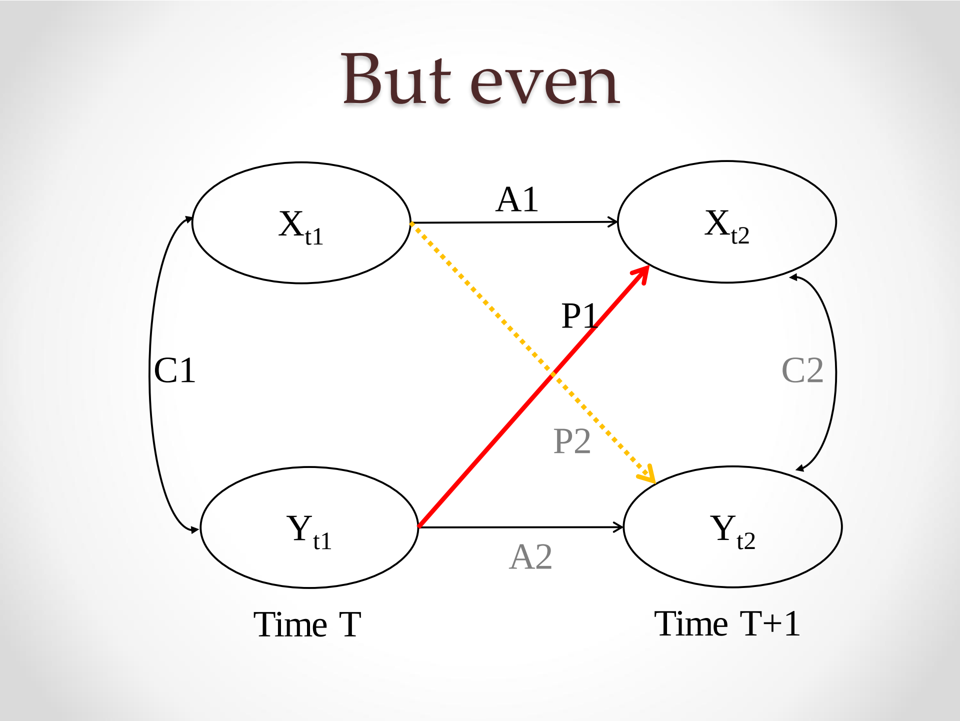
The Classical Longitudinal Model





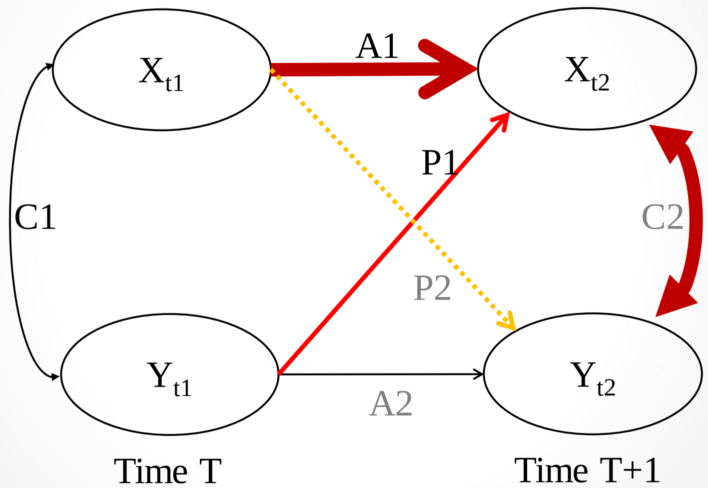


A1
X_{t1}
X_{t2}
P1
C1
C2
P2
Y_{t1}
Y_{t2}
A2
Time T
Time T+1

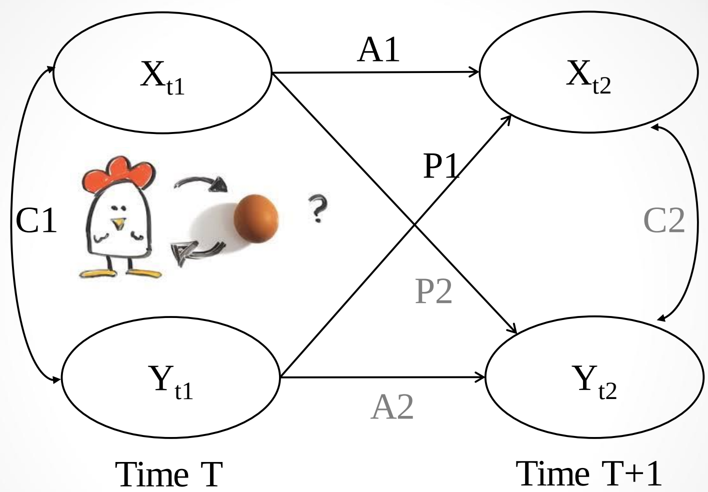


But even
A1
X_{t1}
X_{t2}
P1
C1
C2
P2
Y_{t1}
Y_{t2}
A2
Time T
Time T+1

But even



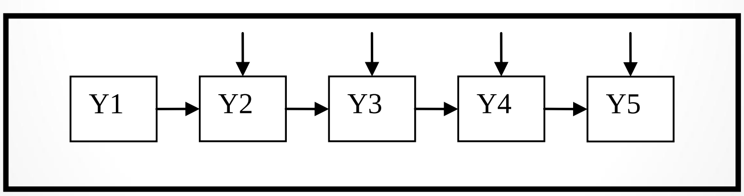
A1
X_{t1}
X_{t2}
P1
C1
C2
P2
Y_{t1}
Y_{t2}
A2
Time T
Time T+1



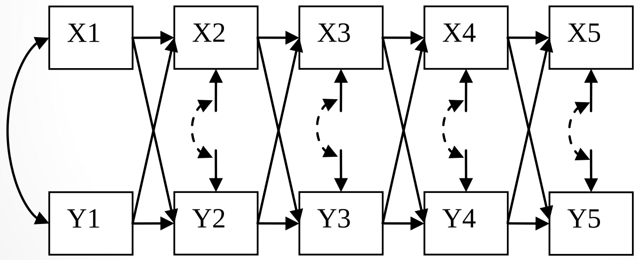
A1
X_{t1}
X_{t2}
P1
C1
?
C2
P2
Y_{t1}
Y_{t2}
A2
Time T
Time T+1

Autoregressive Cross Lagged Model

Autoregressive models allow one to estimate the stability / unstability of autoregressive parameters between adjacent measurement points over time.



Autoregressive cross-lagged models allow one to estimate the stability / unstability of autoregressive parameters between adjacent measurement points over time. They model the influence of “states” on later “states” but fail to consider stable process (traits).



$$y_{it} = \alpha_{yt} + \rho_{y_t, y_{t-1}} y_{i,t-1} + \rho_{y_t, w_{t-1}} w_{i,t-1} + \varepsilon_{yit}$$

$$w_{it} = \alpha_{wt} + \rho_{w_t, y_{t-1}} y_{i,t-1} + \rho_{w_t, w_{t-1}} w_{i,t-1} + \varepsilon_{wit}$$

$$y_{it} = \boxed{\alpha_{yt}} + \rho_{y_t,y_{t-1}} y_{i,t-1} + \rho_{y_t,w_{t-1}} w_{i,t-1} + \varepsilon_{yit}$$
$$w_{it} = \boxed{\alpha_{wt}} + \rho_{w_t,y_{t-1}} y_{i,t-1} + \rho_{w_t,w_{t-1}} w_{i,t-1} + \varepsilon_{wit}$$

Each process is thus a function of:

- **An intercept (average level at Time 1) +**

$$y_{it} = \alpha_{yt} + \boxed{\rho_{y_t,y_{t-1}} y_{i,t-1}} + \rho_{y_t,w_{t-1}} w_{i,t-1} + \varepsilon_{yit}$$
$$w_{it} = \alpha_{wt} + \rho_{w_t,y_{t-1}} y_{i,t-1} + \boxed{\rho_{w_t,w_{t-1}} w_{i,t-1}} + \varepsilon_{wit}$$

Each process is thus a function of:

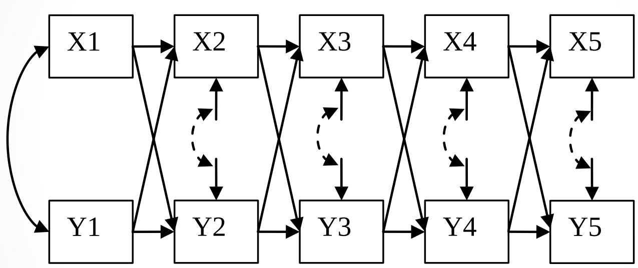
- **An intercept (average level at Time 1) +**
- **A product of the autoregressive parameter and the previous time point of the variable itself +**

$$y_{it} = \alpha_{yt} + \rho_{y_t,y_{t-1}} y_{i,t-1} + \boxed{\rho_{y_t,w_{t-1}} w_{i,t-1}} + \varepsilon_{yit}$$

$$w_{it} = \alpha_{wt} + \boxed{\rho_{w_t,y_{t-1}} y_{i,t-1}} + \rho_{w_t,w_{t-1}} w_{i,t-1} + \varepsilon_{wit}$$

Each process is thus a function of:

- **An intercept (average level at Time 1) +**
- **A product of the autoregressive parameter and the previous time point of the variable itself +**
- **A product of the cross lagged parameter and the previous time point of the other variable.**



```
TITLE: Non latent autoregressive cross lagged model, no constraints.
MODEL:
! Autoregressive part
X1;
X2 ON X1;
X3 ON X2;
X4 ON X3;
X5 ON X4;
Y1;Y2 ON Y1;Y3 ON Y2;Y4 ON Y3;Y5 ON Y4;
! Cross lagged part
X2 ON Y1; X3 ON Y2; X4 ON Y3; X5 ON Y4;
Y2 ON X1;Y3 ON X2;Y4 ON X3;Y5 ON X4;
! Time-specific correlations
X2 WITH Y2; X3 WITH Y3; X4 WITH Y4; X5 WITH Y5;
```

Predictive Equilibrium

- Cole and Maxwell (2003) underscored the importance of tests of predictive equilibrium, which consist of testing the extent to which the observed relations generalize to the various time intervals.
- A predictive system that has reached equilibrium shows relations that can be expected to generalize – i.e. the developmental process under study has stabilized for the period under study.
- Without theses tests, it is impossible to determine whether variations in results reflect random sampling variations or meaningful developmental changes.
- A model that has reached equilibrium is more parsimonious, and thus more stable and has more power.

Predictive Equilibrium

Equilibrium of:

- Autoregressions
- Cross-lagged relations
- Time-specific correlations.

Cole, D.A., & Maxwell, S. E. (2003). Testing meditational models with longitudinal data: Questions and tips in the use of structural equation modeling. *Journal of Abnormal Psychology*, *112*, 558–577.

