UNIVERSITÉ Concordia UNIVERSITY

The Bifactor ESEM framework

A way to see the forest and the trees in psychometric measurement

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Substantive-Methodological Synergy Research Laboratory

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Brief Recap

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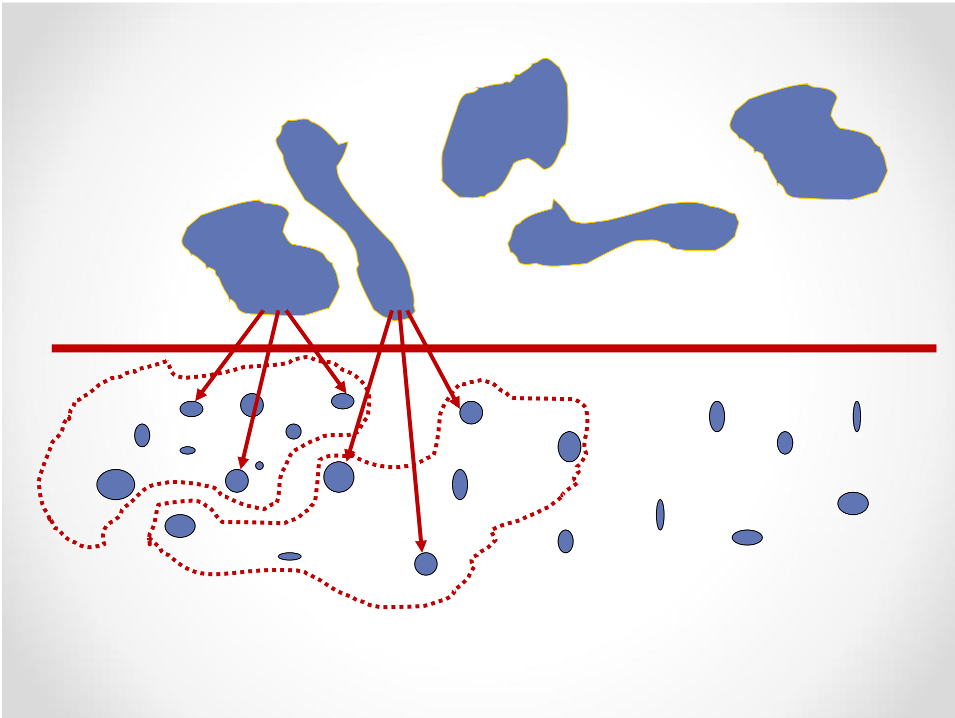
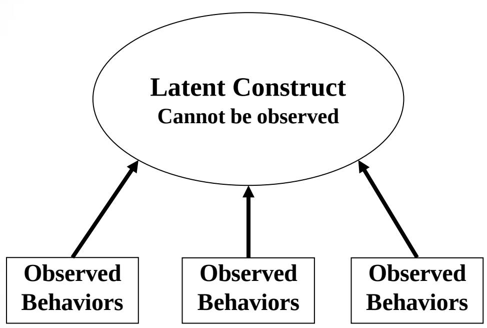


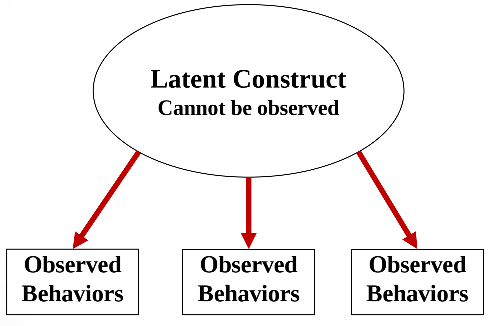
3

Psychological Constructs



4

Psychological Constructs



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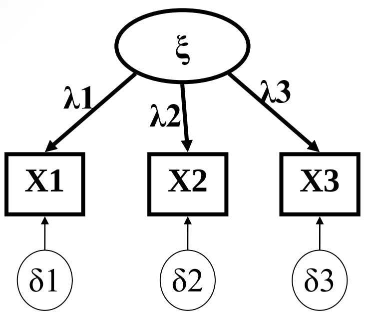

$$X = \Lambda\xi + \delta$$

$$X = \tau + \Lambda\xi + \delta$$

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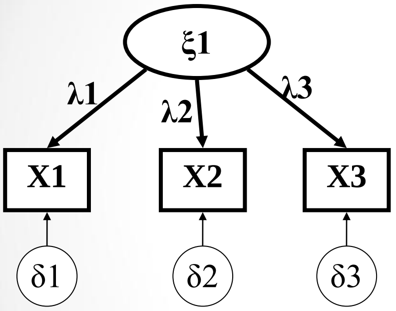
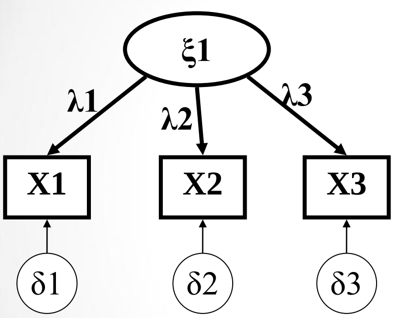
Factor Analysis

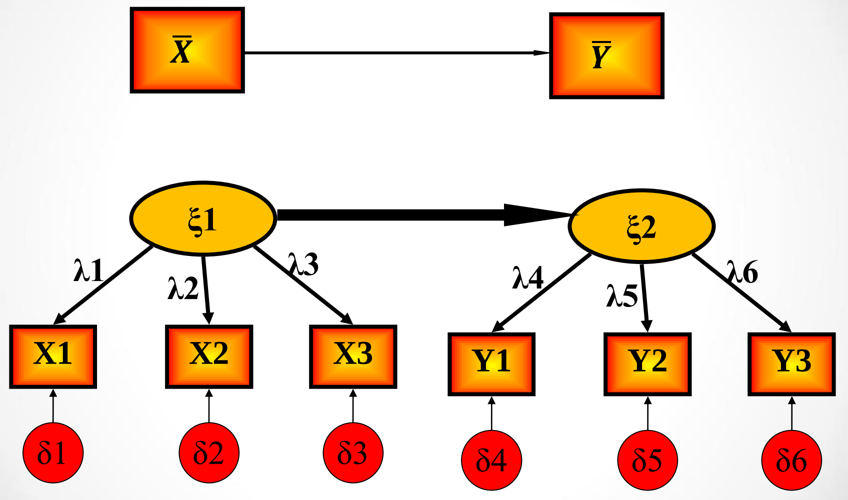


- Focuses on the **<u>covariance</u>** matrix: What is shared among the indicators.
- A **<u>reflective model</u>**: The indicators are seen as providing a reflection of the latent construct.
- The indicators are assumed to have 2 causes: the latent construct, and the uniqueness (which includes random error, and all that is specific to the item).

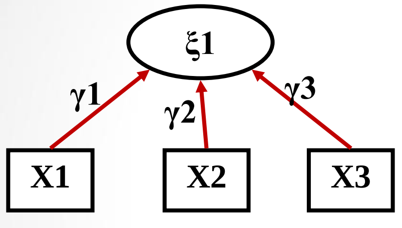
Factor Analysis



- The residual, or uniqueness, describes what is unique to the indicator (not shared with the other indicators).
- What is shared among indicators is completely absorbed in the factor (i.e., covariance).
- The factor is thus corrected for measurement error; perfectly reliable.
- Conditional independence: Factor analysis « assumes » that all of the covariance will be absorbed by the factor (no residual correlations among the uniquenesses) .



Principal Component Analysis

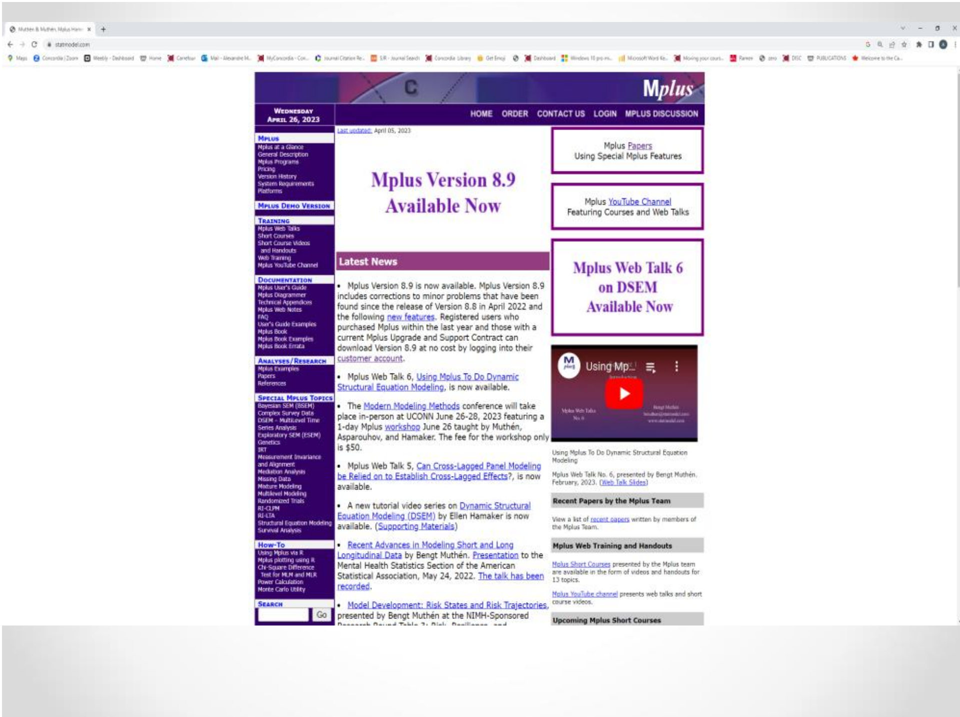


- Aims to reproduces the complete **variance-covariance** matrix, thus what is shared among the indicators and what is unique to them.
- **Formative model**: Indicators "form" the latent variable.
- Useful as a way to obtain a "summary" index of otherwise unrelated indicators (e.g., life events: divorce, marriage, death of a loved one, imprisonment).
- Assume that you are interested in all that is in the indicators.

Lauri Tarkkonen's Blueberry Pie

- You mix the dough, let it rest.
- Go pick the blueberries in your garden.
- Come back, roll the dough, make a base.
- **Principal component analysis:**
  - Drop the content of your picking bucket in directly into the pie plate.
  - This is a good method when you were very careful in the picking process and are growing organic blueberries.
- **Factor analysis :**
  - When you were not as careful, you may prefer to start by extracting leaves, frogs, lizards and spiders from your bucket, and then washing the blueberries. This is like controlling for "measurement errors".

Brief Introduction to Mplus


Mplus
Wednesday April 26, 2023
HOME ORDER CONTACT US LOGIN MPLUS DISCUSSION
Mplus Version 8.9 Available Now
Mplus Papers Using Special Mplus Features
Mplus YouTube Channel Featuring Courses and Web Talks
Mplus Web Talk 6 on DSEM Available Now
Latest News
• Mplus Version 8.9 is now available. Mplus Version 8.9 includes corrections to minor problems that have been found since the release of Version 8.8 in April 2022 and the following new features. Registered users who purchased Mplus within the last year and those with a current Mplus Upgrade and Support Contract can download Version 8.9 at no cost by logging into their customer account.
• Mplus Web Talk 6, Using Mplus To Do Dynamic Structural Equation Modeling, is now available.
• The Modern Modeling Methods conference will take place in-person at UCONN June 26-28, 2023 featuring a 1-day Mplus workshop June 26 taught by Muthén, Asparouhov, and Hamaker. The fee for the workshop only is $50.
• Mplus Web Talk 5, Can Cross-Lagged Panel Modeling be Relied on to Establish Cross-Lagged Effects?, is now available.
• A new tutorial video series on Dynamic Structural Equation Modeling (DSEM) by Ellen Hamaker is now available. (Supporting Materials)
• Recent Advances in Modeling Short and Long Longitudinal Data by Bengt Muthén. Presentation to the Mental Health Statistics Section of the American Statistical Association, May 24, 2022. The talk has been recorded.
Recent Papers by the Mplus Team
Mplus Web Training and Handouts
Upcoming Mplus Short Courses

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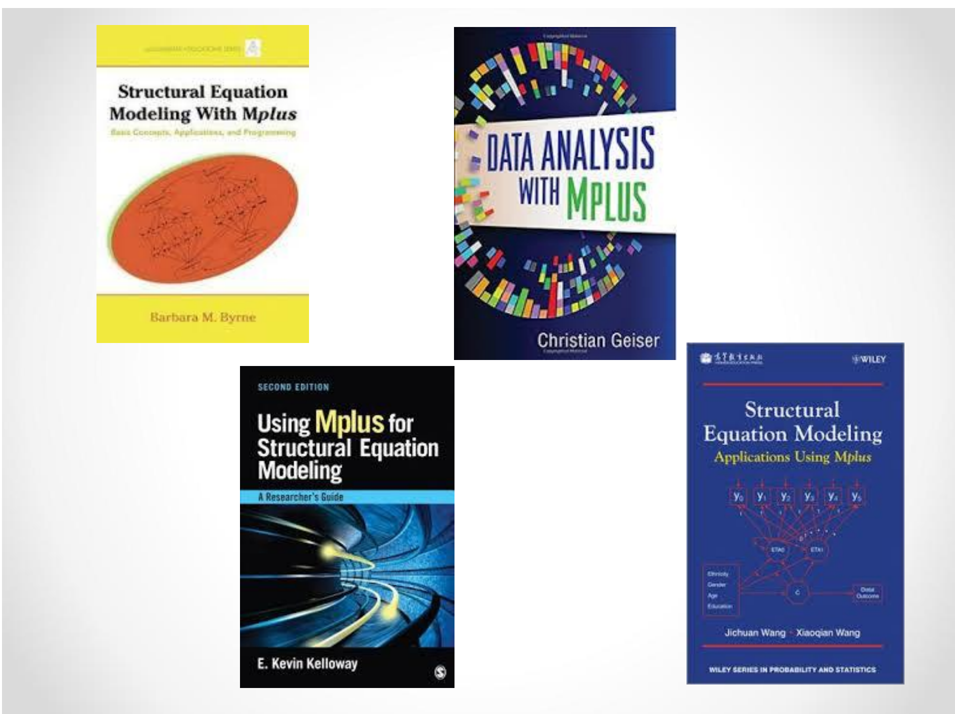
Structural Equation Modeling With Mplus
Basic Concepts, Applications, and Programming
Barbara M. Byrne
DATA ANALYSIS WITH MPLUS
Christian Geiser
SECOND EDITION
Using Mplus for Structural Equation Modeling
A Researcher's Guide
E. Kevin Kelloway
WILEY
Structural Equation Modeling
Applications Using Mplus
Jichuan Wang · Xiaoqian Wang
WILEY SERIES IN PROBABILITY AND STATISTICS

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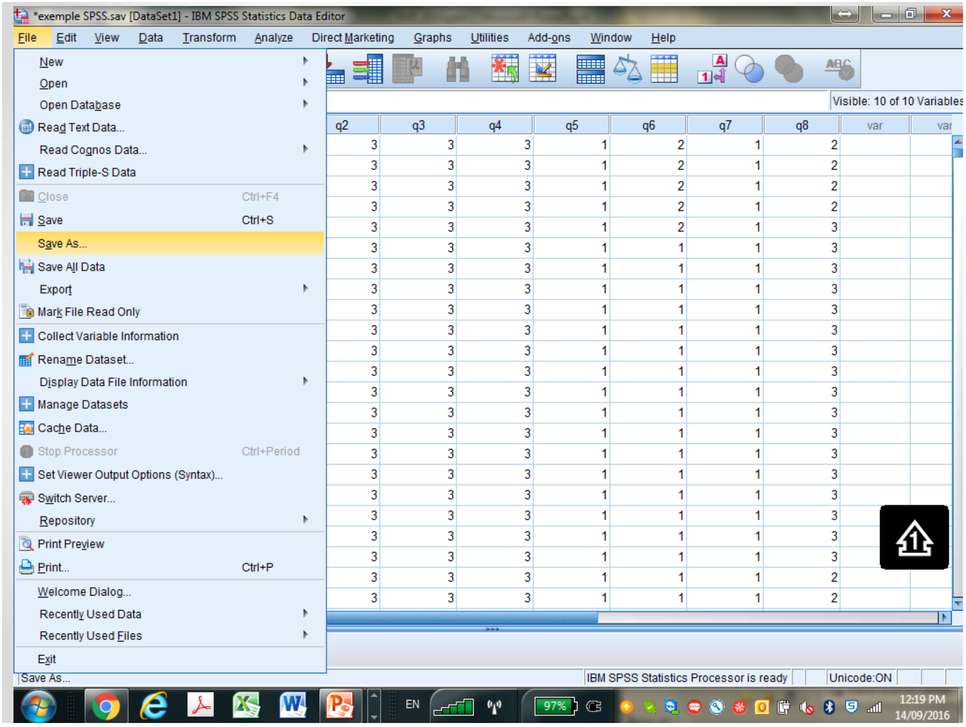
*exemple SPSS.sav [DataSet1] - IBM SPSS Statistics Data Editor
File Edit View Data Transform Analyze Direct Marketing Graphs Utilities Add-ons Window Help
New
Open
Open Database
Read Text Data...
Read Cognos Data...
Read Triple-S Data
Close Ctrl+F4
Save Ctrl+S
Save As...
Save All Data
Export
Mark File Read Only
Collect Variable Information
Rename Dataset...
Display Data File Information
Manage Datasets
Cache Data...
Stop Processor Ctrl+Period
Set Viewer Output Options (Syntax)...
Switch Server...
Repository
Print Preview
Print... Ctrl+P
Welcome Dialog...
Recently Used Data
Recently Used Files
Exit
Visible: 10 of 10 Variables
IBM SPSS Statistics Processor is ready
Unicode:ON

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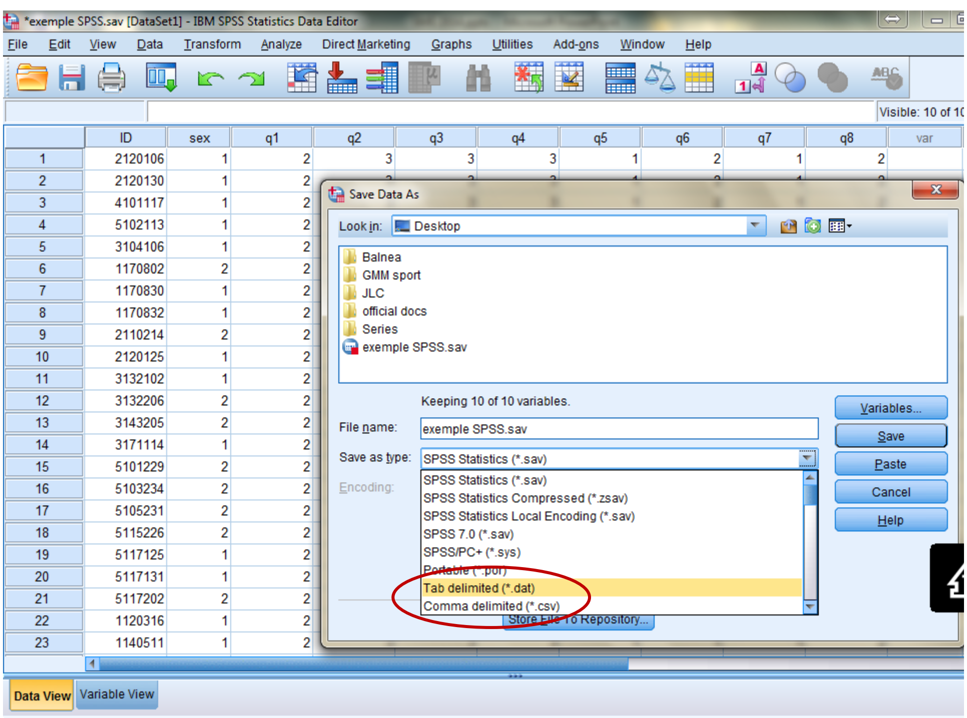
*exemple SPSS.sav [DataSet1] - IBM SPSS Statistics Data Editor
Save Data As
Look in: Desktop
Balnea
GMM sport
JLC
official docs
Series
exemple SPSS.sav
Keeping 10 of 10 variables.
File name: exemple SPSS.sav
Save as type: SPSS Statistics (*.sav)
Encoding:
SPSS Statistics (*.sav)
SPSS Statistics Compressed (*.zsav)
SPSS Statistics Local Encoding (*.sav)
SPSS 7.0 (*.sav)
SPSS/PC+ (*.sys)
Portable (*.por)
Tab delimited (*.dat)
Comma delimited (*.csv)
Variables...
Save
Paste
Cancel
Help
Store File To Repository...
Data View
Variable View
IBM SPSS Statistics Processor is ready
Unicode:ON

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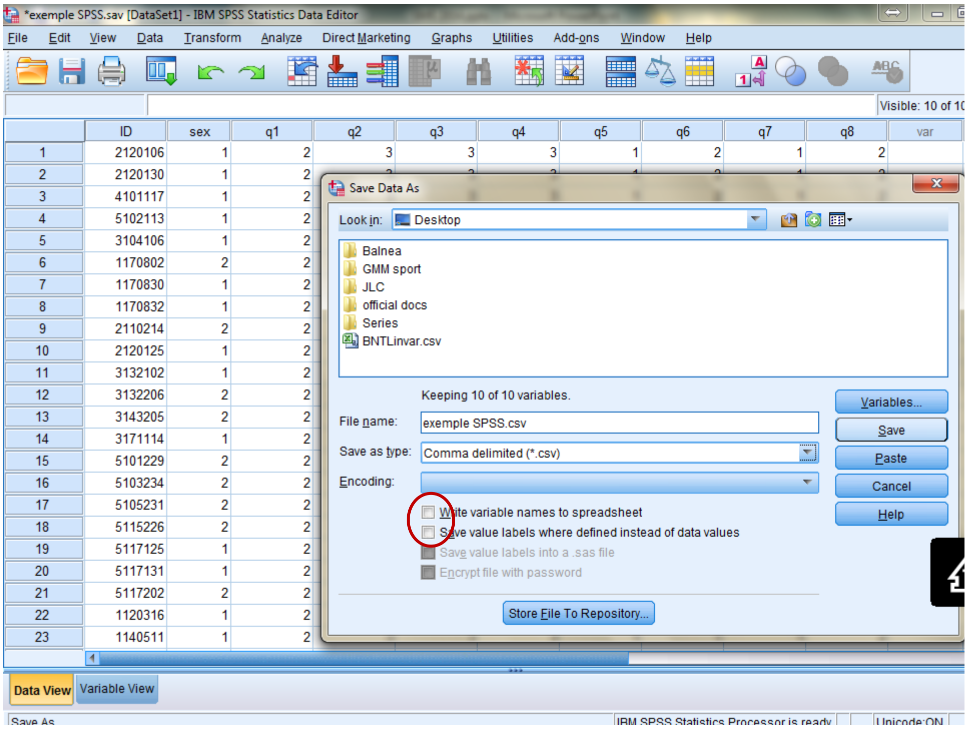
*exemple SPSS.sav [DataSet1] - IBM SPSS Statistics Data Editor
File Edit View Data Transform Analyze Direct Marketing Graphs Utilities Add-ons Window Help
Visible: 10 of 10
ID sex q1 q2 q3 q4 q5 q6 q7 q8 var
Save Data As
Look in: Desktop
Balnea
GMM sport
JLC
official docs
Series
BNTLinvar.csv
Keeping 10 of 10 variables.
File name: exemple SPSS.csv
Save as type: Comma delimited (*.csv)
Encoding:
Write variable names to spreadsheet
Save value labels where defined instead of data values
Save value labels into a .sas file
Encrypt file with password
Store File To Repository...
Variables...
Save
Paste
Cancel
Help
Data View
Variable View
IBM SPSS Statistics Processor is ready
Unicode:ON

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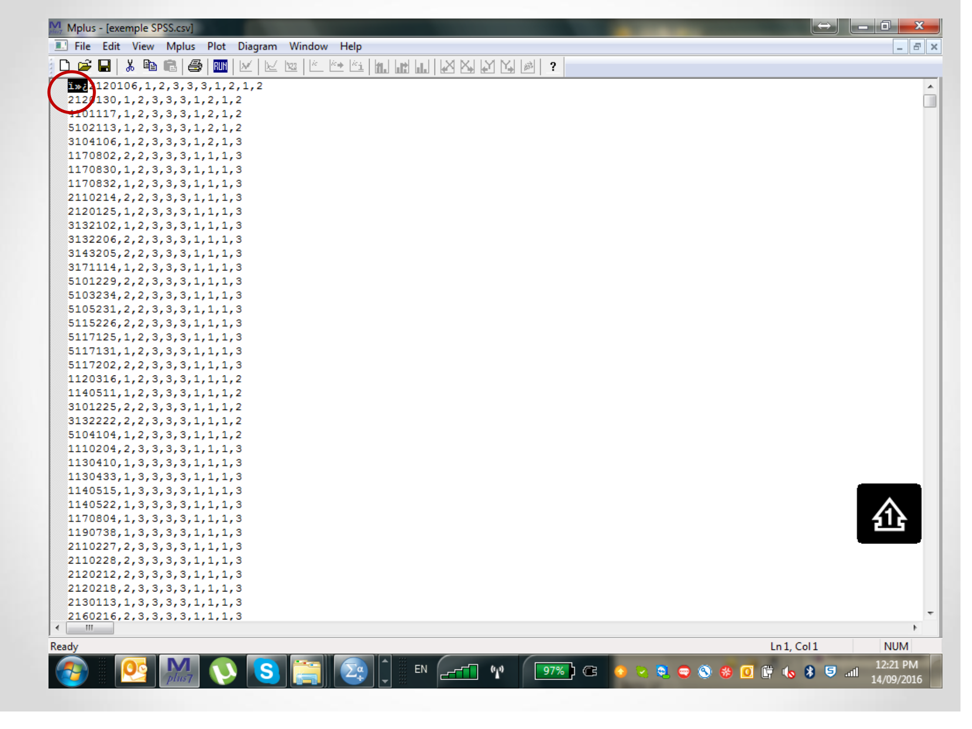
Mplus - [exemple SPSS.csv]
File Edit View Mplus Plot Diagram Window Help
ï»¿2120106,1,2,3,3,3,1,2,1,2
2120130,1,2,3,3,3,1,2,1,2
4101117,1,2,3,3,3,1,2,1,2
5102113,1,2,3,3,3,1,2,1,2
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1170830,1,2,3,3,3,1,1,1,3
1170832,1,2,3,3,3,1,1,1,3
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2110228,2,3,3,3,3,1,1,1,3
2120212,2,3,3,3,3,1,1,1,3
2120218,2,3,3,3,3,1,1,1,3
2130113,1,3,3,3,3,1,1,1,3
2160216,2,3,3,3,3,1,1,1,3
Ready
Ln 1, Col 1
NUM
12:21 PM
14/09/2016

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Title:

Title of the model to be estimated;

! Annotations following "!" are discarded by the program

! The TITLE function is not mandatory.

! All commands end with ";"

! All section titles end with ":"

Data:

File is esemdata.csv;

! The FILE function of the DATA section is used to identify your

! data set. If the data set is in the same folder, then this is fine.

! If the data set is in another folder, then the full link is indicated.

File is D:\DOCUMENTS\LATENT VARIABLE MODELING\esemdata.csv;

VARIABLE:
Names are ID sex q1 q2 q3 q4 q5 q6 q7 q8;
! The NAMES function lists, in order, all variables in the data set.
Usevariables are q1 q2 q3 q4 q5 q6 q7 q8;
! The USEVARIABLES function lists those used in the analysis.
Missing are all (-999);
! The MISSING function identifies the missing data code.
Idvariable = ID;
! The IDVARIABLE identifies the unique identifier.
Auxiliary = sex;
Auxiliary = sex (m);
! Sometimes, one wants to save the results from an analysis to an
! external data file (e.g., scores on the factors). This external data
! file will include all variables included in the analyses + those
! listed as auxiliary. The (m) indicators allows auxiliary variables
! to be taken into account in the missing data process.

ANALYSIS:
TYPE = General;
ESTIMATOR = MLR; ! Or ML, etc.
! MLR is robust to multivariate non-normality
! MLR can be made to be robust to nesting.

MODEL:
!!! This is where everything happens !

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A simple correction for nesting:

VARIABLE:
NAMES = ID sex q1 q2 q3 q4 q5 q6 q7 q8;
USEVARIABLES = q1 q2 q3 q4 q5 q6 q7 q8;
Missing are all (-999);
Idvariable = ID;

CLUSTER = Unit;

Analysis:
TYPE = **COMPLEX**;
ESTIMATOR = MLR;

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MODEL:
!!! This is where everything happens !
OUTPUT:
SAMPSTAT STANDARDIZED MODINDICES CINTERVAL RESIDUAL SVALUES TECH1 TECH3 TECH4 ;

SAMPSTAT: sample descriptive.
STANDARDIZED: Standardized parameter estimates.
CINTERVAL: Confidence intervals for parameter estimates.
RESIDUAL: Residuals for parameter estimates.
MODINDICES: Modification indices.
SVALUES: Starts Values.
TECH1: Parameter specifications and starts values (not for EFA).
TECH3: Correlations and covariances for parameter estimates.
TECH4: Means, Correlations and covariances for the latent variables.

MODEL:

ON: Defines a regression e.g., Y ON X;

WITH: Defines a correlation e.g., X WITH Y;

BY: Defines a factor loading e.g., F1 BY X1 X2 X3;

[ ]: Variable names within brackets define intercepts and means e.g., [X1]; or [F1];

Variable names: By themselves, variable names define variances, uniquenesses and disturbances e.g., X1; or F1;

***:** Is used to request the free estimation of a parameter that would otherwise be constrained e.g., F1 BY X1* X2 X3; or to provide a start value for a parameter e.g., F1 BY X1*.900 X2*.850 X3*800;

@: Is used to constrain a parameter to a specific value e.g., F1 BY X1@1 X2 X3;

(): alphanumeric codes in parentheses following a parameter can be used to constrain parameters to equality, e.g. F1 BY X1* (l1) X2 (l2);

*** WARNING

Data set contains cases with missing on x-variables.

These cases were not included in the analysis.

Number of cases with missing on x-variables: 61

*** WARNING

Data set contains cases with missing on all variables except x-variables. These cases were not included in the analysis.

Number of cases with missing on all variables except x-variables: 30

By explicitly requesting the free Estimation of the variance of the exogenous variables in MODEL: FIML will be activated.

X1 X2 X3;

THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION

Number of Free Parameters	13
Loglikelihood	
H0 Value	-7221.664
H0 Scaling Correction Factor for MLR	1.6859
H1 Value	-7221.604
H1 Scaling Correction Factor for MLR	1.6256
Information Criteria	
Akaike (AIC)	14469.328
Bayesian (BIC)	14537.107
Sample-Size Adjusted BIC (n* = (n + 2) / 24)	14495.811

Chi-Square Test of Model Fit

Value	97.470*
Degrees of Freedom	19
P-Value	0.0000
Scaling Correction Factor for MLR	1.4070

* The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used for chi-square difference testing in the regular way. MLM, MLR and WLSM chi-square difference testing is described on the Mplus website. MLMV, WLSMV, and ULSMV difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.055
90 Percent C.I.	0.045 0.066
Probability RMSEA <= .05	0.203

CFI/TLI

CFI	0.947
TLI	0.922

SRMR (Standardized Root Mean Square Residual)

Value	0.040

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
POSF **BY**				
Q1	1.000	0.000	999.000	999.000
Q2	1.175	0.061	19.371	0.000
POSF **ON**				
ZSELFEST	0.225	0.027	8.272	0.000
ZDEPRESS **ON**				
ZSELFEST	-0.346	0.037	-9.418	0.000
ZSELFEST **WITH**				
ZLONELY	-0.246	0.032	-7.688	0.000
Means				
Intercepts				
Variances				
Residual Variances				
POSF	0.936	0.050	18.751	0.000
Q1	0.883	0.049	17.982	

Unstandardized loading

Unstandardized regression (b)

Covariance

Means, Intercept, Variances

Residuals (Disturbances, uniquenesses)

STANDARDIZED MODEL RESULTS
STDYX Standardization

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
POSF BY				
Q1	0.583	0.028	21.101	0.000
Q2	0.727	0.023	31.835	0.000
POSF ON				
ZSELFEST	0.225	0.027	8.472	0.000
ZDEPRESS ON				
ZSELFEST	-0.346	0.034	-10.305	0.000
ZSELFEST WITH				
ZLONELY	-0.246	0.028	-8.875	0.000
Means				
Intercepts				
Variances				
Residual Variances				
POSF	0.936	0.013	72.868	0.000
Q1	0.882	0.022	39.564	0.000

Standardized loading

Standardized regression (β)

Correlation (r)

NA

Standardized Residuals

R-SQUARE

Observed Variable	Estimate	S.E.	Est./S.E.	
ZGPA	0.064	0.013	4.948	
ZDEPRESS	0.118	0.022	5.297	
Q1	0.340	0.032	10.550	
Q2	0.529	0.033	15.917	0.000
Q3	0.430	0.035	12.212	0.000
Q4	0.551	0.035	15.578	0.000
Q5	0.753	0.071	10.544	0.000

% Explained variance

Communality (h^2)

Standardized or Unstandardized?