Morin, A.J.S., Arens, A.K., Maïano, C., Ciarrochi, J., Tracey, D., Parker, P.D., & Craven, R.G. (In press Accepted, 12 September 2016). Reciprocal Relationships between Teacher Ratings of Internalizing and Externalizing Behaviors in Adolescents with Different Levels of Cognitive Abilities. *Journal of Youth and Adolescence*. Early view doi:10.1007/s10964-016-0574-3

```
MODEL:
! Autoregressive part
X1;
X2 ON X1 (a1); X3 ON X2 (a1);  X4 ON X3 (a1); X5 ON X4 (a1);
Y1;
Y2 ON Y1 (a2); Y3 ON Y2 (a2); Y4 ON Y3 (a2); Y5 ON Y4 (a2);
! Cross lagged part
X2 ON Y1; X3 ON Y2; X4 ON Y3; X5 ON Y4;
Y2 ON X1;Y3 ON X2;Y4 ON X3;Y5 ON X4;
! Time-specific correlations
X2 WITH Y2; X3 WITH Y3; X4 WITH Y4; X5 WITH Y5;
```

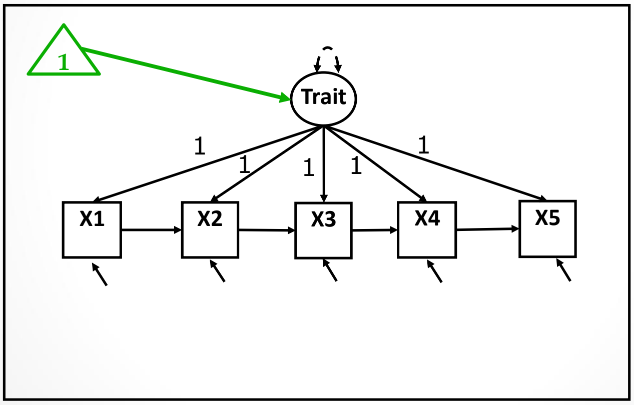
```
MODEL:
! Autoregressive part
X1;
X2 ON X1 (a1); X3 ON X2 (a1);  X4 ON X3 (a1); X5 ON X4 (a1);
Y1;
Y2 ON Y1 (a2); Y3 ON Y2 (a2); Y4 ON Y3 (a2); Y5 ON Y4 (a2);
! Cross lagged part
X2 ON Y1 (CL1);  X3 ON Y2 (CL1); X4 ON  Y3 (CL1); X5 ON Y4 (CL1);
Y2 ON X1(CL2); Y3 ON X2 (CL2); Y4 ON X3 (CL2); Y5 ON X4 (CL2);
! Time-specific correlations
X2 WITH Y2; X3 WITH Y3; X4 WITH Y4; X5 WITH Y5;
```

```
MODEL:
! Autoregressive part
X1;
X2 ON X1 (a1); X3 ON X2 (a1);  X4 ON X3 (a1); X5 ON X4 (a1);
Y1;
Y2 ON Y1 (a2); Y3 ON Y2 (a2); Y4 ON Y3 (a2); Y5 ON Y4 (a2);
! Cross lagged part
X2 ON Y1 (CL1);  X3 ON Y2 (CL1); X4 ON  Y3 (CL1); X5 ON Y4 (CL1);
Y2 ON X1(CL2); Y3 ON X2 (CL2); Y4 ON X3 (CL2); Y5 ON X4 (CL2);
! Time-specific correlations
X2 WITH Y2 (C1); X3 WITH Y3 (C1);
X4 WITH Y4 (C1);  X5 WITH Y5 (C1);
```

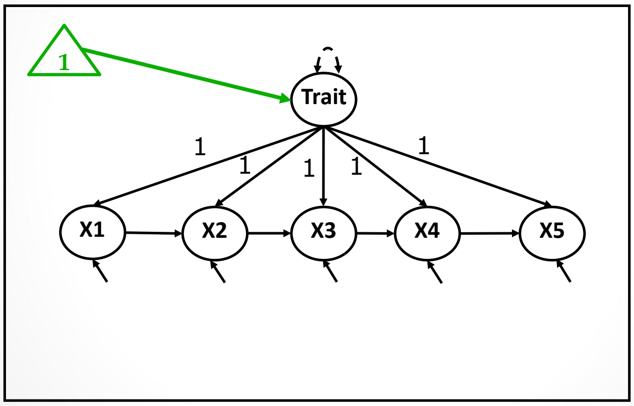
Limitations of the Autoregressive Cross Lagged Model

(1) Models the influence of *states* on later *states* but fail to consider stable developmental processes (*traits*).

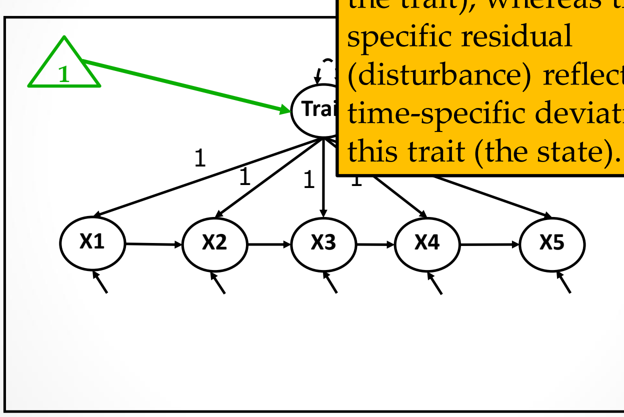
State Trait Models



State-Trait-Error Models?

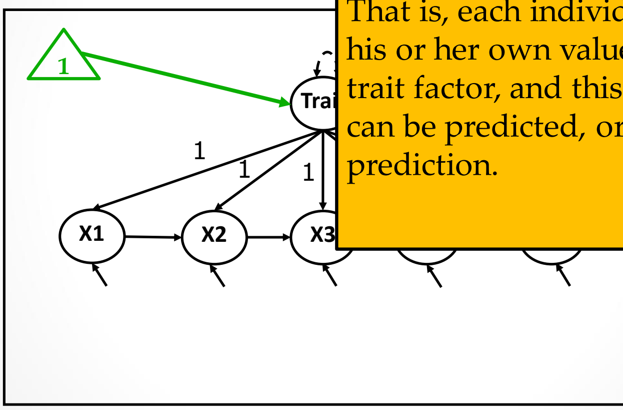


State-Trait-E



In these models the trait factor reflects the average level observed over time in the repeated measures (i.e., the trait), whereas the time specific residual (disturbance) reflects the time-specific deviation from this trait (the state).

State-Trait-E



The trait factor is what is called a "random" effect.

That is, each individual has his or her own value on this trait factor, and this value can be predicted, or used in prediction.

MODEL:

! Trait Factor

Trait BY X1@1 X2@1 X3@1 X4@1 X5@1;

Trait*; [Trait*];

[X1@0 X2@0 X3@0 X4@0 X5@0];

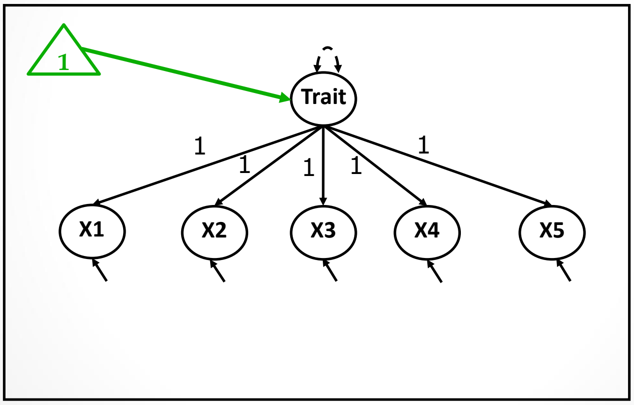
X1 X2 X3 X4 X5;

! Autoregressive part

X2 ON X1; X3 ON X2; X4 ON X3; X5 ON X4;

- Kenny, D.A., & Zautra, A. (1995). The trait-state-error model for multiwave data. *Journal of Consulting and Clinical Psychology*, *63*, 52-59.
- Steyer, R., Schmitt, M., & Eid, M. (1999). Latent State-Trait theory and research in personality and individual differences. *European Journal of Personality*, *13*, 389-408.
- Geiser, C., Keller, B.T., & Lockhart, G. (2013). First- versus second-order latent growth curve models Some insights from latent state-trait theory. *Structural Equation Modeling*, *20*, 479-503.
- Cole, D.A., Martin, N.C., & Steiger, J.H. (2005). Empirical and conceptual problems with longitudinal trait-state- models: Introducing a trait-state-occasion model. *Psychological Methods*, *10*, 3-20.
- Ciesla, J.A., Cole, D.A., & Steiger, J.H. (2007). Extending the trait-state-occasion model: How important is within-wave measurement equivalence. *Structural Equation Modeling*, *14*, 77-97.

[Random Intercept Model]



Limitations of the Autoregressive Cross-Lagged and State-Trait Models

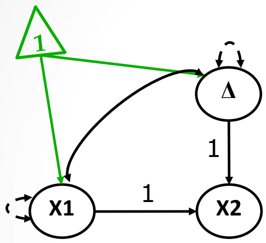
(2) Do not explicitly model developmental *change* over time.

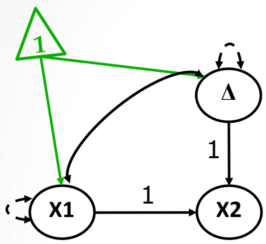
(3) State-Trait Models neglect to account for evolution occurring at the Trait level.

Latent Change Models

- Difference scores (or change scores) have been around for a long time in applied research.
- Change = Time 2 rating – Time 1 rating.
- But these models have come under major criticisms, due to the fact that change scores result in an multiplicative combination of measurement errors.
- Latent changes scores solve these problems.

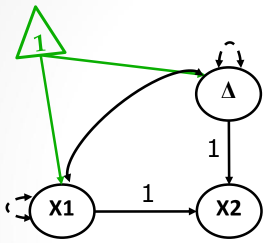
McArdle, J.J. (2009). Latent Variable Modeling of Differences and Changes with Longitudinal Data. *Annual Review of Psychology*, *60*, 577-605.





By fixing the regression of X2 on X1 to be exactly 1, you force the residual of X2 to correspond exactly to the amount of change since T1.

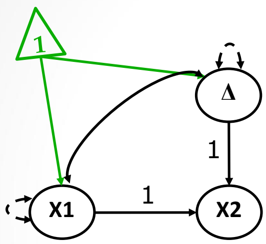
By forcing the loading of X2 on the change factor to be exactly 1 (as well as fixing the intercept and residual of X2 to be exactly 0), you force all information present in X2 to be absorbed in the change factor.



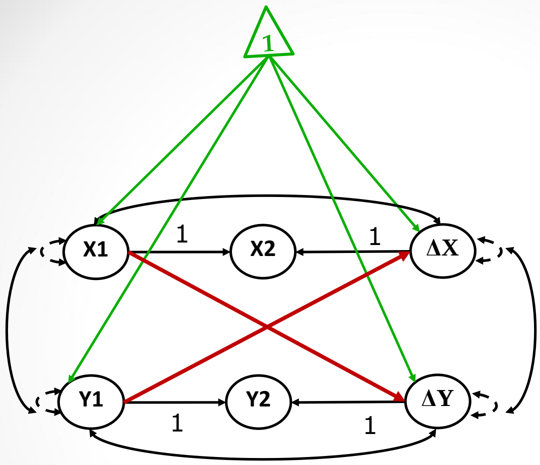
<u>Without the change factor:</u>

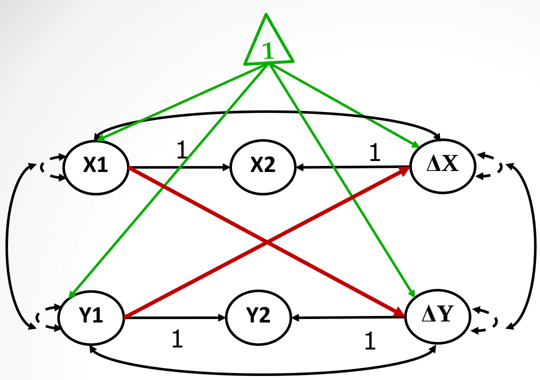
(1) Other variables would still predict X2. With the change factor, they are forced to predict only X2-X1.