Unstandardized:

- Means and Intercepts
- Variances

Both:

- Regressions (on)

Standardized:

- Loadings (by)
- Residuals
- Correlations (with)
- Residual variances

CONFIDENCE INTERVALS OF MODEL RESULTS

	Lower .5%	Lower 2.5%	Lower 5%	Estimate	Upper 5%	Upper 2.5%	Upper .5%
ZGPA ON							
ZSELFEST	0.155	0.172	0.180	0.225	0.270	0.278	0.295
ZDEPRESS	-0.141	-0.122	-0.112	-0.060	-0.009	0.001	0.021
ZDEPRESS ON							
ZSELFEST	-0.441	-0.418	-0.407	-0.346	-0.286	-0.274	-0.252
ZLONELY	-0.087	-0.068	-0.059	-0.009	0.041	0.050	0.069

[…]

CONFIDENCE INTERVALS OF STANDARDIZED MODEL RESULTS

STDYX Standardization

	Lower .5%	Lower 2.5%	Lower 5%	Estimate	Upper 5%	Upper 2.5%	Upper .5%
ZGPA ON							
ZSELFEST	0.157	0.173	0.181	0.225	0.269	0.277	0.293
ZDEPRESS	-0.142	-0.122	-0.112	-0.060	-0.008	0.001	0.021

EFA Vs CFA

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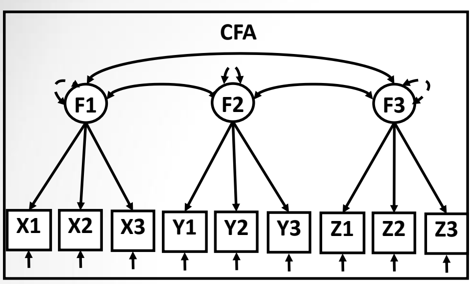

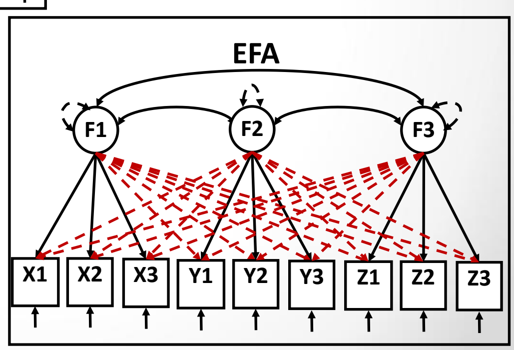

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- Exploratory
- Dust bowl empiricism
- Goodness-of-fit
- Relations among latent variables corrected for measurement errors
- Connection to the SEM framework
- Changes the meaning of the constructs
- Parsimony

- Exploratory
- Dust bowl empiricism
- Goodness-of-fit
- Relations among latent variables corrected for measurement errors
- Connection to the SEM framework
- Changes the meaning of the constructs
- Parsimony

EFA Versus CFA

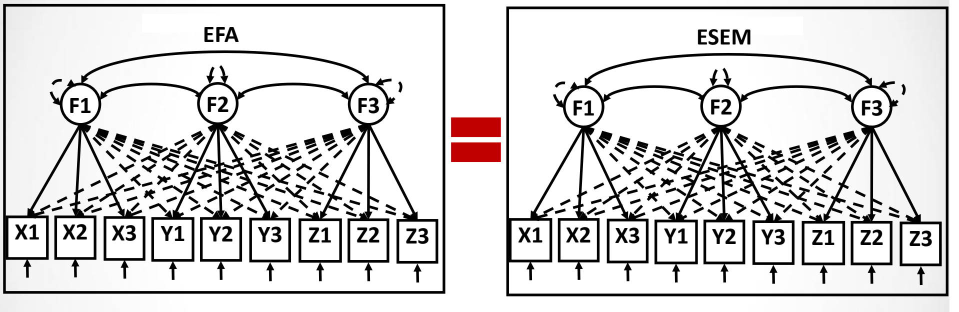
- There is nothing inherently "confirmatory" or "exploratory" about EFA or CFA.
- The method can be "exploratory" (based on the estimation of all relations between indicators and constructs) or confirmatory (based on the estimation of a subset of relations).
- However, both approaches can still be used to address "confirmatory" or "exploratory" research objectives.
  - One can do EFA with clear a priori expectations
  - One can capitalize on chance in CFA (modification indices, post hoc changes, etc.).
- The only true difference is that CFA relies on the independent cluster assumption, whereas EFA incorporates cross-loadings.
- Target rotation makes ESEM fully "confirmatory"

- Exploratory
- Dust bowl empiricism
- Goodness-of-fit
- Relations among latent variables corrected for measurement errors
- Connection to the SEM framework
- Changes the meaning of the constructs
- Parsimony

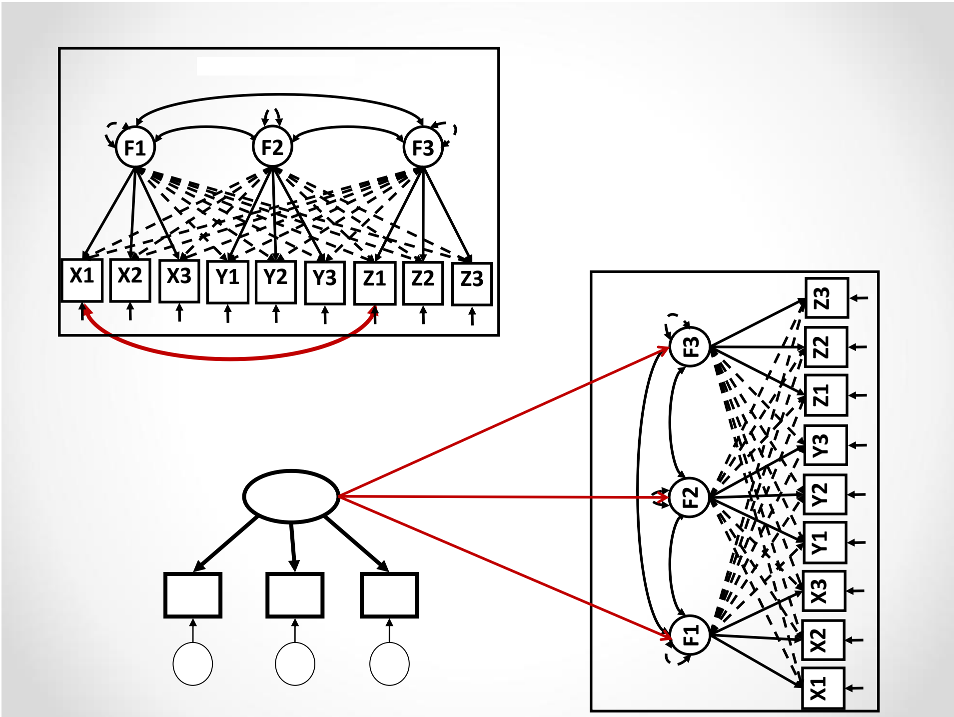
EFA Versus CFA

- Goodness-of-fit indices have been available for EFA models for a while now, just not in SPSS.
- ESEM has recently "fully" connected EFA to the SEM framework.

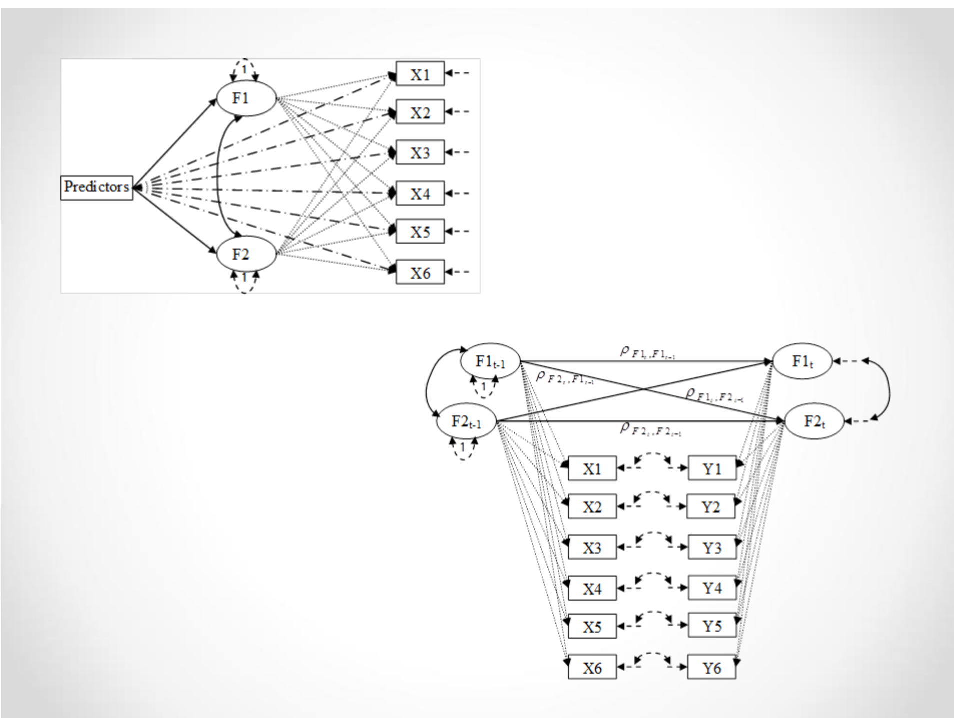
EFA versus ESEM



ESEM is a connection created between EFA and SEM


F1
F2
F3
X1
X2
X3
Y1
Y2
Y3
Z1
Z2
Z3

41


Predictors
F1
F2
1
X1
X2
X3
X4
X5
X6
$F1_{t-1}$
$F2_{t-1}$
$F1_t$
$F2_t$
Y1
Y2
Y3
Y4
Y5
Y6

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EFA

```
ANALYSIS:
TYPE = EFA 1 4;
ESTIMATOR = MLR;
ROTATION = Geomin (.5);
PARALLEL = 1000;
```

ESEM

```
ANALYSIS:
TYPE = general;
ESTIMATOR = MLR;
ROTATION = Geomin (.5);
MODEL:
F1-F2 BY q1 q2 q3 q4 q5 q6
q7 q8 (*1);
```

In the model section, identify the desired number of factors (F1-F2, or F1-F3), with the names being somewhat arbitrary.

All factors forming a single "set" of ESEM factors (with cross loadings being freely estimated within one set, and not across sets) have the same label in parenthesis at the end (*1).

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```
ANALYSIS:
TYPE = general;
ESTIMATOR = MLR;
ROTATION = Geomin (.5);
MODEL:

F1-F2 BY q1 q2 q3 q4 q5 q6 q7 q8 (*1);

F3-F4 BY se1 se2 se3n se4 se5n se6 se7 se8n se9n se10n (*2);

q1 WITH se1;
F1-F2 ON F3-F4;
! And so on
```

Set #1

Set #2

44

Rotations

Geomin (epsilon): A newly developed form of rotation that performs relatively well and is the default in Mplus. The default epsilon values varies across models and we have found that this does not perform so well. We recommend a value of .5 to maximally reduce factor correlations.

Crawford-Ferguson (CF) family:

Minimizes variable complexity (smaller cross-loadings, greater factor correlations)

Minimizes factor complexity (smaller factor correlations, greater cross-loadings)

CF-Quartimax (= to direct quartimin)
CF-Varimax
CF-Equamax
CF-Parsimax
CF-Facparsim

Oblimin, Promax, Varimax: As in SPSS. Seldom used.

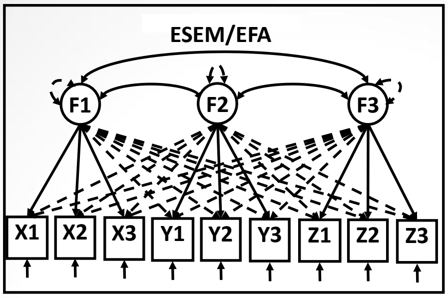
Geomin, all of the CF, and Oblimin are oblique by default, but can be specified as orthogonal.

ROTATION = CF-QUARTIMAX (Oblique);
ROTATION = CF-QUARTIMAX (Orthogonal);

Target Rotation

Target rotation, and bifactor-target rotation, provides a way to use a priori (confirmatory) specifications for factor rotations, with all cross-loadings freely estimated, but targeted to be as close to a pre-specified value (typically 0) as possible.

~0



Analysis:
ESTIMATOR = ML;
ROTATION = TARGET;
Model:
F1 BY X1 X2 X3 Y1~0 Y2~0 Y3~0 Z1~0 Z2~0 Z3~0 (*1);
F2 BY Y1 Y2 Y3 X1~0 X2~0 X3~0 Z1~0 Z2~0 Z3~0 (*1);
F3 BY Z1 Z2 Z3 X1~0 X2~0 X3~0 Y1~0 Y2~0 Y3~0 (*1);

- Exploratory
- Dust bowl empiricism
- Goodness-of-fit
- Relations among latent variables corrected for measurement errors
- Connection to the SEM framework
- Changes the meaning of the constructs
- Parsimony

How to assess the meaning of a construct ?

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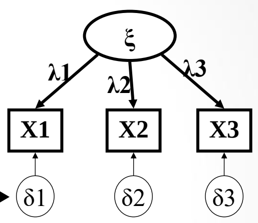
From Psychometrics, you may recall that:

$$\sigma^2{}_X = \sigma^2{}_T + \sigma^2{}_E$$

Reliability, $X = \Lambda\boldsymbol{\xi} + \boldsymbol{\delta}$



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Reliability or Validity

- Factor loadings and uniquenesses are used to separate sources of variance that are unique to the items (uniquenesses, which include random measurement error), from those that are shared with other items (factor loadings, reflecting true score variance).
- But this is all related to reliability.
- Aren't we supposed to assess the meaning of a construct from analyses of validity?
- What is validity ?

$$\sigma^2_X = \sigma^2_T + \sigma^2_E$$

$$\sigma^2_X = \sigma^2_C + \sigma^2_{se} + \sigma^2_{re}$$

- Nomological network
- Relations with other constructs assessing the same, similar, related, or unrelated constructs.
- SO: The meaning of a construct lies in how it relates to other constructs, not in how it relates to its indicators.

- Simulation studies shows that ESEM/EFA best recovers true population correlations and regressions (i.e., relations among constructs) when even negligible cross-loadings (.100) exist in the population model yet remains unbiased when ICM assumptions hold (when there are no cross loadings).
- This means that ESEM solutions should be favored whenever factor correlations differ across models, as long as the fit remains similar.

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Vol. 41 No. 6, September 2015 1561–1577
DOI: 10.1177/0149206315591075

Editorial Commentary

Bayesian Structural Equation Modeling With Cross-Loadings and Residual Covariances: Comments on Stromeyer et al.

Tihomir Asparouhov
Bengt Muthén
Muthén & Muthén
Alexandre J. S. Morin
Australian Catholic University

Structural Equation Modeling: A Multidisciplinary Journal, 00: 1–13, 2018

ISSN: 1070-5511 print / 1532-8007 online
DOI: https://doi.org/10.1080/10705511.2018.1444993

Routledge
Taylor & Francis Group

Check for updates

Comparing Exploratory Structural Equation Modeling and Existing Approaches for Multiple Regression with Latent Variables

Yujiao Mai,[1,2] Zhiyong Zhang,[2] and Zhonglin Wen[1]
[1]*South China Normal University*
[2]*University of Notre Dame*

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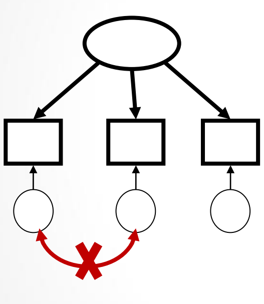
- Exploratory
- Dust bowl empiricism
- Goodness-of-fit
- Relations among latent variables corrected for measurement errors
- Connection to the SEM framework
- Changes the meaning of the constructs
- Parsimony

Parcimony?

Yes

1. When both models provide equivalent results, parsimony should be favored
2. Goodness of fit assessment should not be the sole source of information in the selection of the best model
3. Goodness-of-fit indices corrected for parsimony (TLI, RMSEA) should be given more weight.

Construct-Relevant Psychometric Multidimensionality



Conditional independence:
« Assumes » that all of the covariance will be absorbed by the factors (no residual correlations among the uniquenesses) .

Psychometric Multidimensionality

When the indicators tap into more than one source of true score variance.

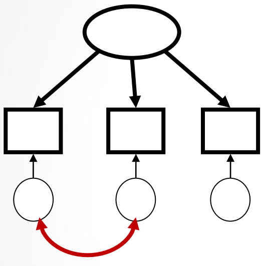
1. Construct-Irrelevant Psychometric Multidimensionality:
   - Involves the assessment of sources of multidimensionality that have no substantive interest.
   - Involves the assessment of sources of multidimensionality that need to be controlled to avoid letting them bias parameter estimates.
   - Methodological artefacts: Wording effects, rater effects, etc.
2. Construct-Relevant Psychometric Multidimensionality:
   - Involves the assessment of sources of multidimensionality that have a substantive interest.
   - Involves the assessment of sources of multidimensionality that need to be controlled to avoid letting them bias parameter estimates.
   - Involves the presence of valid associations between the indicator and more than one construct.

Construct-Irrelevant Psychometric Multidimensionality:

When indicators share something that is not related to the construct of interest.

1. Negatively-worded items
2. Type of informant: Self-report, parental report, peer reports, supervisor reports.
3. Items with parallel wording

Solution 1: Correlated Uniquenesses



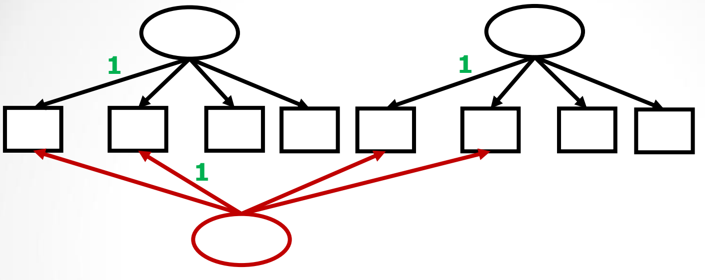
```
MODEL:
F1 BY X1* X2 X3;
F1@1;
X1 WITH X2;
```

Schweizer (2012) describes correlated uniquenesses, especially *post hoc* ones, as a disaster for applied research as they change the meaning of the factors.

Correlated uniquenesses simply "float around", providing an implicit control for multidimensionality while bringing nothing new to the model.

They simply take something out.

Solution 2: Method Factor



- Method factors provide an explicit control for multidimensionality by bringing something new to the model
- It makes no sense to allow the method factors to correlate with the trait factors.
- Be careful not to "double dip": Do not re-use referent indicators.
- Not always realistic to use method factors (e.g., to control for parallel wording among 10 pairs of items).