(2) It would impossible to assess the correlation between the initial level (X1) and the change factor (reflecting the fact that the rate of change might be more of less pronounced depending on initial levels).

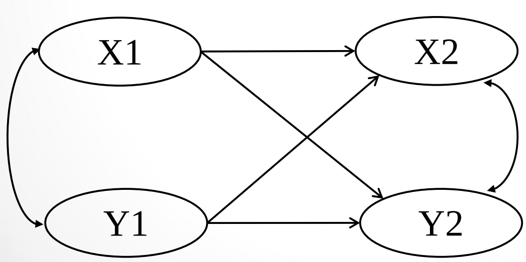


The change factor is random; i.e. it has a mean (reflecting the average change in the total sample) and a variance (reflecting inter-individual differences in rates of change).





Versus?



Illustration

- Using the longitudinal data set, focusing on Time 1 and Time 2 measures of loneliness and self-esteem, and starting with a model of strong invariance based on the referent indicator approach.
- **Autoregressive cross-lagged model**:

- **Latent change model**:

The latent part of both models
Referent indicator method

```
LON_t1 BY Lon1_t1@1
Lon2_t1 Lon3_t1 Lon4_t1 Lon5_t1 (l2-l5);
LON_T2 BY Lon1_t2@1
Lon2_t2 Lon3_t2 Lon4_t2 Lon5_t2 (l2-l5);

[Lon1_t1@0];
[Lon2_t1 Lon3_t1 Lon4_t1 Lon5_t1] (i1-i5);
[Lon1_t2@0];
[Lon2_t2 Lon3_t2 Lon4_t2 Lon5_t2] (i1-i5);

Lon1_t1 Lon2_t1 Lon3_t1 Lon4_t1 Lon5_t1;! Add (u1-u5) for strict invariance
Lon1_t2 Lon2_t2 Lon3_t2 Lon4_t2 Lon5_t2;! Add (u1-u5) for strict invariance

Lon1_t1 Lon2_t1 Lon3_t1 Lon4_t1 Lon5_t1 PWITH Lon1_t2
Lon2_t2 Lon3_t2 Lon4_t2 Lon5_t2;
```

```
se_t1 BY se1_t1@1
se2_t1 se3_t1 se4_t1 se5_t1 (l12-l15);
se_T2 BY se1_t2@1
 se2_t2 se3_t2 se4_t2 se5_t2 (l12-l15);

[se1_t1@0];
[se2_t1 se3_t1 se4_t1 se5_t1] (i12-i15);
[se1_t2@0];
[se2_t2 se3_t2 se4_t2 se5_t2] (i12-i15);

se1_t1 se2_t1 se3_t1 se4_t1 se5_t1; ! Add (u11-u15) for strict invariance
se1_t2 se2_t2 se3_t2 se4_t2 se5_t2; ! Add (u11-u15) for strict invariance

se1_t1 se2_t1 se3_t1 se4_t1 se5_t1 PWITH se1_t2 se2_t2
se3_t2 se4_t2 se5_t2;
```

Autoregressive-Cross Lagged

```
LON_t1*;
LON_T2*;
[LON_t1*];
[LON_T2*];

se_t1*;
se_T2*;
[se_t1*];
[se_T2*];

se_T2 ON se_t1 LON_t1;
LON_T2 ON LON_t1 se_t1;
LON_t1 WITH se_t1;
LON_t2 WITH se_t2;
```

Latent Change Model

```
se_T2 ON se_t1@1;
changese BY se_T2@1;

LON_T2 ON LON_t1@1;
changelon BY LON_T2@1;

changese WITH changelon se_t1 ;
changelon WITH  LON_t1;
se_t1 WITH LON_t1;

changese ON LON_t1;
changelon ON se_t1;

[changese*] ;
[changelon*];
changese*;
changelon*;

se_T2@0;
LON_T2@0;
[se_T2@0];
[LON_T2@0];

se_T1*;
LON_T1*;

[se_T1*];
[LON_T1*];
```

Illustration

- **Autoregressive cross-lagged model**:

χ^2 = 534.445; df = 170; CFI = .935; TLI = .927; RMSEA = .038.

- **Latent change model**:

χ^2 = 534.445; df = 170; CFI = .935; TLI = .927; RMSEA = .038

Illustration

- **Autoregressive cross-lagged model**:

χ^2 = 534.445; df = 170; CFI = .935; TLI = .927; RMSEA = .038.

Self-Esteem (t1) → Loneliness (t2): β = -.045

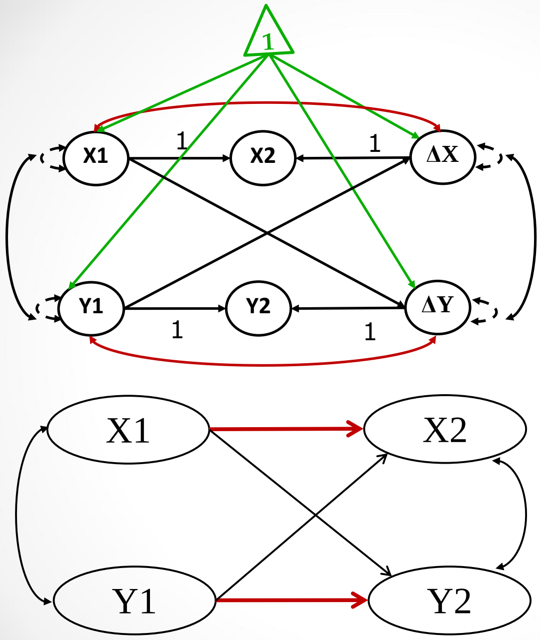
Loneliness (t1) → Self-Esteem (t2): β = .070*

- **Latent change model**:

χ^2 = 534.445; df = 170; CFI = .935; TLI = .927; RMSEA = .038

Self-Esteem (t1) → ΔLoneliness (t1-t2): β = .186**

Loneliness (t1) → ΔSelf-Esteem (t1-t2): β = .179**



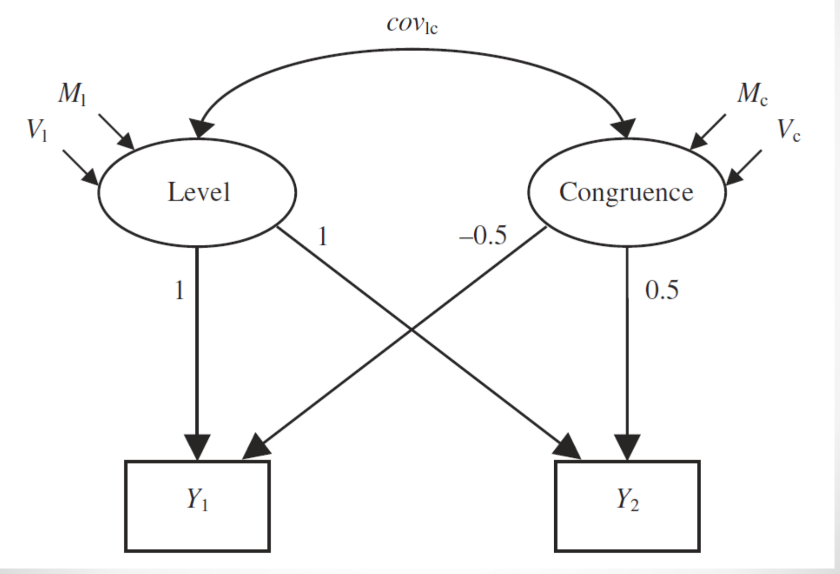
BUT ?

Parenthesis: Other Applications of Latent Change Scores with Cross Sectional Data

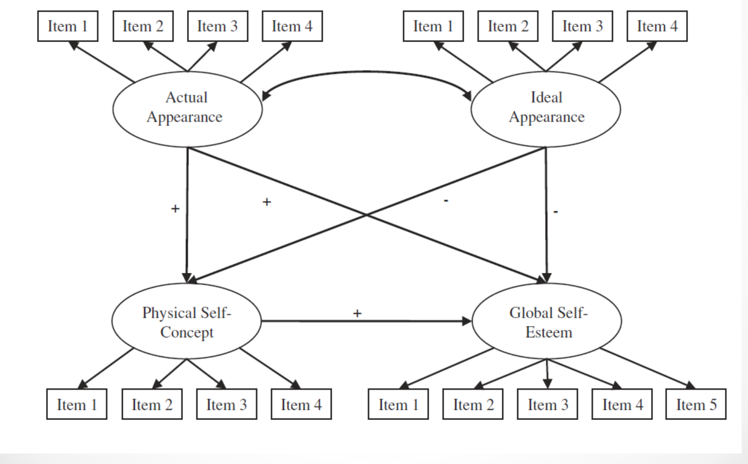
Utility

- Person-Environment fit
- Actual-Ideal Discrepancies
- Etc.
- Any domain when there is a reason to believe that discrepancies between one construct and another is a meaningful variable to consider

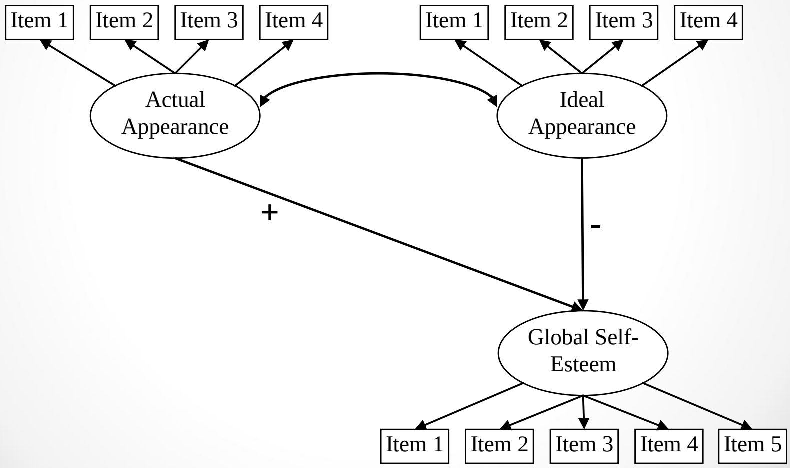
Latent Congruence (Cheung, 2009)



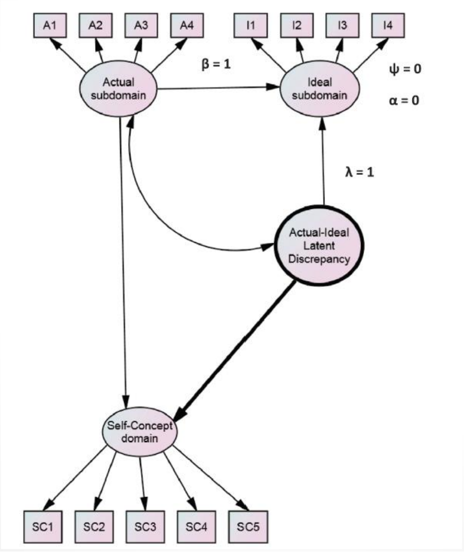
Actual-Ideal Discrepancies (Morin et al., 2015)



Actual-Ideal Discrepancies (Morin et al., 2015)



Or Latent Discrepancy Model (Scalas et al., 2014) ?