```
MODEL:
F1 BY X1* X2 X3 X4;
F2 BY Y1* Y2 Y3 Y4;
F1@1;
F2@1;
F1 WITH F2* ;
MF BY X1* X2 Y1 Y2;
MF@1;
MF WITH F1@0;
MF WITH F2@0;
```

```
MODEL:
F1 BY X1@1 X2 X3 X4;
F2 BY Y1@1 Y2 Y3 Y4;
F1*;
F2*;
F1 WITH F2* ;
MF BY X1* X2@1 Y1 Y2;
MF*;
MF WITH F1@0;
MF WITH F2@0;
```

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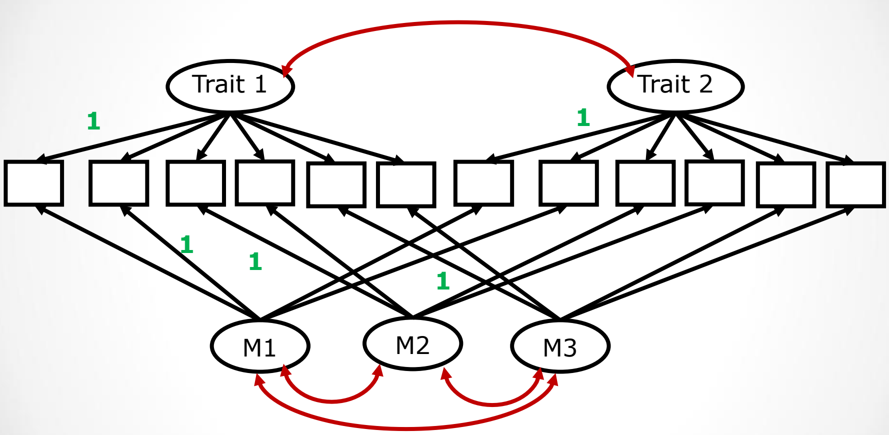
Multitrait-Multimethod



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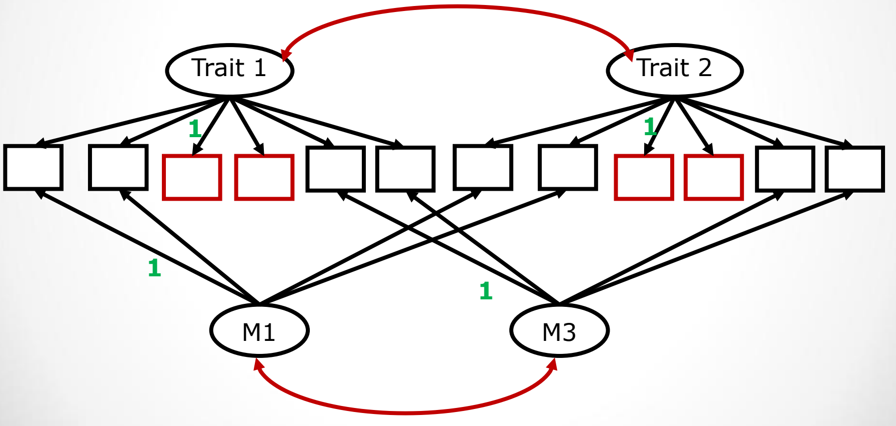
Correlated Traits Correlated Methods – 1: CT-C(M-1)



Psychometric Multidimensionality

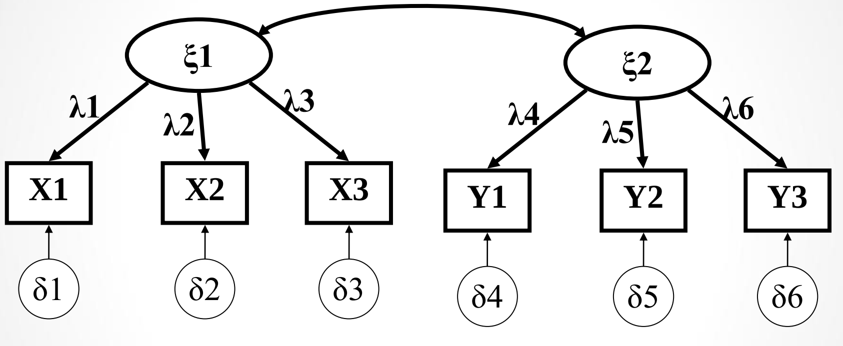
When the indicators tap into more than one source of true score variance.

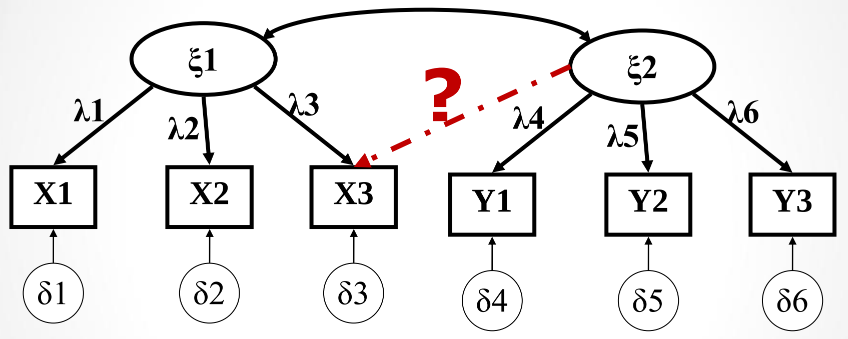
1. Construct-Irrelevant Psychometric Multidimensionality:
   - Involves the assessment of sources of multidimensionality that have no substantive interest.
   - Involves the assessment of sources of multidimensionality that need to be controlled to avoid letting them bias parameter estimates.
   - Methodological artefacts: Wording effects, rater effects, etc.
2. Construct-Relevant Psychometric Multidimensionality:
   - Involves the assessment of sources of multidimensionality that have a substantive interest.
   - Involves the assessment of sources of multidimensionality that need to be controlled to avoid letting them bias parameter estimates.
   - Involves the presence of valid associations between the indicator and more than one construct.

Construct-Relevant Psychometric Multidimensionality:

When indicators share something that is substantively relevant.

1. **Conceptually-related constructs**: Exploratory Structural Equation Modeling (ESEM)
2. **Hierarchically-ordered constructs**: Higher-Order Factor Modeling, Bifactor Modeling.
3. **Both**: Higher-order-ESEM, Bifactor- ESEM





Construct Relevant Psychometric Multidimensionality

The expectation that each item will correspond to a single source of true score variance that is an implicit part of CFA models is unrealistic, and has never been part of psychometric test theory.

Conceptually-Related Constructs

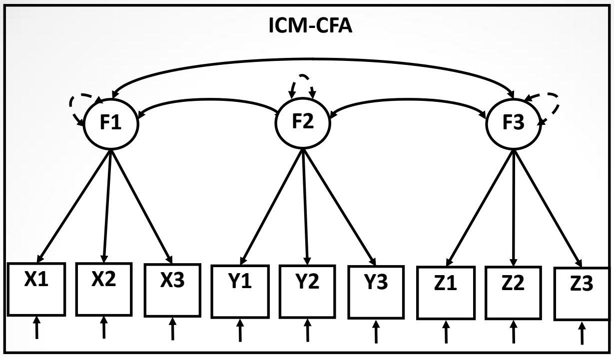
- Insomnia: Depression, anxiety, Drug Abuse, Burnout, etc.
- "I am Good Looking":
  - Physical Self-Concept
  - Peer-Self-Concept, *beauty is partly in the eye of the beholder*.

Construct Relevant Psychometric Multidimensionality

Hierarchically-Ordered Constructs

- "I am Good Looking":
  - Physical Self-Concept
  - Peer-Self-Concept
  - **Global Self-Concept**

Hierarchically-Ordered Constructs



! Model section of Mplus input

MODEL:

F1 BY X1 X2 X3 X4;

F2 BY Y1 Y2 Y3 Y4;

F3 BY Z1 Z2 Z3 Z4;

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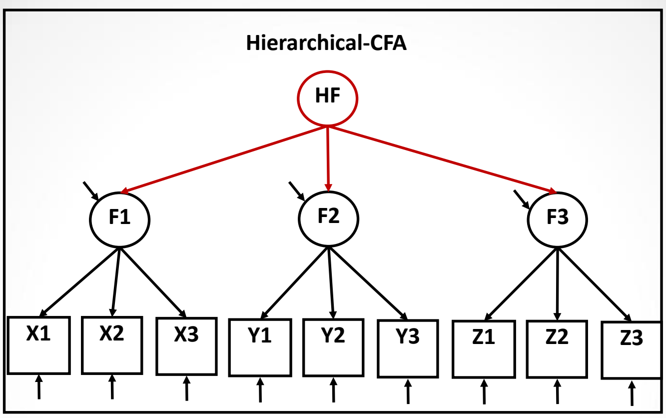

MODEL:

F1 BY X1 X2 X3 X4;

F2 BY Y1 Y2 Y3 Y4;

F3 BY Z1 Z2 Z3 Z4;

HF BY F1 F2 F3;

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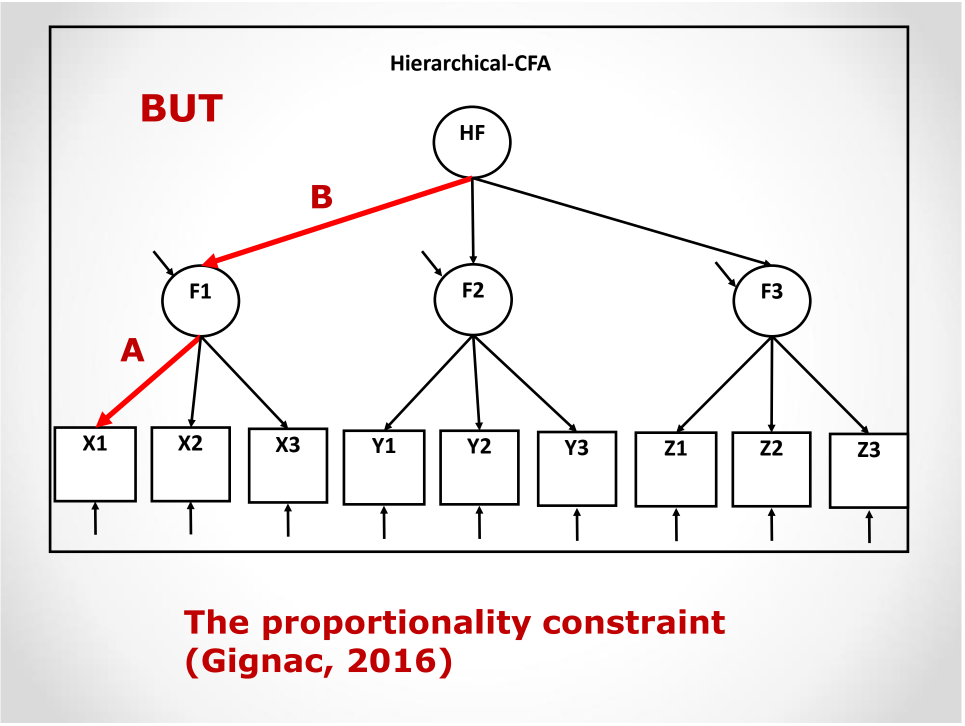
Hierarchical-CFA
BUT
HF
B
F1
F2
F3
A
X1
X2
X3
Y1
Y2
Y3
Z1
Z2
Z3
The proportionality constraint
(Gignac, 2016)

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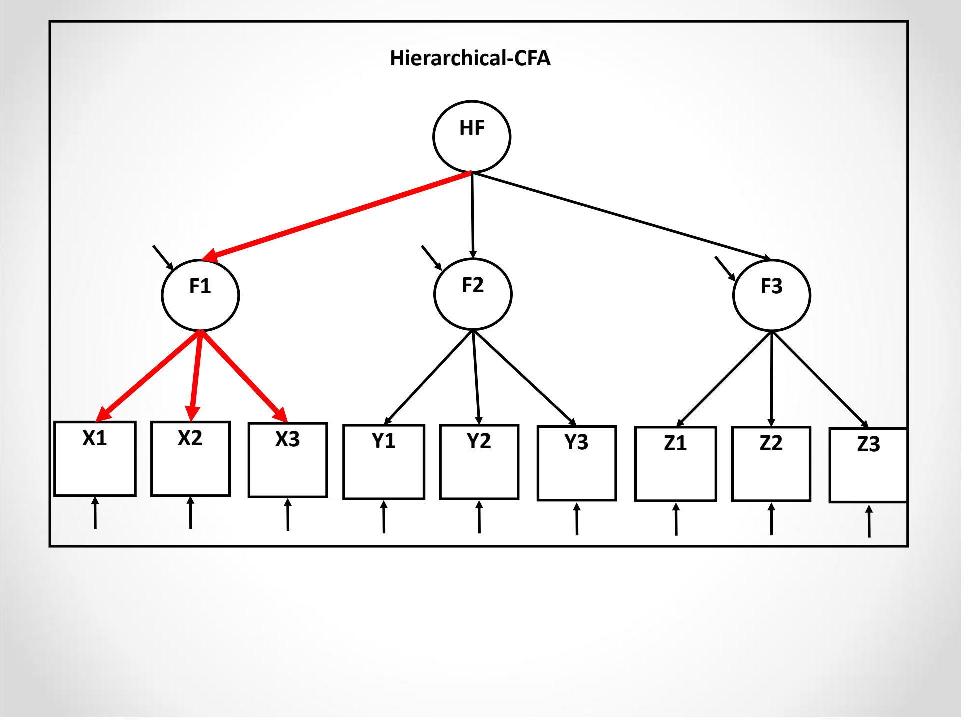
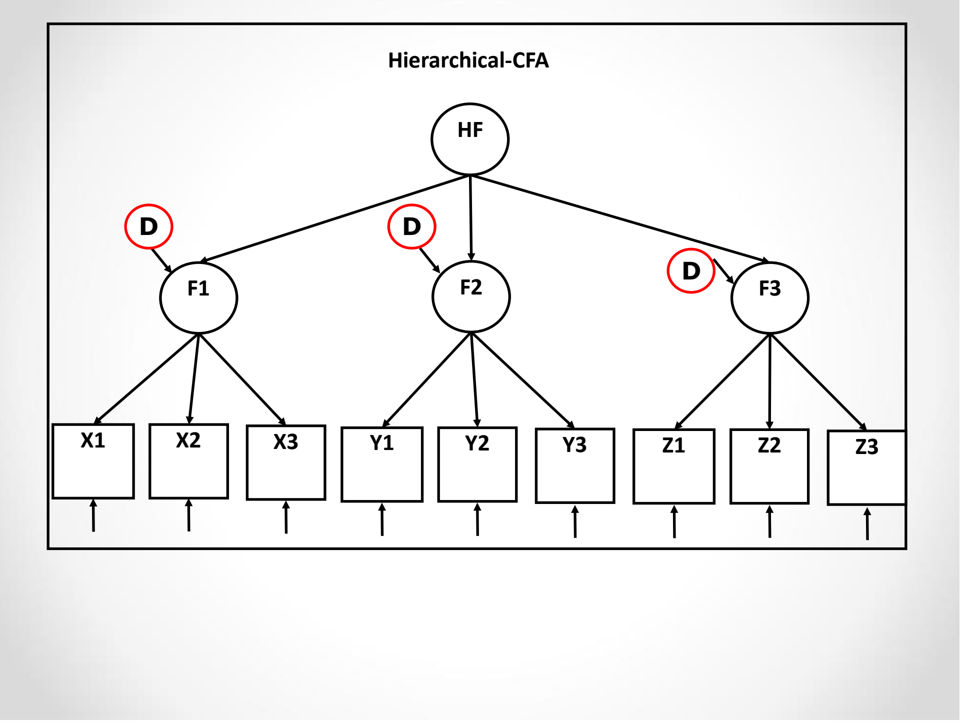
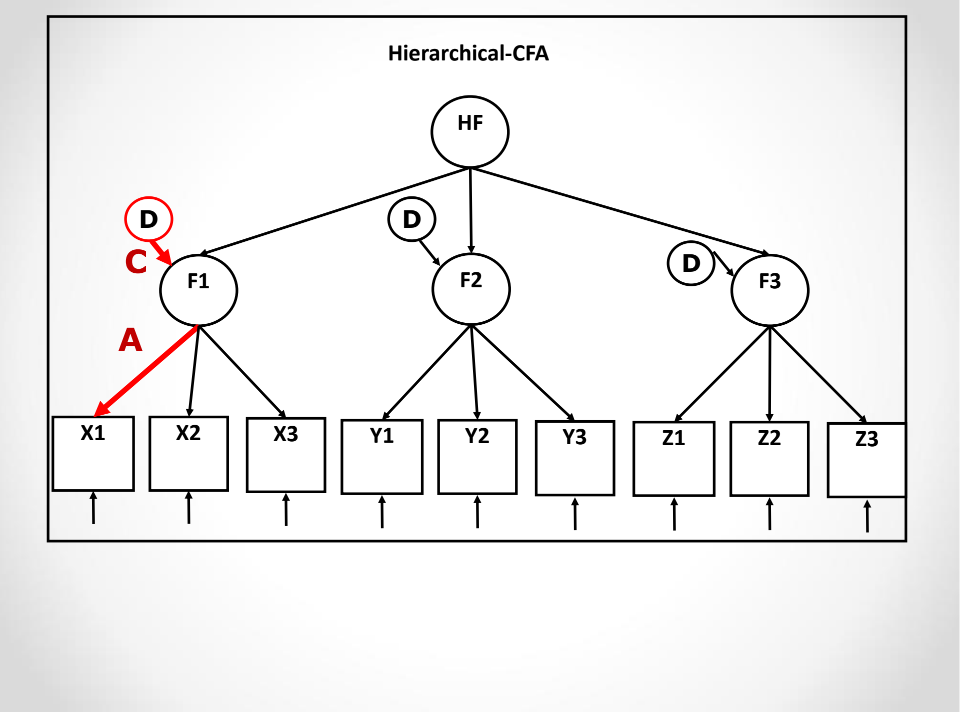

Hierarchical-CFA
HF
F1
F2
F3
X1
X2
X3
Y1
Y2
Y3
Z1
Z2
Z3

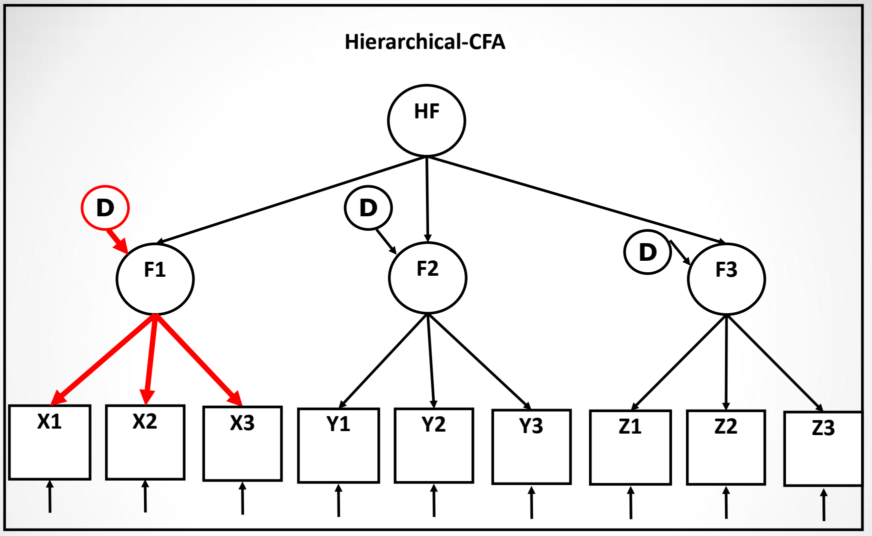
76


Hierarchical-CFA
HF
D
D
D
F1
F2
F3
X1
X2
X3
Y1
Y2
Y3
Z1
Z2
Z3

77


Hierarchical-CFA
HF
D
D
D
C
F1
F2
F3
A
X1
X2
X3
Y1
Y2
Y3
Z1
Z2
Z3

78



Proportionality Constraint

The ratio of global (A*B) to specific (A*C) variance will be the same for all items associated with the same first order factor:

AB/AC = B/C

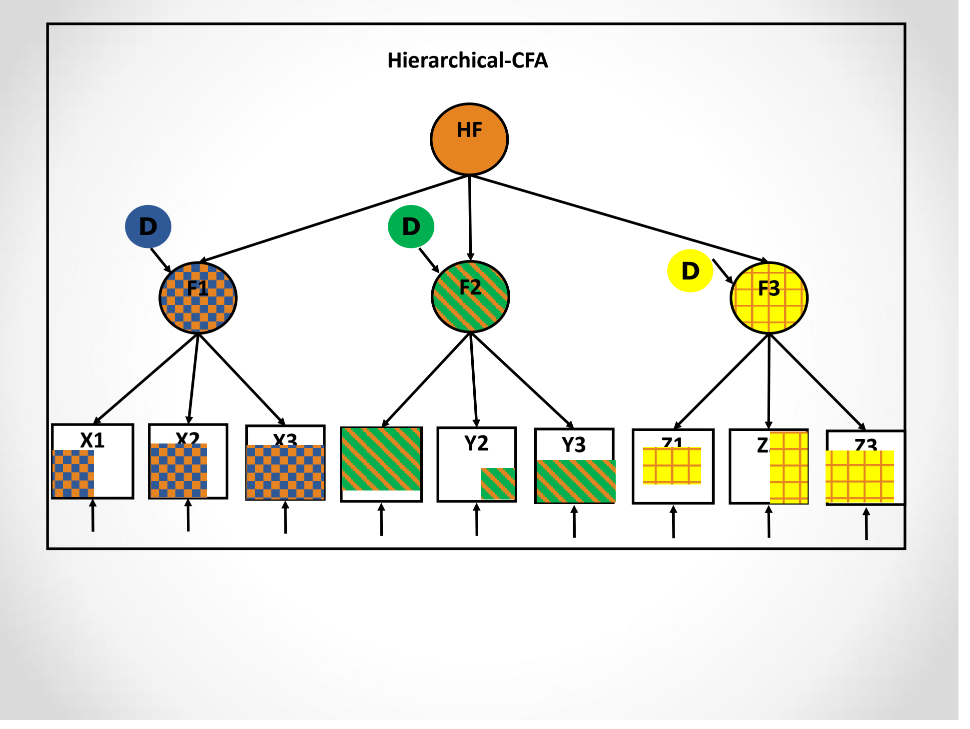
Lets say that A (loading of item 1 on Factor 1) is 2, B is also 2, and C is 3.

2 *2 / 2 * 3 = 4 / 6 = .6667 (67%)

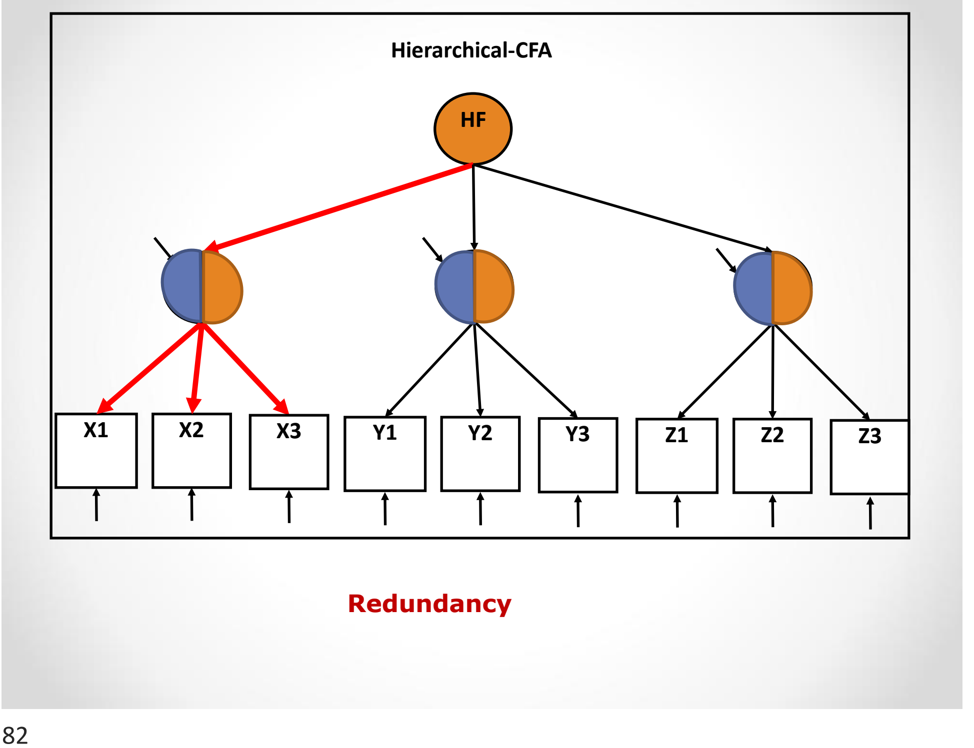
Now lets say that A is 3 for item 2, and 1.5 for item 3.

3*2 / 3*3 = 6 / 9 = .6667 (67%)

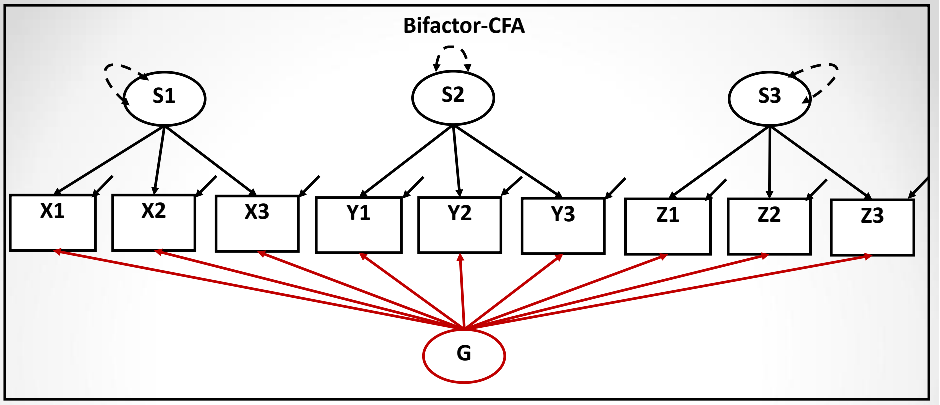
1.5*2 / 1.5 *3 = 3 / 4.5 = .6667 (67%)


Hierarchical-CFA
HF
D
D
D
F1
F2
F3
X1
Y2
Y3

81


Hierarchical-CFA
HF
X1
X2
X3
Y1
Y2
Y3
Z1
Z2
Z3
Redundancy

82



Model:

FG BY X1* X2@1 X3 X4 Y1 Y2 Y3 Y4 Z1 Z2 Z3 Z4;

FS1 BY X1 X2 X3 X4;

FS2 BY Y1 Y2 Y3 Y4;

FS3 BY Z1 Z2 Z3 Z4;

FG WITH FS1@0 FS2@0 FS3@0;

FS1 WITH FS2@0 FS3@0;

FS2 WITH FS3@0;

Bifactor Models

Because of their orthogonality, bifactor models partition the total covariance among the items into a G component underlying all items, and *f*-1 S components explaining the residual covariance not explained by the G-factor.

Conceptually-Related Constructs

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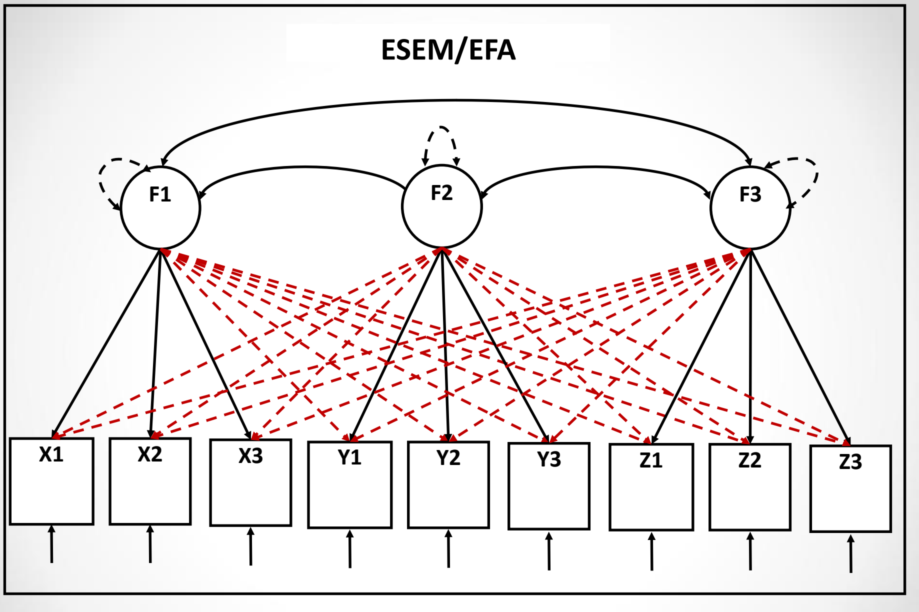

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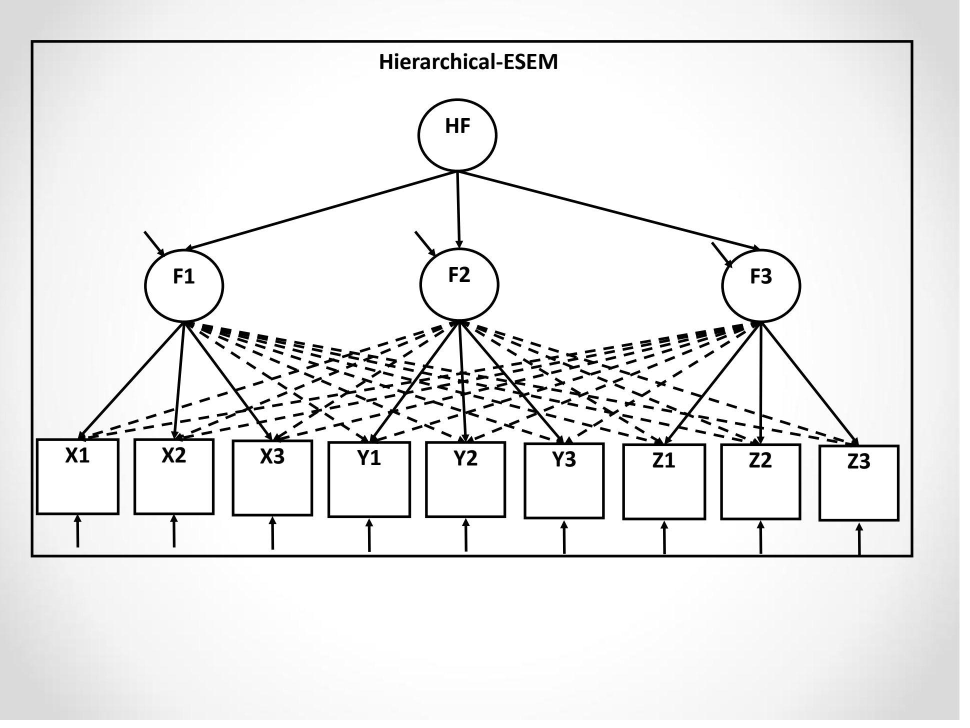
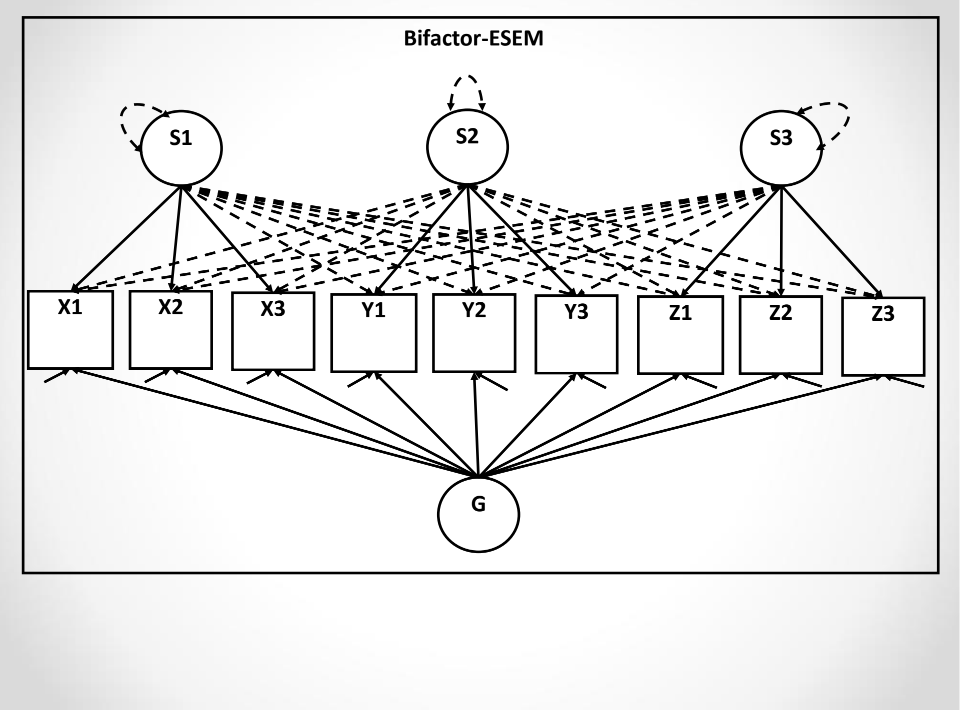
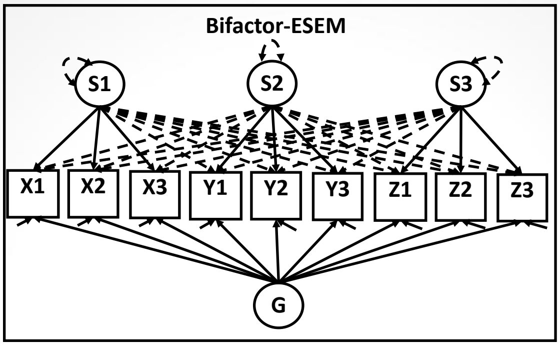

Hierarchical-ESEM
HF
F1
F2
F3
X1
X2
X3
Y1
Y2
Y3
Z1
Z2
Z3

87

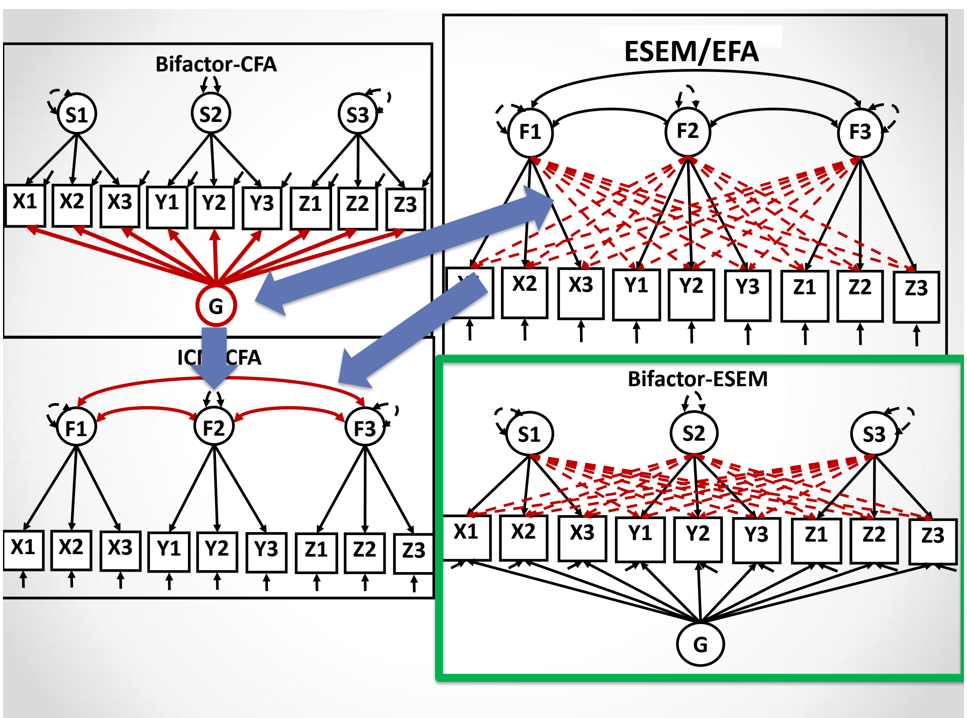

Bifactor-ESEM
S1
S2
S3
X1
X2
X3
Y1
Y2
Y3
Z1
Z2
Z3
G

88



Analysis:
ESTIMATOR = ML;
ROTATION = TARGET (orthogonal);
Model:
FG BY X1 X2 X3 X4 Y1 Y2 Y3 Y4 Z1 Z2 Z3 Z4 (*1);
FS1 BY X1 X2 X3 X4 Y1~0 Y2~0 Y3~0 Y4~0 Z1~0 Z2~0 Z3~0 Z4~0 (*1);
FS2 BY Y1 Y2 Y3 Y4 X1~0 X2~0 X3~0 X4~0 Z1~0 Z2~0 Z3~0 Z4~0 (*1);
FS3 BY Z1 Z2 Z3 Z4 X1~0 X2~0 X3~0 X4~0 Y1~0 Y2~0 Y3~0 Y4~0 (*1);

89



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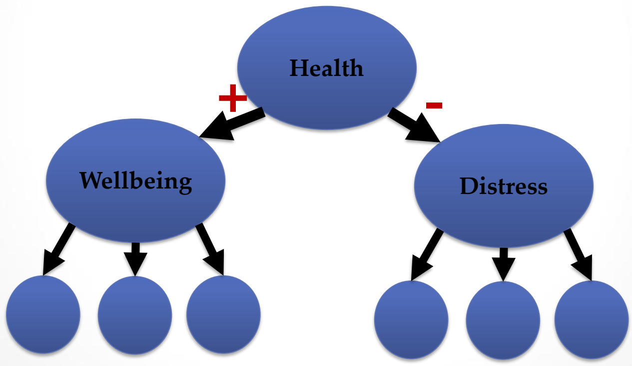
Application of the framework:

1. CFA versus ESEM:
   1. Goodness of fit.
   2. Main loadings: Well-defined factors ?
   3. Cross-loadings: Small enough? Larger ones suggestive of the presence of an underlying global factor ?
   4. Factor correlations: Substantially reduced with ESEM?
2. CFA versus Bifactor-CFA:
   1. Goodness of fit.
   2. Well-defined G-factor? S-factors?
3. Bifactor-ESEM:
   1. Goodness of fit.
   2. Main loadings: Well-defined G-factor ? S-factor?
   3. Cross-loadings: Redued when compared to ESEM?
4. All S-factors do not have to be well-defined.

Psychological Health

Morin, A.J.S., Boudrias, J.-S., Marsh, H.W., Madore, I., & Desrumaux, P. (2016). Further reflections on disentangling shape and level effects in person-centered analyses: An illustration aimed at exploring the dimensionality of psychological health. *Structural Equation Modeling*, *23*, 438-454.

The World Health Organization (2014) defines psychological health as a state characterized not only by the absence of psychological distress, but also by the presence of psychological wellbeing.



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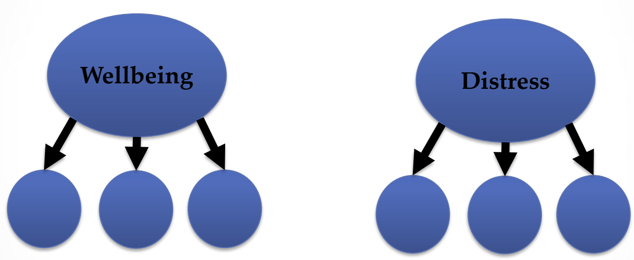
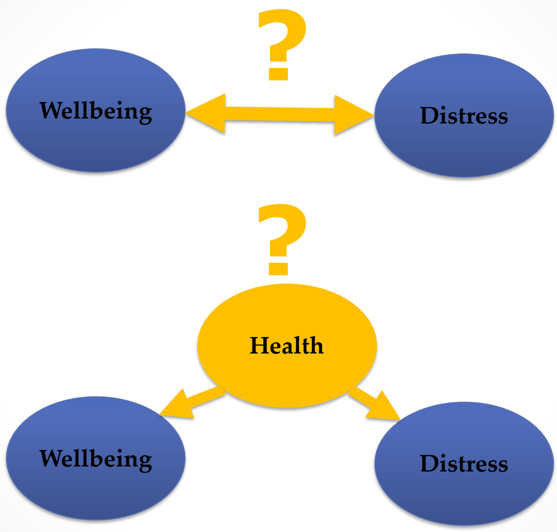


94



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	Harmony (λ)	Serenity (λ)	Involvement (λ)	Irritability (λ)	Anx./Dep. (λ)	Distance (λ)	G-Factor (λ)
D1	.118**	-.060*	.067**	**.585****	-.114**	.042	**-.598****
D8	-.416**	.222**	.262**	**.354****	.059	.047	**-.506****
D12	-.328**	.179**	.127**	**.410****	.066	-.025	**-.543****
D5	.099**	-.019	.131**	**.493****	-.046	-.029	**-.694****
D22	-.131**	.089**	-.038	**.309****	.304**	.110*	**-.613****
D15	.166**	-.121**	-.033	**.412****	.075*	.032	**-.722****
D2	-.138**	.115**	.144**	**.141****	.109**	.082**	**-.680****
D14	.226**	-.123**	.135**	.011	**.256****	-.003	**-.810****
D13	.138**	-.086**	.028	-.046	**.324****	-.177**	**-.689****
D20	.022	-.056**	.126**	.032	**.178****	-.019	**-.858****
D16	.081**	-.022	.119**	-.018	**.174****	.095**	**-.879****
D4	.131**	-.065**	.017	.089**	**.058**	-.023	**-.783****
D21	.202**	-.211**	.132**	.097**	**.239****	.125**	**-.706****
D10	.039	.085**	.169**	-.081**	**.075**	.140**	**-.890****
D23	.089**	-.088**	-.027	.247**	**.382****	.145**	**-.669****
D11	-.222**	.130**	.219**	-.029	**.292****	.032	**-.761****
D19	.055**	.205**	-.241**	.063**	.054*	**.383****	**-.744****
D9	.051*	.104**	-.006	-.032	.011	**.359****	**-.792****
D7	.004	.168**	.092**	.084**	.004	**.248****	**-.803****
D18	-.011	.180**	-.076**	.008	.152**	**.179****	**-.738****
D17	.072**	.128**	-.266**	.066*	.301**	**.045**	**-.745****
D6	.085**	.121**	-.039	.110**	.062**	**.308****	**-.748****
D3	.121**	.145**	-.137**	.017	-.109**	**.241****	**-.736****

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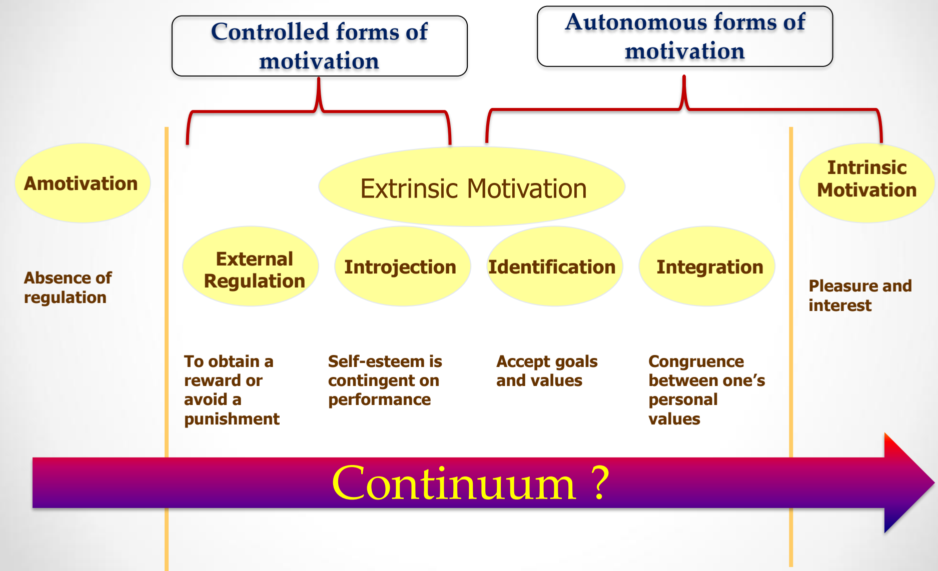
	Harmony (λ)	Serenity (λ)	Involvement (λ)	Irritability (λ)	Anx./Dep. (λ)	Distance (λ)	G-Factor (λ)
W11	**.495****	.142**	.159**	-.181**	.025	.024	**.371****
W18	**.665****	.075**	-.138**	-.218**	.055*	.060*	**.388****
W9	**.338****	.170**	.090**	.033	.142**	-.020	**.628****
W10	**.313****	.286**	.107**	.005	.176**	.121**	**.587****
W5	**.442****	.041	.066**	.083**	.072*	-.027	**.607****
W12	**.273****	.246**	.404**	-.051	-.099*	.076	**.293****
W21	**.326****	.141**	.084**	.134**	-.024	.016	**.358****
W23	.021	**.464****	.007	.089**	-.038	.074**	**.605****
W24	-.027	**.394****	.196**	.041	.16**	.347**	**.629****
W22	.043*	**.483****	.028	.102**	.020	.230**	**.685****
W25	.017	**.430****	.105**	.129**	.145**	.251**	**.743****
W17	.082**	**.431****	.189**	-.059*	.072*	.173**	**.518****
W4	.089**	**.369****	.107**	.091**	.078**	.148**	**.672****
W15	.185**	**.670****	-.064**	.059*	-.201**	-.207**	**.443****
W16	.076**	**.486****	-.016	-.241**	.076*	.059	**.346****
W7	.154**	**.596****	.030	.031	-.318**	-.134**	**.357****
W19	.093**	**.542****	-.155**	-.018	-.012	-.115**	**.318****
W3	.049*	-.011	**.690****	.128**	-.043	.144**	**.402****
W14	.079**	.084**	**.499****	.105**	.229**	-.315**	**.592****
W20	.009	.131**	**.381****	.121**	.264**	-.409**	**.638****
W6	.202**	.033	**.611****	-.010	-.043	.047	**.456****
W2	.036	.079**	**.495****	.128**	.054*	.086**	**.552****

Self-Determination Theory

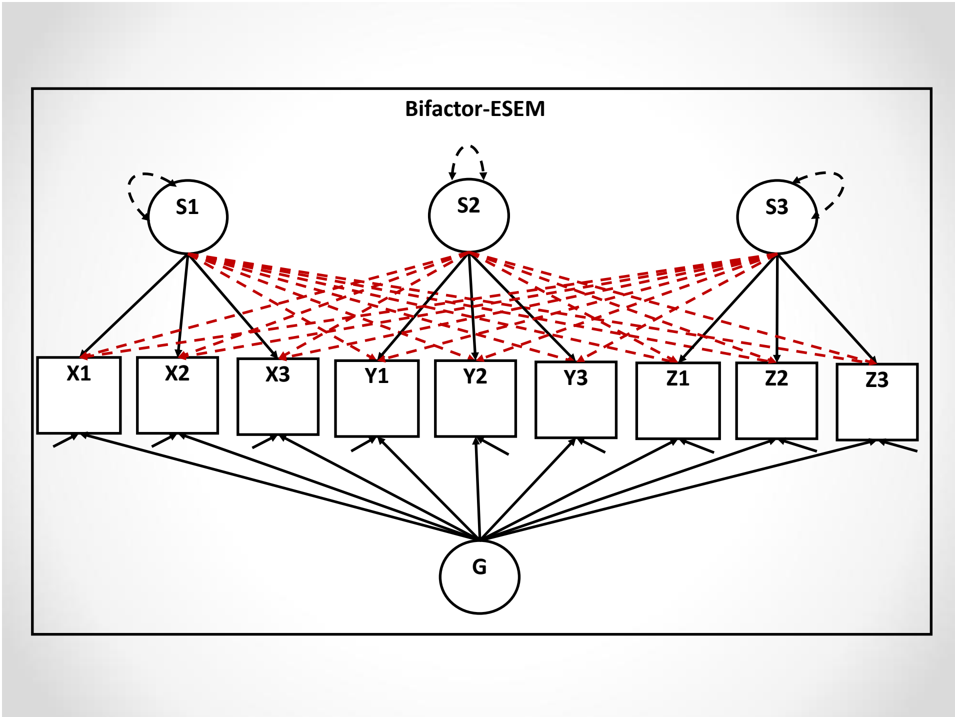
Howard, J., Gagné, M., Morin, A.J.S., Wang, Z.N., & Forest, J. (2018). Using bifactor exploratory structural equation modeling to test for a continuum structure of motivation. *Journal of Management, 44 (7),* 2638-2664.

Types of Motivation



A Continuum of Relative Autonomy ?

- CFA correlations: Inconclusive.
- Rasch analyses based on higher-order logic: No continuum (Chemolli, & Gagné, 2014), but still based on CFA-like correlations.
- ESEM factor correlations: Satisfactory support for the continuum (Guay, Morin et al., 2015; Litalien, Guay & Morin, 2015).



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Items	G-Factor	S-Factor 1	S-Factor 2	S-Factor 3	S-Factor 4	S-Factor 5	S-Factor 6
1. Intrinsic							
Item 1	0.73	0.41	-0.01	-0.05	-0.07	-0.05	-0.04
Item 2	0.71	0.54	0.02	-0.01	-0.02	-0.03	-0.05