Morin et al. (2015)

- **Classical Actual-Ideal Discrepancy Model**

χ^2 = 789.21, df = 113; CFI = 0.94; TLI = 0.93; RMSEA = 0.06

Ideal → PSC: *b* = -0.079; *p* ≤ .01

Ideal → GSE: *b* = -0.006, ns

- **Latent Discrepancy Model**

χ^2 = 789.21, df = 113; CFI = 0.94; TLI = 0.93; RMSEA = 0.06

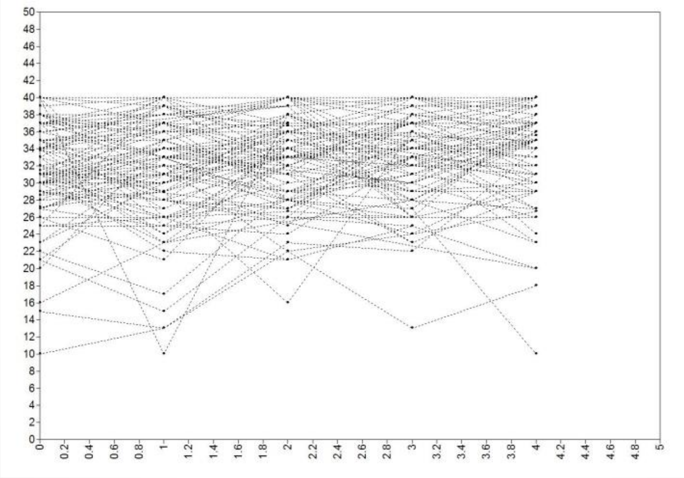
Discrepancy → PSC: *b* = -0.079; *p* ≤ .01

Discrepancy → GSE: *b* = -0.006, ns

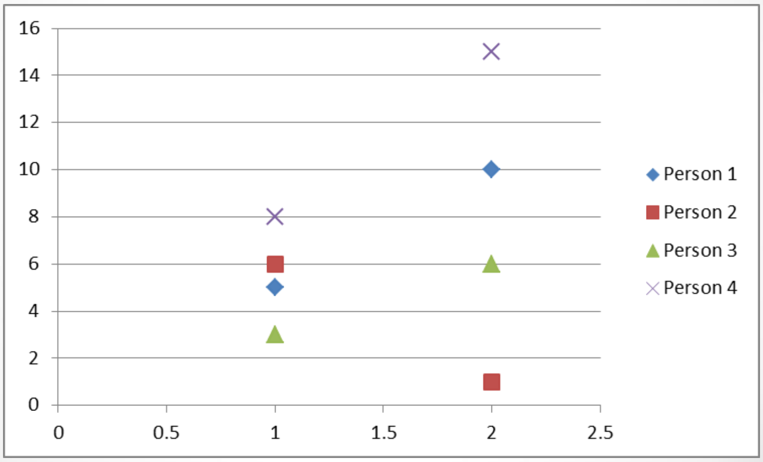
Limitations of the Latent Change Models

(4) Are unable to depict the shape of the growth trajectory over time. They do model "change", but not the process of change

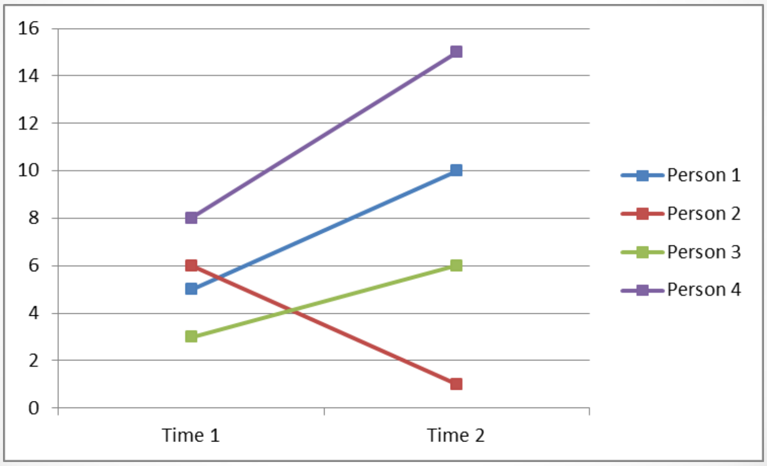
Individual trajectories



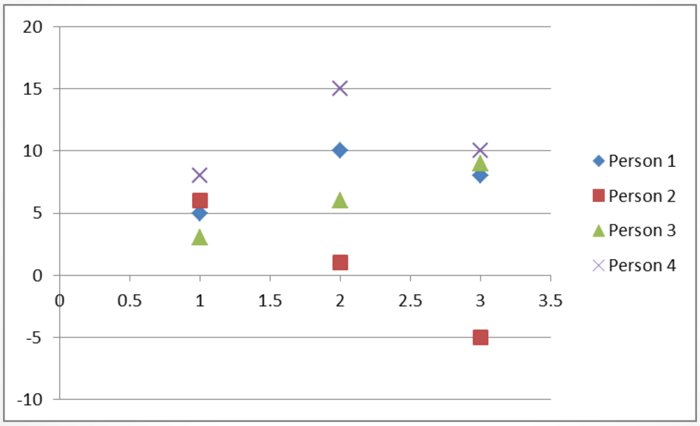
Two time points



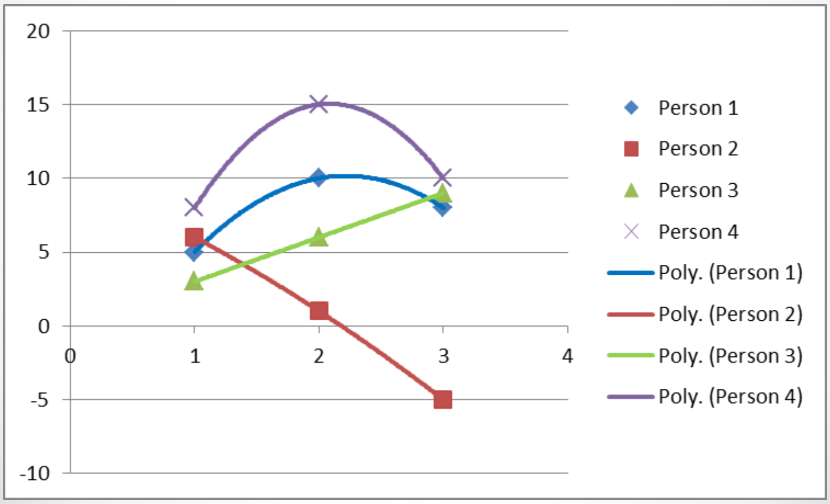
Two time points



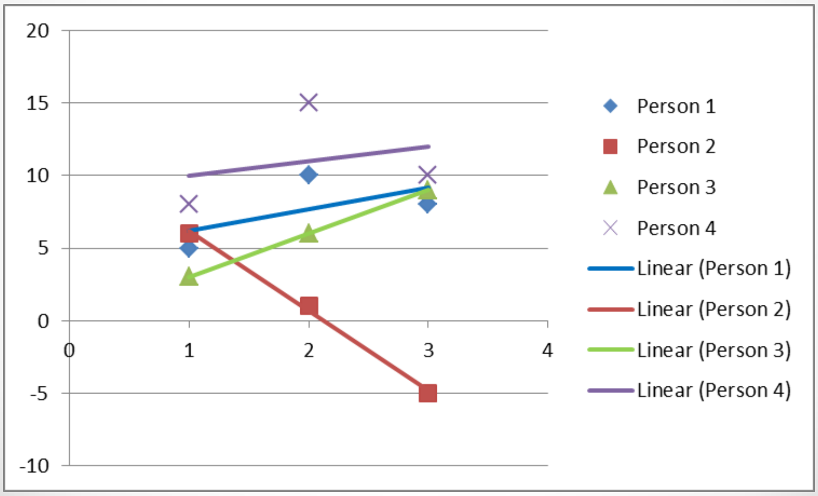
Three Time points



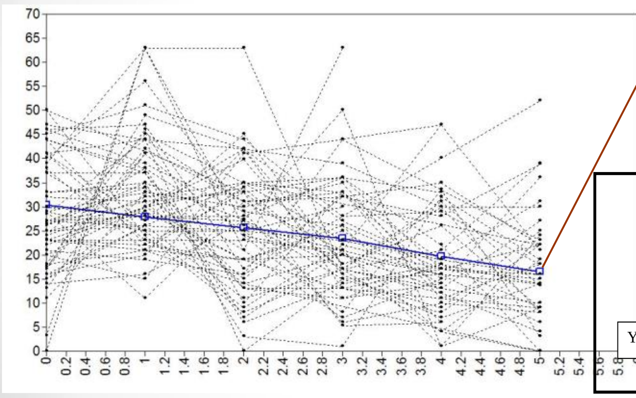
Three Time points



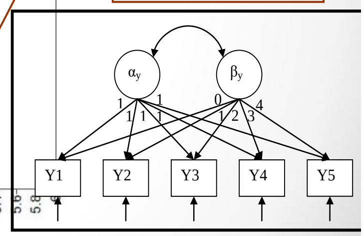
Three Time points



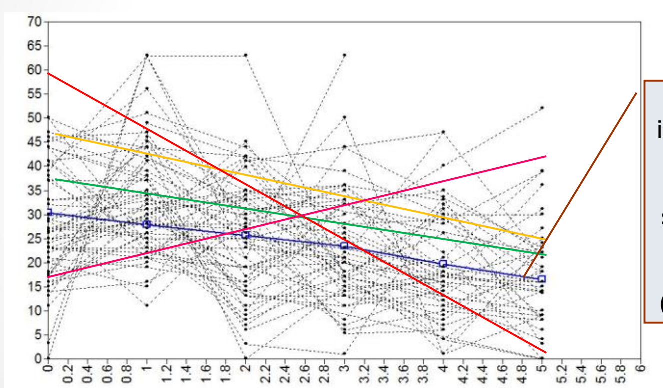
Latent curve models allow one to synthesize individual trajectories with few latent parameters through a restricted factor model.



An average trajectory with a random intercept and slope is estimated

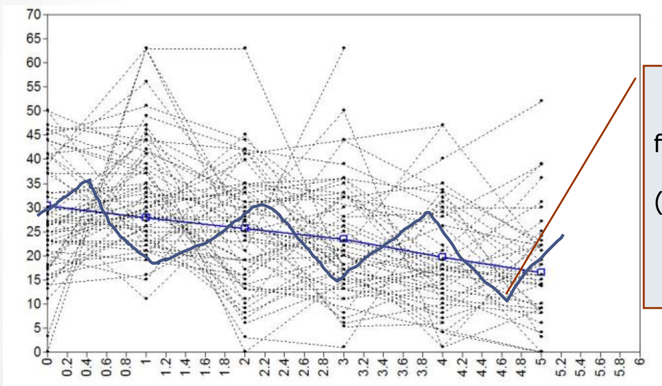


Latent curve models allow one to synthesize individual trajectories with few latent parameters through a restricted factor model.

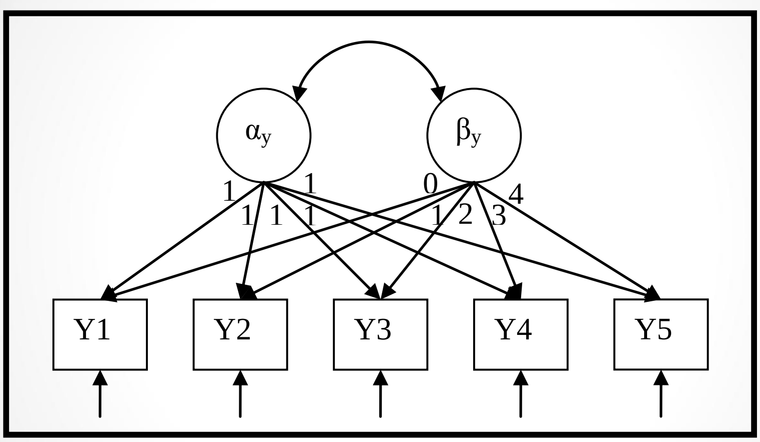


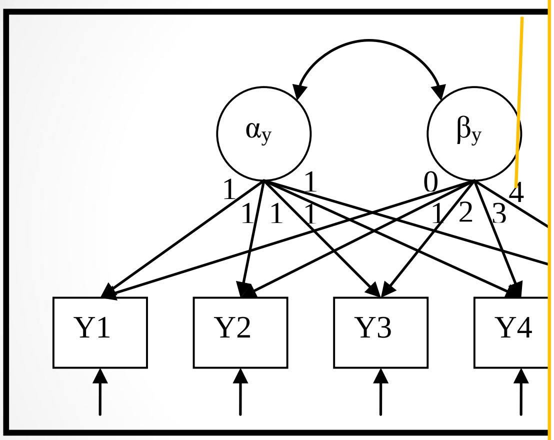
Random: Each individual has its own trajectory (intercept and slope variance). Model-based time-structured (trait) variability

Latent curve models allow one to synthesize individual trajectories with few latent parameters through a restricted factor model.