Item 3	0.75	0.48	0.04	-0.09	-0.03	-0.17	-0.05
2. Identified							
Item 1	0.56	0.04	0.27	0.31	-0.02	0.02	-0.10
Item 2	0.79	0.04	0.26	-0.10	-0.04	<0.01	0.05
Item 3	0.73	0.01	0.34	0.01	-0.04	-0.01	0.05
3. Introjected							
Item 1	0.33	-0.03	0.03	0.38	0.28	0.10	0.03
Item 2	0.61	-0.02	0.08	0.33	0.06	0.04	-0.05
Item 3	0.28	-0.05	0.06	0.55	0.18	0.12	0.09
Item 4	0.26	-0.05	-0.02	0.55	0.05	0.03	<0.01
4. Ext-social							
Item 1	0.21	-0.03	-0.09	0.08	0.61	0.22	0.09
Item 2	0.18	-0.05	-0.01	0.13	0.59	0.17	0.09
Item 3	0.02	-0.03	0.06	0.21	0.59	0.25	0.07
5. Ext-material							
Item 1	0.25	-0.13	-0.32	-0.07	0.06	0.78	0.11
Item 2	0.18	-0.03	0.09	0.13	0.33	0.47	0.11
Item 3	-0.07	0.07	0.35	0.22	0.32	0.59	<0.01
6. Amotivation							
Item 1	-0.35	-0.05	-0.04	0.01	0.07	0.08	0.62
Item 2	-0.30	<0.01	0.03	<0.01	0.04	0.09	0.59
Item 3	-0.31	-0.06	0.02	0.06	0.12	0.06	0.62

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Relations With Covariates: Standardized Coefficients

	Quantity Only		Quantity and Quality							
Covariates	G-Factor	R^2	Amotivation	External Material	External Social	Introjection	Identified	Intrinsic	G-Factor	R^2
Affective Commitment	.61 (< .01)	.38	−.11 (.01)	−.12 (< .01)	.10 (.02)	−.14 (.03)	.20 (.04)	.17 (.02)	.55 (< .01)	.42
Continuance Commitment	−.02 (.64)	.00	.15 (.01)	−.02 (.63)	.23 (< .01)	.09 (.33)	.24 (.08)	−.09 (.37)	−.02 (.80)	.14
Autonomy	.39 (< .01)	.15	−.21 (< .01)	−.01 (.93)	−.06 (.21)	−.14 (.02)	−.15 (.04)	.16 (< .01)	.38 (< .01)	.26
Competence	.09 (.03)	.01	−.07 (.21)	−.10 (.06)	−.09 (.14)	.06 (.43)	−.11 (.18)	.02 (.76)	.12 (.01)	.05
Relatedness	.39 (< .01)	.15	−.19 (< .01)	.01 (.83)	−.04 (.51)	−.14 (.02)	−.15 (.04)	.17 (< .01)	.37 (< .01)	.25

Note: Probability (p) values are shown in parentheses. G-factor = global factor representing the global quantity of self-determined motivation.

Litalien, D., Morin, A.J.S., Gagné, M., Vallerand, R.J., Losier, G., Ryan, R.M. (2017). Evidence of a continuum structure of academic self-determination: A two-study test using a Bifactor-ESEM representation of academic motivation. *Contemporary Educational Psychology, 51*, 67-82

Bifactor-ESEM

λ (*SE*)							
G-Factor	S-Factor 1	S-Factor 2	S-Factor 3	S-Factor 4	S-Factor 5	S-Factor 6	S-Factor 7
.609(.041)**	.427(.076)**	.115(.031)**	.004(.043)	.026(.058)	-.055(.044)	-.076(.041)	-.122(.037)**
.713(.038)**	.418(.062)**	.146(.033)**	.127(.054)*	-.015(.044)	-.027(.033)	-.169(.038)**	.066(.028)*
.725(.032)**	.235(.075)**	.057(.036)	-.051(.077)	-.055(.057)	-.118(.040)**	-.063(.035)	-.046(.043)
.713(.036)**	.267(.082)**	.004(.037)	-.119(.054)*	.074(.067)	-.099(.041)*	-.076(.038)*	-.064(.036)
.549(.046)**	.138(.061)*	.256(.054)**	**.189(.061)****	.049(.068)	-.023(.046)	-.099(.043)*	.126(.034)**
.480(.043)**	.118(.038)**	.681(.042)**	.013(.043)	.013(.038)	.033(.028)	-.067(.029)*	.107(.026)**
.471(.047)**	-.014(.038)	.700(.045)**	.009(.053)	-.083(.036)*	-.037(.029)	-.084(.028)**	.174(.028)**
.644(.037)**	.043(.051)	.448(.049)**	-.049(.050)	-.089(.048)	-.025(.033)	-.117(.039)**	.134(.031)**
.620(.044)**	.078(.055)	.118(.046)*	.261(.117)*	-.082(.060)	.048(.038)	-.158(.038)**	-.032(.033)
.739(.037)**	-.024(.051)	-.032(.041)	.237(.116)*	-.143(.052)**	.168(.041)**	-.013(.031)	.004(.031)
.739(.035)**	-.016(.049)	.118(.039)**	.150(.116)	-.114(.043)**	.032(.042)	-.104(.031)**	.116(.031)**
.828(.020)**	.065(.047)	-.023(.033)	-.025(.124)	-.110(.054)*	-.005(.039)	-.062(.029)*	.004(.032)
.462(.049)**	.134(.052)*	-.055(.031)	.016(.077)	.516(.077)**	.008(.036)	.147(.043)**	-.224(.041)**
.376(.049)**	.000(.049)	-.064(.034)	.001(.055)	.457(.109)**	.035(.044)	.361(.063)**	-.180(.046)**
.492(.047)**	-.082(.064)	-.020(.040)	-.048(.077)	.305(.118)*	-.011(.048)	.262(.058)**	-.120(.053)*
.493(.054)**	-.082(.059)	-.032(.042)	-.161(.068)*	.397(.087)**	-.001(.047)	.186(.048)**	-.150(.041)**
.382(.047)**	.061(.057)	.037(.039)	**.151(.053)****	-.052(.048)	.647(.059)**	.134(.037)**	.002(.037)
.512(.057)**	-.125(.075)	.028(.050)	**.277(.067)****	.114(.070)	.332(.067)**	.154(.043)**	.060(.039)
.580(.040)**	-.139(.047)**	-.011(.035)	-.006(.085)	.052(.050)	.517(.049)**	.090(.042)*	.102(.036)**
.599(.046)**	-.023(.061)	-.072(.033)	-.195(.089)	-.023(.060)	.663(.087)**	.076(.033)*	.013(.031)
.131(.057)*	.140(.062)*	.021(.042)	.043(.076)	.163(.051)**	.092(.042)*	.709(.039)**	.026(.038)
.327(.058)**	.030(.051)	-.124(.030)**	-.001(.045)	.170(.064)**	.072(.033)*	.742(.040)**	-.080(.031)*
.377(.055)**	-.176(.056)**	-.090(.033)**	-.034(.056)	.125(.056)*	.086(.038)*	.633(.045)**	.013(.035)
.331(.061)**	-.211(.052)**	-.068(.042)	-.095(.060)	.106(.056)	.087(.042)*	.703(.053)**	-.091(.032)
-.400(.048)**	-.106(.048)*	.028(.031)	-.058(.062)	-.159(.050)**	.031(.031)	-.025(.037)	.659(.039)**
-.310(.051)**	.067(.049)	.128(.032)**	.072(.078)	.019(.058)	.073(.034)*	-.004(.038)	.666(.040)**
-.381(.049)**	-.027(.032)	.179(.030)**	-.016(.038)	-.150(.044)**	.014(.024)	-.077(.028)**	.732(.035)**
-.403(.046)**	-.010(.035)	.068(.025)**	.015(.042)	-.120(.042)**	.005(.027)	-.028(.030)	.775(.034)**

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	G-factor only		Global and specific factors								
	β (*SE*)	R^2	G-factor β (*SE*)	Int. Know. β (*SE*)	Int. Stim. β (*SE*)	Int. Acc. β (*SE*)	Identified β (*SE*)	Introjected β (*SE*)	External β (*SE*)	Amotivation β (*SE*)	R^2