Residuals:
Few persons follow a perfectly linear (curvilinear, etc.) trajectory.
Residual state-like variability.





Time codes (λ): Reflects the passage of time.

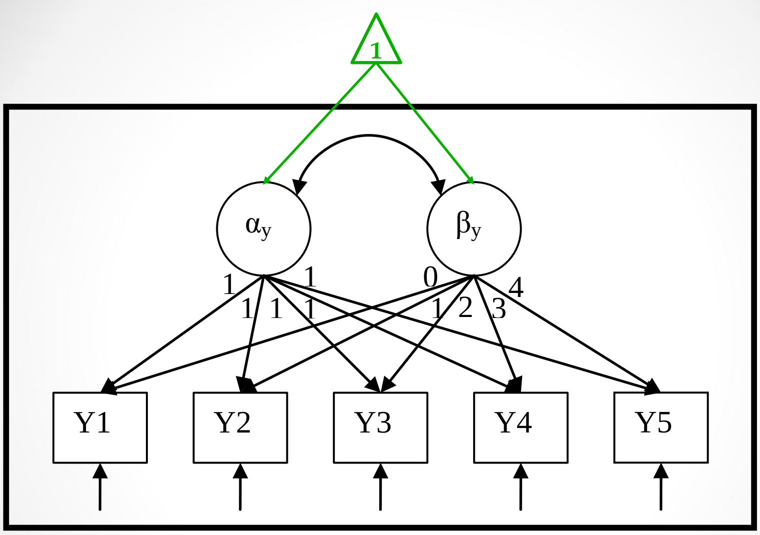
Linear: 0-1-2-3-4-5 (or some other reflecting the true passage of time, e.g. 0-1-3-4-6).

Quadratic: Addition of a second slope factor with squared time codes (0-1-4-9-16-25).

Piecewise: Two slopes with loadings reflecting the transition point:

- 0-1-2-3-3-3-3
- 0-0-0-0-1-2-3

Multibase: Free loadings reflecting the % of change between 0 and 1.



Linear LCM

$$Y_{it} = \alpha_{iy} + \beta_{iy}\lambda_t + \varepsilon_{yit}$$

$$\alpha_{iy} = \mu_\alpha + \zeta_{\alpha yi}$$

$$\beta_{iy} = \mu_\beta + \zeta_{\beta yi}$$

Requires 3 time points

Slope factor

Time specific residuals

$$Y_{it} = \alpha_{iy} + \beta_{iy}\lambda_t + \varepsilon_{yit}$$

Intercept factor

Time codes (λ): Reflects the passage of time.

Linear: 0-1-2-3-4-5 (or some other reflecting the true passage of time, e.g. 0-1-3-4-6).

$$\alpha_{iy} = \boxed{\mu_\alpha} + \zeta_{\alpha yi}$$

$$\beta_{iy} = \boxed{\mu_\beta} + \zeta_{\beta yi}$$

Average intercept and slope observed in the total sample

$$\alpha_{iy} = \mu_\alpha + \boxed{\zeta_{\alpha yi}}$$

$$\beta_{iy} = \mu_\beta + \boxed{\zeta_{\beta yi}}$$

Disturbances (zeta) reflecting the variability of the estimated intercepts and slopes across cases within latent trajectory classes. These disturbances have a mean of zero and a variance/covariance matrix represented by Phi:

$$\Phi_y = \begin{bmatrix} \psi_{\alpha\alpha y} & \\ \psi_{\alpha\beta y} & \psi_{\beta\beta y} \end{bmatrix}$$

Quadratic LCM

$$Y_{it} = \alpha_{iy} + \beta 1_{iy}\lambda_t + \beta 2_{iy}\lambda_t^2 + \varepsilon_{yit}$$

$$\alpha_{iy} = \mu_\alpha + \zeta_{\alpha yi}$$

$$\beta 1_{iy} = \mu_{\beta 1} + \zeta_{\beta 1yi}$$

$$\beta 2_{iy} = \mu_{\beta 2} + \zeta_{\beta 2yi}$$

Requires 4 time points

$$Y_{it} = \alpha_{iy} + \beta 1_{iy}\lambda_t + \beta 2_{iy}\lambda_t^2 + \varepsilon_{yit}$$

Time codes (λ): Reflects the passage of time.

Linear: 0-1-2-3-4-5 (or some other reflecting the true passage of time, e.g. 0-1-3-4-6).

Quadratic: Addition of a second slope factor with squared time codes (0-1-4-9-16-25).

$$\alpha_{iy} = \mu_{\alpha} + \boxed{\zeta_{\alpha yi}}$$

$$\beta 1_{iy} = \mu_{\beta 1} + \boxed{\zeta_{\beta 1 yi}}$$

$$\beta 2_{iy} = \mu_{\beta 2} + \boxed{\zeta_{\beta 2 yi}}$$

Disturbances (zeta) reflecting the variability of the estimated intercepts and slopes across cases within latent trajectory classes. These disturbances have a mean of zero and a variance/covariance matrix represented by Phi:

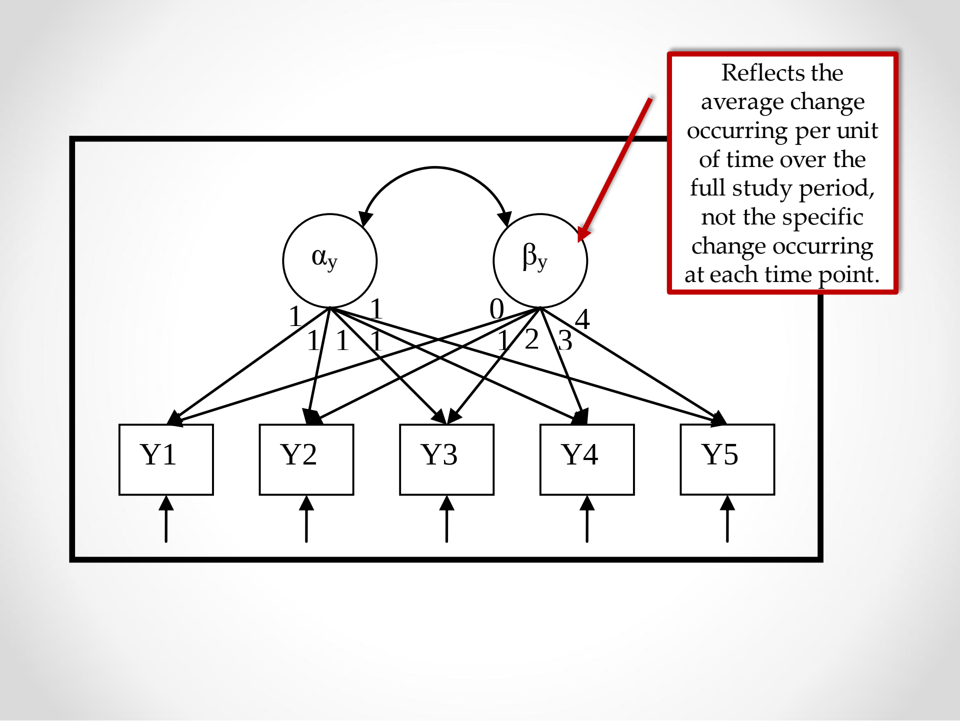
$$\Phi_y = \begin{bmatrix} \psi_{\alpha\alpha y} & & \\ \psi_{\alpha\beta 1 y} & \psi_{\beta 1 \beta 1 y} & \\ \psi_{\alpha\beta 2 y} & \psi_{\beta 1 \beta 2 y} & \psi_{\beta 2 \beta 2 y} \end{bmatrix}$$

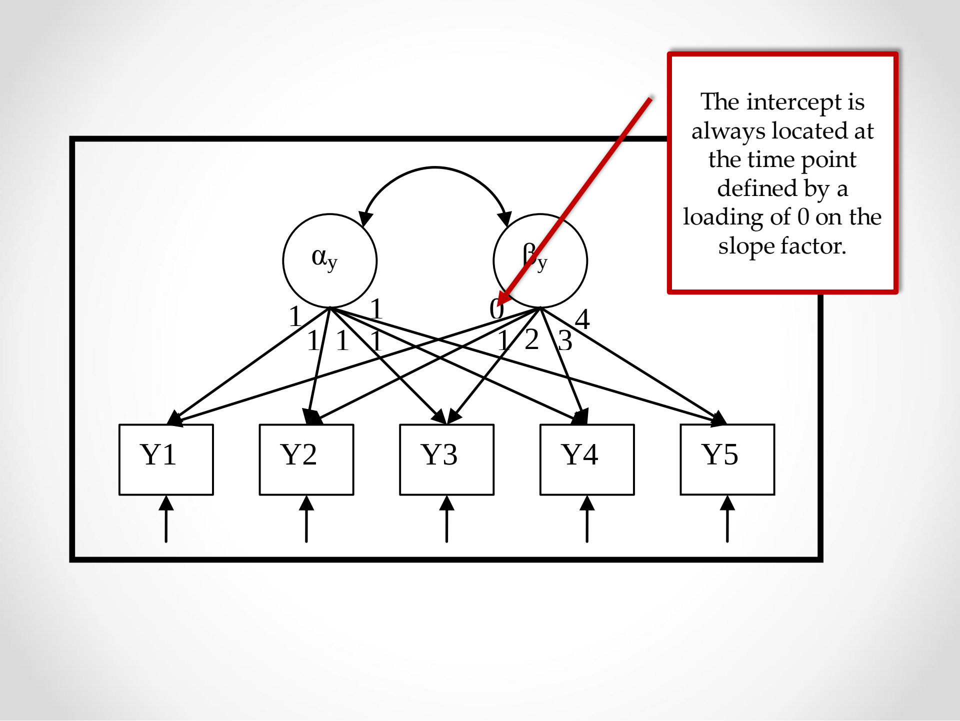
Time

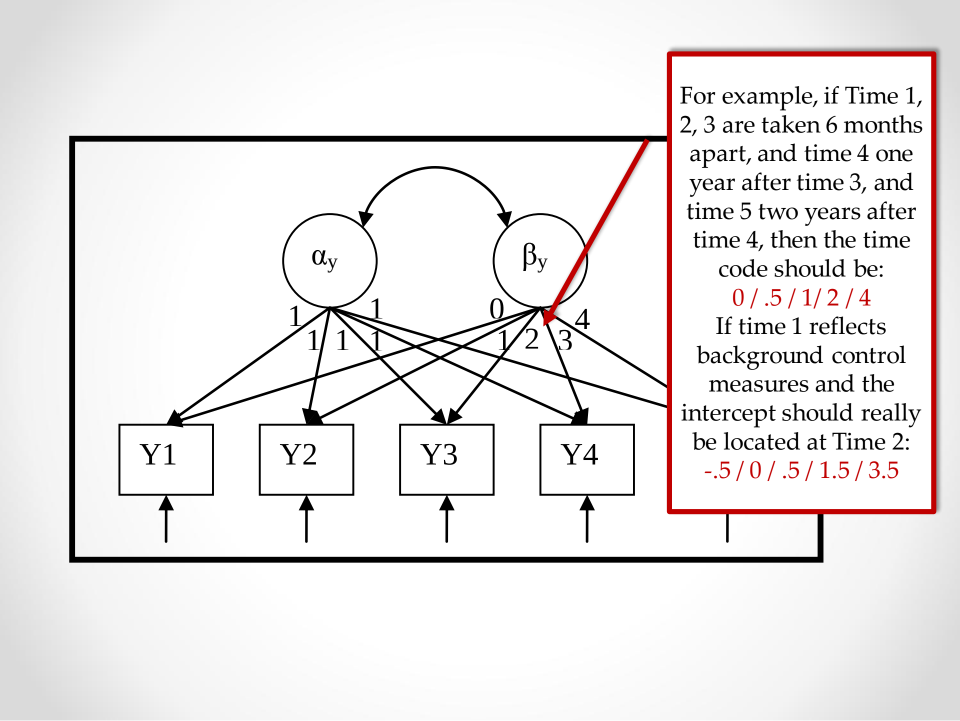
- In any applications of LCM time must be a meaningful variable: Age, Grade, Tenure, etc.
- Simply having a sample of participants of various age/grade/tenure level measured repeatedly over time is not sufficient. One needs to expect that time plays a meaningful role in order to support the needs to rely on analyses of trajectories.
- The argument that LCM are the best way to asses change over time is fallacious, unless time is meaningful.
- The position of the intercept should never be arbitrary, given that it will have a great impact on the results.
- The time codes are also not arbitrary, and need to reflect the passage of time.

Mehta, P. D., & West, S. G. (2000). Putting the individual back into individual growth curves. *Psychological Methods, 5,* 23–43.

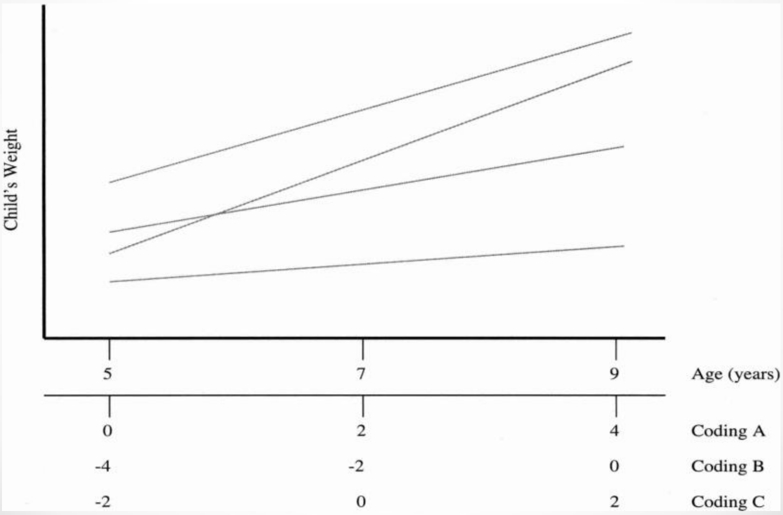
Biesanz, J.C., Deeb-Soosa, N., Papadakis, A.A., Bollen, K.A., & Curran, P.J. (2004). The role of coding time in estimating and interpreting growth curve models. *Psychological Methods, 9,* 30–52.


Reflects the average change occurring per unit of time over the full study period, not the specific change occurring at each time point.
α_y
β_y
1 1 1 1 1
0 1 2 3 4
Y1
Y2
Y3
Y4
Y5


The intercept is always located at the time point defined by a loading of 0 on the slope factor.
α_y
β_y
1 1 1 1 1
0 1 2 3 4
Y1
Y2
Y3
Y4
Y5



Biesanz et al., (2004)

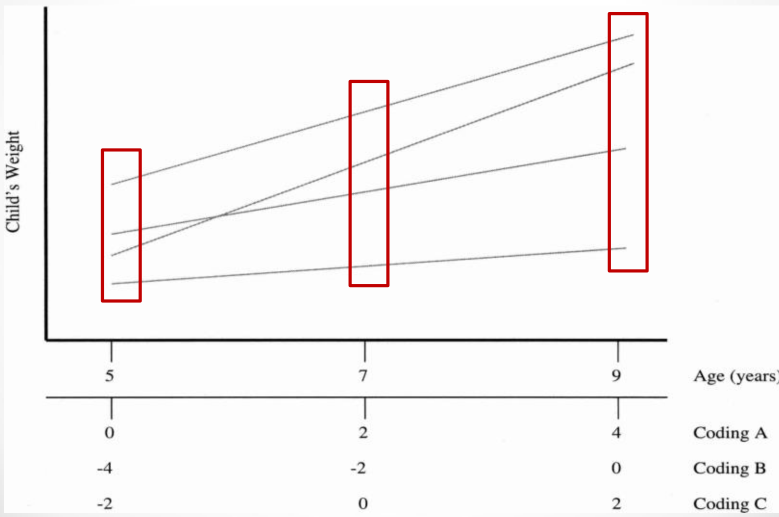


$$\Lambda_A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 4 \end{bmatrix}, \Lambda_B = \begin{bmatrix} 1 & -4 \\ 1 & -2 \\ 1 & 0 \end{bmatrix}, \Lambda_C = \begin{bmatrix} 1 & -2 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}.$$

	Model					
	A		B		C	
Parameter	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*
Variance						
Intercept	28.776	6.143	260.032	33.037	111.599	13.522
Linear	8.201	1.470	8.201	1.470	8.201	1.470
Covariance						
Intercept–linear	12.505	2.676	45.309	6.382	28.907	3.912
M						
Intercept	39.457	0.487	71.707	1.369	55.582	0.877
Linear	8.063	0.267	8.063	0.267	8.063	0.267
Unique variance						
Age 5	8.319	5.280	8.319	5.280	8.319	5.280
Age 7	12.094	4.880	12.094	4.880	12.094	4.880
Age 9	57.168	15.992	57.168	15.992	57.168	15.992
Model $T_{ML}(df = 1)$	2.131		2.131		2.131	

$$\Lambda_A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 4 \end{bmatrix}, \Lambda_B = \begin{bmatrix} 1 & -4 \\ 1 & -2 \\ 1 & 0 \end{bmatrix}, \Lambda_C = \begin{bmatrix} 1 & -2 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}.$$

	Model					
	A		B		C	
Parameter	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*
Variance						
Intercept	28.776	6.143	260.032	33.037	111.599	13.522
Linear	8.201	1.470	8.201	1.470	8.201	1.470
Covariance						
Intercept–linear	12.505	2.676	45.309	6.382	28.907	3.912
M						
Intercept	39.457	0.487	71.707	1.369	55.582	0.877
Linear	8.063	0.267	8.063	0.267	8.063	0.267
Unique variance						
Age 5	8.319	5.280	8.319	5.280	8.319	5.280
Age 7	12.094	4.880	12.094	4.880	12.094	4.880
Age 9	57.168	15.992	57.168	15.992	57.168	15.992
Model $T_{ML}(df = 1)$	2.131		2.131		2.131	



$$\mathbf{\Lambda}_A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 4 \end{bmatrix}, \mathbf{\Lambda}_B = \begin{bmatrix} 1 & -4 \\ 1 & -2 \\ 1 & 0 \end{bmatrix}, \mathbf{\Lambda}_C = \begin{bmatrix} 1 & -2 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}.$$

	Model					
	A		B		C	
Parameter	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*
Variance						
Intercept	28.776	6.143	260.032	33.037	111.599	13.522
Linear	8.201	1.470	8.201	1.470	8.201	1.470
Covariance						
Intercept–linear	12.505	2.676	45.309	6.382	28.907	3.912
M						
Intercept	39.457	0.487	71.707	1.369	55.582	0.877
Linear	8.063	0.267	8.063	0.267	8.063	0.267
Unique variance						
Age 5	8.319	5.280	8.319	5.280	8.319	5.280
Age 7	12.094	4.880	12.094	4.880	12.094	4.880
Age 9	57.168	15.992	57.168	15.992	57.168	15.992
Model $T_{ML}(df = 1)$	2.131		2.131		2.131	

$$\Lambda_A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 4 \end{bmatrix}, \Lambda_B = \begin{bmatrix} 1 & -4 \\ 1 & -2 \\ 1 & 0 \end{bmatrix}, \Lambda_C = \begin{bmatrix} 1 & -2 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}.$$

	Model					
	A		B		C	
Parameter	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*
Variance						
Intercept	28.776	6.143	260.032	33.037	111.599	13.522
Linear	8.201	1.470	8.201	1.470	8.201	1.470
Covariance						
Intercept–linear	12.505	2.676	45.309	6.382	28.907	3.912
M						
Intercept	39.457	0.487	71.707	1.369	55.582	0.877
Linear	8.063	0.267	8.063	0.267	8.063	0.267
Unique variance						
Age 5	8.319	5.280	8.319	5.280	8.319	5.280
Age 7	12.094	4.880	12.094	4.880	12.094	4.880
Age 9	57.168	15.992	57.168	15.992	57.168	15.992
Model $T_{ML}(df = 1)$	2.131		2.131		2.131	

Piecewise LCM

$$Y_{it} = \alpha_{iy} + \beta 1_{iy}\lambda_{1t} + \beta 2_{iy}\lambda_{2t} + \varepsilon_{yit}$$

$$\alpha_{iy} = \mu_\alpha + \zeta_{\alpha yi}$$

$$\beta 1_{iy} = \mu_{\beta 1} + \zeta_{\beta 1yi} \qquad \Phi_y = \begin{bmatrix} \psi_{\alpha\alpha y} & & \\ \psi_{\alpha\beta 1y} & \psi_{\beta 1\beta 1y} & \\ \psi_{\alpha\beta 2y} & \psi_{\beta 1\beta 2y} & \psi_{\beta 2\beta 2y} \end{bmatrix}$$

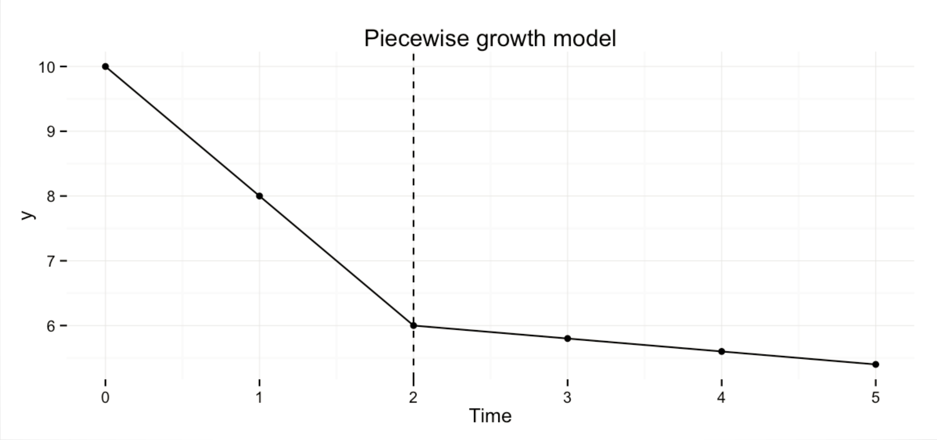
$$\beta 2_{iy} = \mu_{\beta 2} + \zeta_{\beta 2yi}$$

Requires 5 time points, with at least 2 before and after the turning point.

$$Y_{it} = \alpha_{iy} + \beta 1_{iy}\lambda_{1t} + \beta 2_{iy}\lambda_{2t} + \varepsilon_{yit}$$

Time codes (λ): Reflects the passage of time.