Study 1: Outcomes											
Vitality	0.225 (0.048)**	0.051	0.213 (0.053)**	0.119 (0.074)	0.095 (0.058)	−0.055 (0.068)	0.084 (0.065)	−0.099 (0.057)	−0.166 (0.053)**	−0.132 (0.049)**	0.133
Ill-being	−0.124 (0.047)**	0.015	−0.108 (0.051)*	−0.137 (0.058)*	0.035 (0.051)	0.041 (0.073)	−0.131 (0.068)	0.112 (0.048)**	0.150 (0.043)**	0.353 (0.044)**	0.210
Study 2: Outcomes											
Academic Ach.	0.097 (0.051)	0.009	0.081 (0.051)	−0.017 (0.057)	0.112 (0.054)*	0.112 (0.059)	−0.064 (0.049)	−0.145 (0.048)**	−0.126 (0.048)**	−0.169 (0.049)**	0.102
Dropout Intent.	−0.312 (0.057)**	0.097	−0.190 (0.066)**	−0.071 (0.070)	−0.064 (0.070)	−0.116 (0.101)	−0.167 (0.046)**	0.058 (0.042)	−0.067 (0.037)	0.634 (0.045)**	0.497
Satisfaction	0.412 (0.046)**	0.170	0.285 (0.059)**	0.147 (0.072)*	0.075 (0.066)	0.178 (0.099)	0.178 (0.051)**	−0.048 (0.044)	0.002 (0.046)	−0.441 (0.039)**	0.369
Study 2: Predictors											
SN Competence	0.082 (0.055)	–	0.070 (0.070)	−0.012 (0.100)	−0.069 (0.097)	0.278 (0.088)**	−0.111 (0.070)	−0.178 (0.059)**	0.224 (0.066)**	−0.034 (0.063)	–
SN Autonomy	0.379 (0.054)**	–	0.388 (0.064)**	0.000 (0.110)	−0.012 (0.118)	−0.231 (0.101)*	0.160 (0.075)*	−0.073 (0.069)	−0.234 (0.077)**	−0.484 (0.078)**	–
SN Relatedness	0.024 (0.053)	–	0.020 (0.055)	−0.016 (0.078)	−0.049 (0.084)	0.087 (0.076)	0.082 (0.066)	0.069 (0.063)	0.069 (0.066)	0.067 (0.050)	–
R^2 in Mot. from the Predictors	0.199	–	0.197	0.001	0.012	0.061	0.034	0.040	0.043	0.215	–

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Garn, A.C., Morin, A.J.S., & Lonsdale, C. (2019). Basic psychological need satisfaction toward learning: A longitudinal test of mediation using bifactor exploratory structural equation modeling. *Journal of Educational Psychology*, *111, 354-372*

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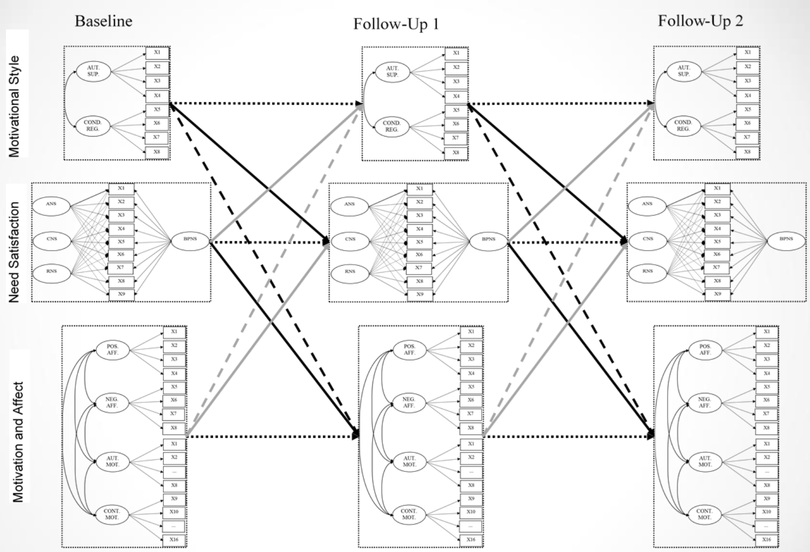

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Setting the Scale

Referent Indicator:

F1 BY X1@1 X2 X3;

F1*;

[X1@0 X2 X3];

[F1*];

Standardized Factors (the only one available in ESEM):

F1 BY X1* X2 X3;

F1@1;

[X* X2 X3];

[F1@0];

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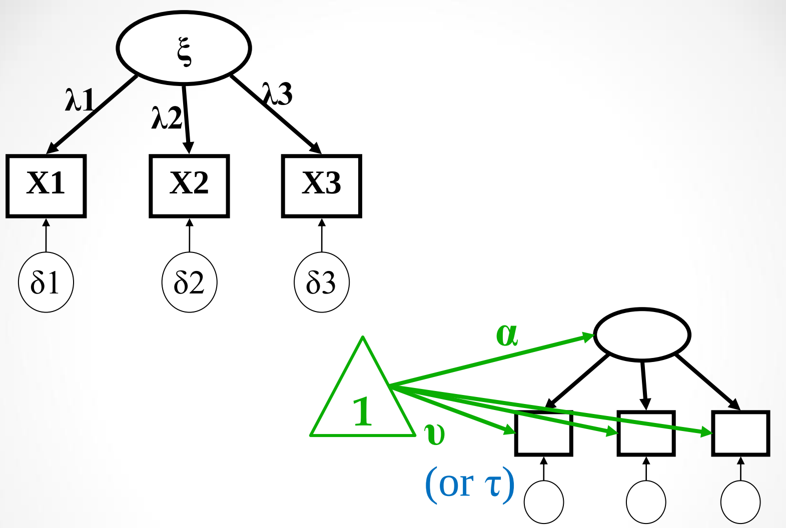

$$X = \tau + \Lambda\xi + \delta$$

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Measurement Invariance

Measurement invariance addresses the key question of whether the latent constructs have the same meaning across samples (multi-group invariance), situations (longitudinal invariance), or even testing procedure (e.g., online versus paper, which is a form of multi-group invariance).

Non-invariance indicates that the constructs are not comparable, and thus that means, or relations among constructs, cannot be compared across samples, situations, etc.

Type	Meaning
Configural Invariance	• The same **model** (including the same number of factors, the same type of relations among factors, and the same item-factor correspondence) is consistent with the data across samples . • Lack of invariance precludes any form of comparisons across samples.
Weak (metric, pattern) Invariance	• The same **factor loadings** reflect the item-factor relations across samples. • The constructs have the same meaning, are manifested in the same way, across samples. • This form of invariance is a prerequisite to the comparison of latent variances, latent covariances, or latent relations across groups. • A lack of invariance indicates that the instrument does not measure the same construct across groups, precluding any other form of comparison.

Type	Meaning
Strong (scalar) Invariance	• The factor loadings and **item intercepts** are the same across groups. • This form of invariance is a prerequisite to the comparison of latent means across groups. • Lack of invariance shows that the different groups use the items' response scale differently, that the groups can score higher or lower on the various indicators irrespective of group differences occurring at the latent factor level.
Strict Invariance	• The factor loadings, item intercepts, and **item uniquenesses** are the same across groups. • This form of invariance is a prerequisite to any form of comparison based on manifest scores across groups. • Lack of invariance shows that the constructs are assessed with different levels of reliability (measurement error across groups).

Type	Meaning
Latent Variance & Covariance Invariance	• Test whether the variances and covariances between the constructs are equivalent across groups.
Latent Means Invariance	• Test for the presence of latent mean differences(or lack thereof) across subgroups.

Remember:

ANALYSIS:
TYPE = general; ESTIMATOR = MLR;
ROTATION = Geomin (.5);
MODEL:
F1-F2 BY q1 q2 q3 q4 q5 q6 q7 q8 (*1);

ANALYSIS:
TYPE = general; ESTIMATOR = MLR;
ROTATION = Target (Oblique);
MODEL:
POSF BY q1 q2 q3 q4 q8 q5~0 q6~0 q7~0 (*1);
NEGF BY q5 q6 q7 q1~0 q2~0 q3~0 q4~0 q8~0 (*1);

Remember:

ANALYSIS:
TYPE = general; ESTIMATOR = MLR;
ROTATION = Geomin (.5);
MODEL:
F1-F2 BY q1 q2 q3 q4 q5 q6 q7 q8 (*1);

ANALYSIS:
TYPE = general; ESTIMATOR = MLR;
ROTATION = Target (Oblique);
MODEL:
POSF BY q1 q2 q3 q4 q8 q5~0 q6~0 q7~0 (*1);
NEGF BY q5 q6 q7 q1~0 q2~0 q3~0 q4~0 q8~0 (*1);

These two blocks are treated interchangeably, and specified in the same manner throughout.