Linear: 0-1-2-3-4-5 (or some other reflecting the true passage of time, e.g. 0-1-3-4-6).
Quadratic: Addition of a second slope factor with squared time codes (0-1-4-9-16-25).
Piecewise: Two slopes with loadings reflecting the transition point:
- **0-1-2-3-3-3-3**
- **0-0-0-0-1-2-3**



Piecewise time codes

- **Approach 1:**

0-1-2-3-3-3-3: The first slope reflects change up to the 4th measurement occasion. Linear growth.

0-0-0-0-1-2-3: The second slope reflect change occurring after the 4th measurement occasion. Linear growth.

- **Approach 2: Added Rate model**

0-1-2-3-4-5-6: The first slope reflects a linear rate of change occurring throughout the study. Linear Growth

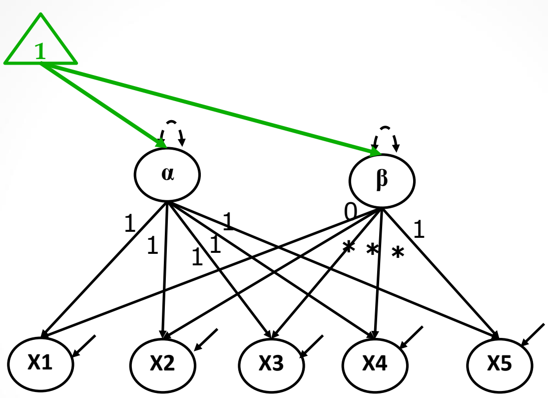
0-0-0-0-1-2-3: The second slope reflect the difference in the rate of change occurring between the two periods (before and after after the 4th measurement occasion). Additive growth.

Turning Points:

- Piecewise modeling is useful when the turning point is known a priori (e.g., intervention, change, retirement, transition, puberty).
- The turning point needs to be common to all participants (not random).
- When the turning point is not known a priori but suspected to exist (e.g., imagine you want to know how long, on average, after retirement it takes for people to change their life rhythm), then it is possible to estimate a fully linear trajectory and to examine the modification indices associated with the loadings on the slope factor to locate the turning point (see Kwok et al., 2010).
- Convergence problems are associated with having only 2 measurement occasions before, but not after, the turning point (Diallo et al., 2015)

Latent Basis / Multibase Models

- So far, we have seen LCM in which all factor loadings representing the time codes are fixed to specific values.
- For identification purposes, only two of them need to be fixed to the pre-specified values of 0 and, typically, 1.
- As always, zero serves to locate the intercept.
- The slope factor will then reflect the total amount of change occurring between the 0 and 1 time points.



$$Y_{it} = \alpha_{iy} + \beta_{iy}\lambda_t + \varepsilon_{yit}$$

$$\alpha_{iy} = \mu_\alpha + \zeta_{\alpha yi} \qquad \beta_{iy} = \mu_\beta + \zeta_{\beta yi}$$

Freely estimated time codes

0 * * * * 1:

Here the slope reflects the total amount of change occurring between the first and last time point (over the course of the study).

The freely estimated unstandardized loadings reflect the proportion of this change that has occurred at each time point.

e.g., 0 .5 .6 .7 .8 1: 50% of the change has occurred by Time 2, 60% by Time 3, etc.

Imagine a slope of 100 and an intercept of 0.

e.g., 0 .5 1.5 .9 .8 1: Non-linear trajectory. 50% of the change has occurred by Time 2 (50), 150% by Time 3 (150), then levels go down by Time 4 (80) and 5 (90), and then up again to 100 by the end.

Freely estimated time codes

0 1* * * * :

Here the slope reflects the total amount of change occurring between the first and second time point.

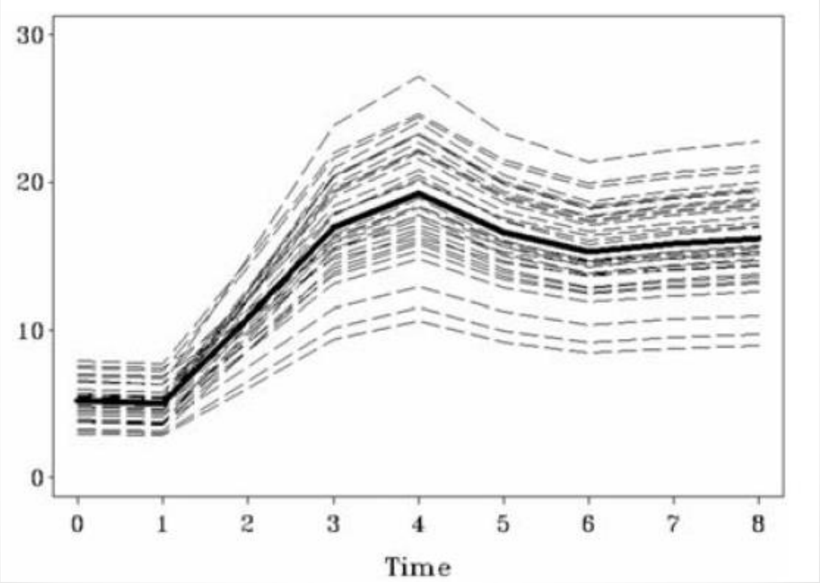
The freely estimated unstandardized loadings reflect how much change (in relation to this initial change level) occurs at each subsequent time point.

Imagine a slope of 10 and an intercept of 0.

e.g., 0 1 2 2.5. 2 3 : By time 3, twice the amount of change that has occurred by Time 1 has occurred (20), then 25, then down to 20, then 30.

Limitations of multibase models

- As in all LCM seen so far, the intercept and slope(s) factors are random parameters, free to differ across participants (each participant has his or her own linear, quadratic, or piecewise trajectory).
- In multibase models:
  - The variance of the intercept factor reflects inter-individual variations in initial levels.
  - The variance of the slope factor reflects inter-individual variations in the total amount of change.
  - The loadings are the same for everyone: Each participant's trajectories have the exact same shape.



Example: Linear

Five measurement points, observed (non latent) equally repeated measures (X1 to X5)

Model:

```
i s | X1@0 X2@1 X3@2 X4@3 X5@4 ;
X1 X2 X3 X4 X5;
```

The | symbol is a shortcut to define a latent curve model.

The **I S** labels can be changed as need. With two of them, a linear LCM will be estimated.

Without the shortcut, the full input would be:

Model:

```
i BY X1@1 X2@1 X3@1 X4@1 X5@1;
s BY X1@0 X2@1 X3@2 X4@3 X5@4 ;
i s ;
i with s;
[i s];
[X1@0 X2@0 X3@0 X4@0 X5@0];
X1 X2 X3 X4 X5;
```

Example: Quadratic

Model:

i s **q** | X1@0 X2@1 X3@2 X4@3 X5@4 ;

X1 X2 X3 X4 X5;

OR:

Model:

i BY X1@1 X2@1 X3@1 X4@1 X5@1;

s BY X1@0 X2@1 X3@2 X4@3 X5@4 ;

q BY X1**@0** X2**@1** X3**@4** X4**@9** X5**@16** ;

i s q; i with s q; s with q;

[i s q];

[X1@0 X2@0 X3@0 X4@0 X5@0];

X1 X2 X3 X4 X5;

Example: Piecewise

Model:

i s1 | X1@0 X2@1 X3@2 X4@2 X5@2 ;

i s2 | X1@0 X2@0 X3@0 X4@1 X5@2 ;

X1 X2 X3 X4 X5;

OR:

Model:

i BY X1@1 X2@1 X3@1 X4@1 X5@1;

s1 BY X1@0 X2@1 X3@2 X4@2 X5@2 ;

s2 BY X1@0 X2@0 X3@0 X4@1 X5@2 ;

i s1 s2; i with s1 s2; s1 with s2;

[i s1 s2]; [X1@0 X2@0 X3@0 X4@0 X5@0];

X1 X2 X3 X4 X5;

Example: Multibase

MODEL:

i s | X1@0 X2* X3* X4* X5@1 ;

X1 X2 X3 X4 X5;

Or

i s | X1@0 X2@1 X3* X4* X5 * ;

X1 X2 X3 X4 X5;

Or

Model:

i BY X1@1 X2@1 X3@1 X4@1 X5@1;

s BY X1@0 X2@1 X3* X4* X5* ;

i s ; i with s;

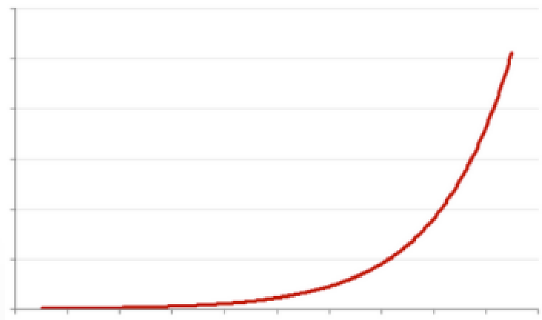
[i s];[X1@0 X2@0 X3@0 X4@0 X5@0];

X1 X2 X3 X4 X5;

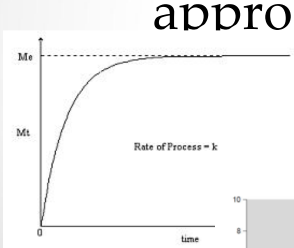
Other non-linear trajectories

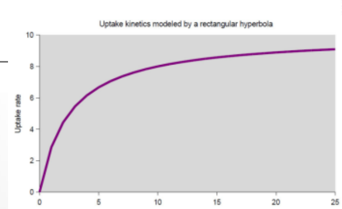
- Complex.
- Not often useful.
- Often include non-random parameters (that do not vary across individuals, and yet should).
- Exponential, Logistic, negative exponential, rectangular hyperbolic, Gompertz.
- See:

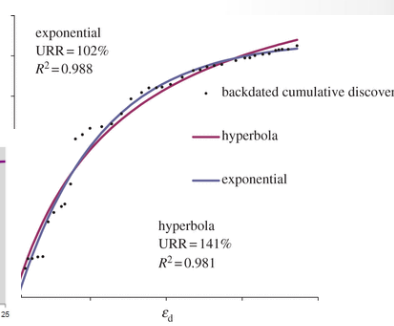
Exponential:
Accelerating rate of change



Negative Exponential / Rectangular Hyperbolic:
* Quick growth reaching a ceiling
* Rate of change that increases as one approaches the asymptote

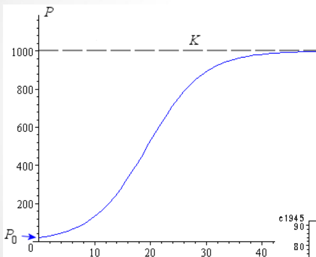


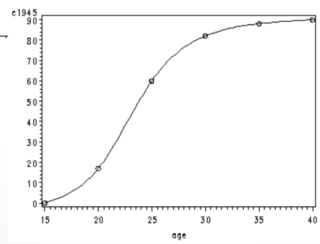


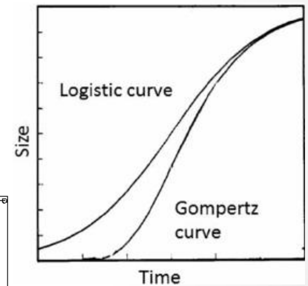


Logistic/ Gompertz:

* Difference: The logistic is symmetric, the Gompertz is not







See:

- Wu, W., Selig, J.P., & Little, T.D. (2013). Longitudinal Data analysis. In T.D. Little (Ed.) *The Oxford Handbook of Quantitative Methods in Psychology*, *Volume 2* (pp. 387-410). New York: Oxford University Press.
- Hancock, G.R., Harring, J.R., & Lawrence, F.R. (2013). Using latent growht modelign to evaluate longitudinal change. In G. R. Hancock & R. O. Mueller (Eds), *Structural Equation Modeling: A Second Course*, 2nd *edition* (pp. 309-342). Greenwich, CO: IAP.
- Ram, N., & Grimm, K. (2007). Using simple and complex growth models to articulate developmental change: Matching theory to method. International Journal of Behavioral Development, 31, 303-316.
- Grimm, K.J., & Ram, N. (2009). Nonlinear growth models in Mplus and SAS. *Structural Equation Modeling*, *16*, 676-701.
- Preacher, K.J., & Hancock, G.R. (2015). Meaningful aspects of change as novel random coefficients: A general method for reparametrizing longitudinal models. *Psychological Methods*, *20*, 84-101.

Conditional Models

- **Time Invariant Covariates**

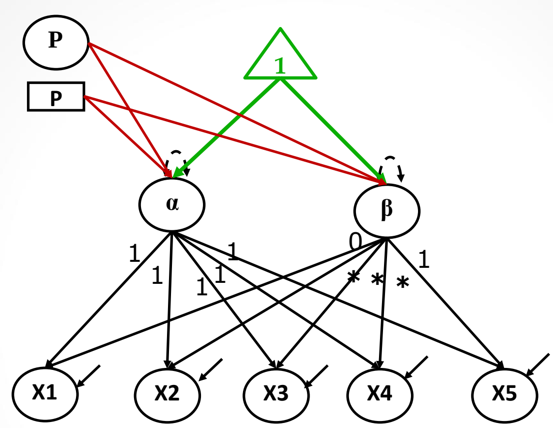
Incorporation of predictors that are either only measured at the beginning of the study, or outcomes that are only measured at the end of the study, or of predictors/outcomes that do not change over time (sex, etc.).
Typically, these are only allowed to predict, or be predicted, by the intercept and slope(s) factors.

- **Time Varying Covariates**

Incorporation of predictors/outcomes that differ across time points.
Typically, these are only allowed to predict, or be predicted, but the time-varying observations. The invariance over time of these predictions can be tested.

- **Multiple Processes**

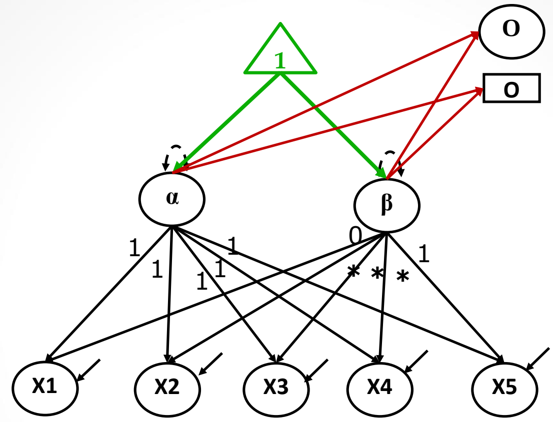
Incorporation of more than one LCM in the same model.



$$Y_{it} = \alpha_{iy} + \beta_{iy}\lambda_t + \varepsilon_{yit}$$

$$\alpha_{iy} = \mu_\alpha + \gamma_{\alpha 1}x_{1i} + \zeta_{\alpha yi}$$

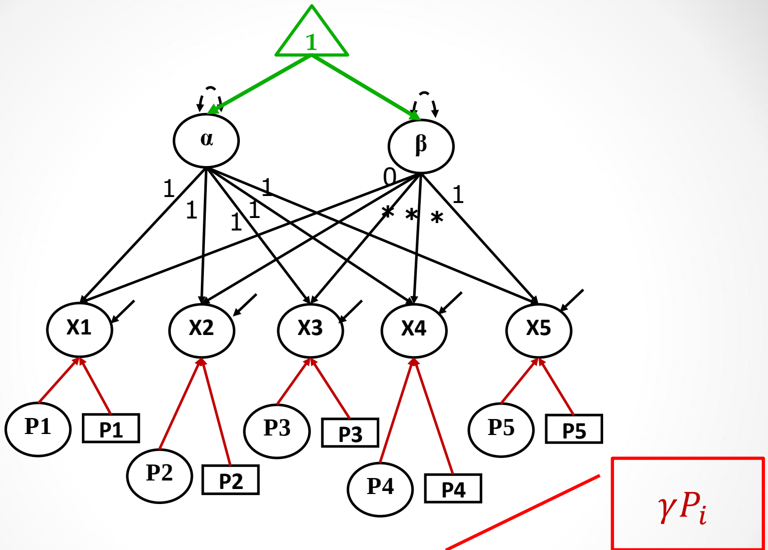
$$\beta_{iy} = \mu_\beta + \gamma_{\beta 1}x_{1i} + \zeta_{\beta yi}$$



$$\boldsymbol{O_i = \alpha_O + \beta_O\alpha_{iy} + \beta_O\beta_{iy}\varepsilon_{oi}}$$

```
Model:
i BY X1@1 X2@1 X3@1  X4@1 X5@1;
s BY X1@0 X2@1 X3@2  X4@3 X5@4 ;
i s ;  i with s;
[i s]; [X1@0 X2@0 X3@0 X4@0 X5@0];
X1 X2 X3 X4 X5;
i s ON P1 P2;
```

```
Model:
i BY X1@1 X2@1 X3@1  X4@1 X5@1;
s BY X1@0 X2@1 X3@2  X4@3 X5@4 ;
i s ;  i with s;
[i s]; [X1@0 X2@0 X3@0 X4@0 X5@0];
X1 X2 X3 X4 X5;
O1 O2 ON i s;
```



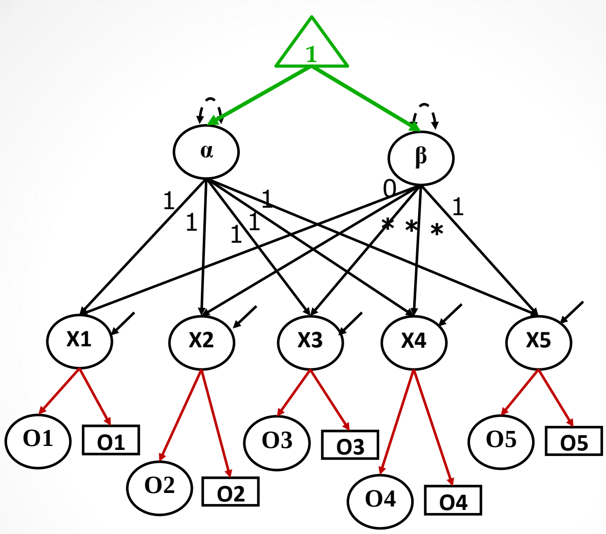
$$Y_{it} = \alpha_{iy} + \beta_{iy}\lambda_t + \gamma_t P_{it} + \varepsilon_{yit}$$

$$\alpha_{iy} = \mu_\alpha + \zeta_{\alpha yi} \qquad \beta_{iy} = \mu_\beta + \zeta_{\beta yi}$$

```
Model:
i BY X1@1 X2@1 X3@1  X4@1 X5@1;
s BY X1@0 X2@1 X3@2  X4@3 X5@4 ;
i s ;  i with s;
[i s]; [X1@0 X2@0 X3@0 X4@0 X5@0];
X1 X2 X3 X4 X5;
X1 ON P1;
X2 ON P2;
X3 ON P3;
X4 ON P4;
X5 ON P5;
```

X1-X5 PON P1-P5;

```
Model:
i BY XI@I X2@I X3@I  X4@I X5@I;
s BY XI@0 X2@I X3@2  X4@3 X5@4 ;
i s ;  i with s;
[i s]; [XI@0 X2@0 X3@0 X4@0 X5@0];
XI X2 X3 X4 X5;
XI ON PI  (a);
X2 ON P2 (a);
X3 ON P3 (a);
X4 ON P4 (a);
X5 ON P5 (a);
```

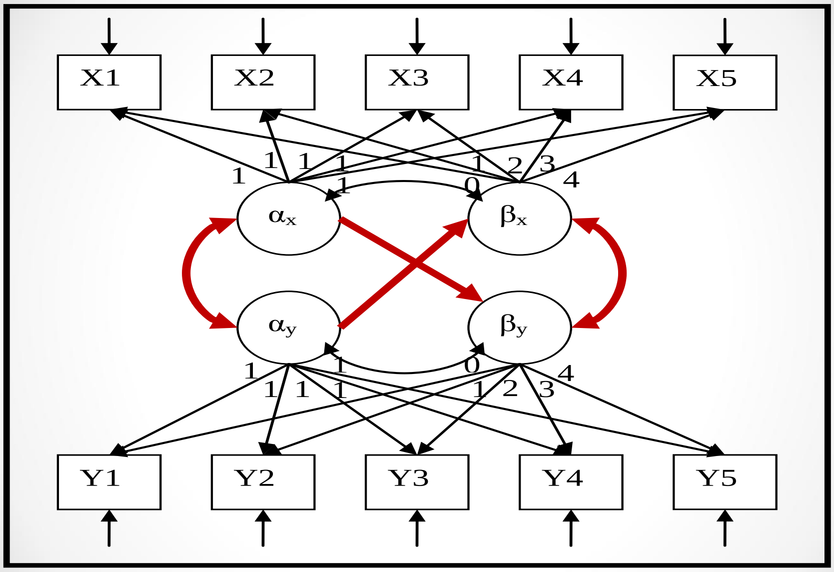


$$O_{it} = \alpha_{Ot} + \rho_{O_t} X_{it} + \varepsilon_{oit}$$

```
Model:
i BY X1@1 X2@1 X3@1  X4@1 X5@1;
s BY X1@0 X2@1 X3@2  X4@3 X5@4 ;
i s ;  i with s;
[i s]; [X1@0 X2@0 X3@0 X4@0 X5@0];
X1 X2 X3 X4 X5;
O1 ON X1;
O2 ON X2;
O3 ON X3;
O4 ON X4;
O5 ON X5;
```

O1-O5 PON X1-X5;

```
Model:
i BY X1@1 X2@1 X3@1  X4@1 X5@1;
s BY X1@0 X2@1 X3@2  X4@3 X5@4 ;
i s ;  i with s;
[i s]; [X1@0 X2@0 X3@0 X4@0 X5@0];
X1 X2 X3 X4 X5;
O1 ON X1 (a);
O2 ON X2 (a)
O3 ON X3 (a);
O4 ON X4 (a);
O5 ON X5 (a);
```




```
Model:
ix BY X1@1 X2@1 X3@1  X4@1 X5@1;
sx BY X1@0 X2@1 X3@2  X4@3 X5@4 ;

iy BY Y1@1 Y2@1 Y3@1  Y4@1 Y5@1;
sy BY Y1@0 Y2@1 Y3@2  Y4@3 Y5@4 ;

ix sx iy sy ;
[ix sx iy sy];
ix with sx; iy with sy;
ix with iy; sx with sy;

[X1@0 X2@0 X3@0 X4@0 X5@0];
[Y1@0 Y2@0 Y3@0 Y4@0 Y5@0];
X1 X2 X3 X4 X5 Y1 Y2 Y3 Y4 Y5;

sx ON iy;
sy ON ix;
```

Thank you!