```
MODEL:
POSF BY q1 q2 q3 q4 q8 q5~0 q6~0 q7~0 (*1);
NEGF BY q5 q6 q7 q1~0 q2~0 q3~0 q4~0 q8~0 (*1);
POSF@1; NEGF@1;
q1-q8;
[q1-q8];
[POSF@0];
[NEGF@0];

MODEL FEM:
POSF BY q1 q2 q3 q4 q8 q5~0 q6~0 q7~0 (*1);
NEGF BY q5 q6 q7 q1~0 q2~0 q3~0 q4~0 q8~0 (*1);
POSF@1; NEGF@1;
q1-q8;
[q1-q8];
[POSF@0];
[NEGF@0];
```

Variances are fixed to 1 by default.

With older versions of Mplus, ESEM does not work when variances specification are given in the general section.

WEAK

```
MODEL:
POSF BY q1 q2 q3 q4 q8 q5~0 q6~0 q7~0 (*1);
NEGF BY q5 q6 q7 q1~0 q2~0 q3~0 q4~0 q8~0 (*1);
POSF@1; NEGF@1;
q1-q8;
[q1-q8];
[POSF@0];
[NEGF@0];

MODEL FEM:
!!!!!
POSF*; NEGF*;
q1-q8;
[q1-q8];
[POSF@0];
[NEGF@0];
```

Loadings are invariant by default. There is no other way to test weak invariance than to simply take out the loadings from all subsequent groups.

MODEL:
POSF BY q1 q2 q3 q4 q8 q5~0 q6~0 q7~0 (*1 1);
NEGF BY q5 q6 q7 q1~0 q2~0 q3~0 q4~0 q8~0 (*1 1);
POSF@1; NEGF@1;
q1-q8;
[q1-q8] (i1-i8);
[POSF@0];
[NEGF@0];

STRONG

MODEL FEM:
POSF*; NEGF*;
q1-q8;
[q1-q8] (i1-i8);
[POSF*];
[NEGF*];

MODEL:
POSF BY q1 q2 q3 q4 q8 q5~0 q6~0 q7~0 (*1 1);
NEGF BY q5 q6 q7 q1~0 q2~0 q3~0 q4~0 q8~0 (*1 1);
POSF@1; NEGF@1;
q1-q8 (u1-u8);
[q1-q8] (i1-i8);
[POSF@0];
[NEGF@0];

STRICT

MODEL FEM:
POSF*; NEGF*;
q1-q8 (u1-u8);
[q1-q8] (i1-i8);
[POSF*];
[NEGF*];

```
MODEL:
POSF BY q1 q2 q3 q4 q8 q5~0 q6~0 q7~0 (*1 1);
NEGF BY q5 q6 q7 q1~0 q2~0 q3~0 q4~0 q8~0 (*1 1);
POSF@1; NEGF@1;
POSF WITH NEGF (c1);
q1-q8 (u1-u8);
[q1-q8] (i1-i8);
[POSF@0];
[NEGF@0];

MODEL FEM:
POSF@1; NEGF@1;
POSF WITH NEGF (c1);
q1-q8 (u1-u8);
[q1-q8] (i1-i8);
[POSF*];
[NEGF*];
```

Var
Covar


```
MODEL:
POSF BY q1 q2 q3 q4 q8 q5~0 q6~0 q7~0 (*1 1);
NEGF BY q5 q6 q7 q1~0 q2~0 q3~0 q4~0 q8~0 (*1 1);
POSF@1; NEGF@1;
POSF WITH NEGF (c1);
q1-q8 (u1-u8);
[q1-q8] (i1-i8);
[POSF@0];
[NEGF@0];

MODEL FEM:
POSF@1; NEGF@1;
POSF WITH NEGF (c1);
q1-q8 (u1-u8);
[q1-q8] (i1-i8);
[POSF@0];
[NEGF@0];
```

MEANS

Special Considerations
ESEM estimation (WLSMV)

Follow the guidelines provided for **ESEM** for the factor **loadings, variances, and covariances**.

However, different guidelines apply to the **thresholds, means, and uniquenesses**.

For an example with syntax, see:
Guay, F., Morin, A.J.S, Litalien, D., Valois, P., & Vallerand, R.J. (2015). Application of Exploratory Structural Equation Modeling to Evaluate the Academic Motivation Scale. *Journal of Experimental Education, 83* (1), 51-82. DOI: 10.1080/00220973.2013.876231
And
Morin (2023) in Hoyle's

Special Considerations
Bifactor-CFA and Bifactor-ESEM

Follow the guidelines provided for **CFA** for tests of invariance using **bifactor-CFA** models.

Follow the guidelines provided for **ESEM** for **Bifactor-ESEM** models. Because constraints are imposed on non-rotated parameters, equality constraints still need to be imposed on the covariance structure (factor correlations) even if the rotated solution is orthogonal (i.e., they are not @0, but rotated to 0).

For examples of Bifactor-ESEM with syntax, see:
MLR: Morin, A.J.S., Arens, A.K., & Marsh, H.W. (2016). A Bifactor Exploratory Structural Equation Modeling Framework for the Identification of Distinct Sources of Construct-Relevant Psychometric Multidimensionality. *Structural Equation Modeling, 23,* 116-139.
WLSMV: Morin, A.J.S., Arens, A.K., Tran, A., & Caci, H. (2016). Exploring Sources of Construct-Relevant Multidimensionality in Psychiatric Measurement: A Tutorial and Illustration using the Composite Scale of Morningness. *International Journal of Methods in Psychiatric Research*, *25*, 277-288
And
Morin (2023) in Hoyle's

ESEM and Bifactor-ESEM Multi-Group Invariance:

We just created a new online tool to help generate Mplus syntax for tests of invariance across groups for exploratory structural equation models (ESEM), and bifactor-ESEM models using polytomous data (continuous with MLR estimation or ordinal with WLSMV estimation). The tool can be accessed at https://statstools.app/b_esem/
Problems should be reported to : leondb@gmail.com

Citation to use for this tool:

De Beer, L.T., & Morin, A.J.S (2022). (B)ESEM invariance syntax generator for Mplus. Retrieved from https://statstools.app/b_esem/ doi: 10.6084/m9.figshare.19360808

Special Considerations

Higher-Order Models: WLSMV or MLR

- Start with a first-order measurement model, with no higher-order structure.
- Conduct all steps of invariance.
- Starting with the most invariant model, up to strict invariance only, add the higher-order factor structure.
- Test the invariance of the higher-order factor structure.

For an example with syntax applied to WSLMV, see:
Morin A.J.S., Moullec G., Maïano C., Layet, L., Just, J.-L., & Ninot G. (2011). Psychometric properties of the Center for Epidemiologic Studies Depression Scale (CES-D) in French Clinical and Non-Clinical Adults. *Epidemiology and Public Health/Revue d'Épidémiologie et de Santé Publique, 59* (5), 327-340.

Longitudinal Invariance with ESEM

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FX_T1 BY X1_t1 X2_t1 X3_t1	FX_T2 BY X1_t2 X2_t2 X3_t2
Y1_t1~0 Y2_t1~0 Y3_t1~0 (*t1);	Y1_t2~0 Y2_t2~0 Y3_t2~0 (*t2);
FY_T1 BY Y1_t1 Y2_t1 Y3_t1	FY_T2 BY Y1_t2 Y2_t2 Y3_t2
X1_t1~0 X2_t1~0 X3_t1~0 (*t1);	X1_t2~0 X2_t2~0 X3_t2~0 (*t2);
[X1_t1 X2_t1 X3_t1];	[X1_t2 X2_t2 X3_t2];
[Y1_t1 Y2_t1 Y3_t1];	[Y1_t2 Y2_t2 Y3_t2];
X1_t1 X2_t1 X3_t1;	X1_t2 X2_t2 X3_t2;
Y1_t1 Y2_t1 Y3_t1;	Y1_t2 Y2_t2 Y3_t2;
FX_T1@1; FY_T1@1;	FX_T2@1; FY_T2@1;
[FX_T1@0]; [FY_T1@0];	[FX_T2@0]; [FY_T2@0];

```
X1_t1 X2_t1 X3_t1 pwith X1_t2 X2_t2 X3_t2;
```

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Weak

FX_T1 BY X1_t1 X2_t1 X3_t1	FX_T2 BY X1_t2 X2_t2 X3_t2
Y1_t1~0 Y2_t1~0 Y3_t1~0 (*t1 1);	Y1_t2~0 Y2_t2~0 Y3_t2~0 (*t2 1);
FY_T1 BY Y1_t1 Y2_t1 Y3_t1	FY_T2 BY Y1_t2 Y2_t2 Y3_t2
X1_t1~0 X2_t1~0 X3_t1~0 (*t1 1);	X1_t2~0 X2_t2~0 X3_t2~0 (*t2 1);
[X1_t1 X2_t1 X3_t1];	[X1_t2 X2_t2 X3_t2];
[Y1_t1 Y2_t1 Y3_t1];	[Y1_t2 Y2_t2 Y3_t2];
X1_t1 X2_t1 X3_t1;	X1_t2 X2_t2 X3_t2;
Y1_t1 Y2_t1 Y3_t1;	Y1_t2 Y2_t2 Y3_t2;
FX_T1@1; FY_T1@1;	FX_T2*; FY_T2*;
[FX_T1@0]; [FY_T1@0];	[FX_T2@0]; [FY_T2@0];

X1_t1 X2_t1 X3_t1 pwith X1_t2 X2_t2 X3_t2;

Strong

FX_T1 BY X1_t1 X2_t1 X3_t1	FX_T2 BY X1_t2 X2_t2 X3_t2
Y1_t1~0 Y2_t1~0 Y3_t1~0 (*t1 1);	Y1_t2~0 Y2_t2~0 Y3_t2~0 (*t2 1);
FY_T1 BY Y1_t1 Y2_t1 Y3_t1	FY_T2 BY Y1_t2 Y2_t2 Y3_t2
X1_t1~0 X2_t1~0 X3_t1~0 (*t1 1);	X1_t2~0 X2_t2~0 X3_t2~0 (*t2 1);
[X1_t1 X2_t1 X3_t1] (i1-i3);	[X1_t2 X2_t2 X3_t2] (i1-i3);
[Y1_t1 Y2_t1 Y3_t1] (i4-i6);	[Y1_t2 Y2_t2 Y3_t2] (i4-i6);
X1_t1 X2_t1 X3_t1;	X1_t2 X2_t2 X3_t2;
Y1_t1 Y2_t1 Y3_t1;	Y1_t2 Y2_t2 Y3_t2;
FX_T1@1; FY_T1@1;	FX_T2*; FY_T2*;
[FX_T1@0]; [FY_T1@0];	[FX_T2*]; [FY_T2*];

X1_t1 X2_t1 X3_t1 pwith X1_t2 X2_t2 X3_t2;

Strict

FX_T1 BY X1_t1 X2_t1 X3_t1	FX_T2 BY X1_t2 X2_t2 X3_t2
Y1_t1~0 Y2_t1~0 Y3_t1~0 (*t1 1);	Y1_t2~0 Y2_t2~0 Y3_t2~0 (*t2 1);
FY_T1 BY Y1_t1 Y2_t1 Y3_t1	FY_T2 BY Y1_t2 Y2_t2 Y3_t2
X1_t1~0 X2_t1~0 X3_t1~0 (*t1 1);	X1_t2~0 X2_t2~0 X3_t2~0 (*t2 1);
[X1_t1 X2_t1 X3_t1] (i1-i3);	[X1_t2 X2_t2 X3_t2] (i1-i3);
[Y1_t1 Y2_t1 Y3_t1] (i4-i6);	[Y1_t2 Y2_t2 Y3_t2] (i4-i6);
X1_t1 X2_t1 X3_t1 (u1-u3);	X1_t2 X2_t2 X3_t2 (u1-u3);
Y1_t1 Y2_t1 Y3_t1 (u4-u6);	Y1_t2 Y2_t2 Y3_t2 (u4-u6);
FX_T1@1; FY_T1@1;	FX_T2*; FY_T2*;
[FX_T1@0]; [FY_T1@0];	[FX_T2*]; [FY_T2*];

X1_t1 X2_t1 X3_t1 pwith X1_t2 X2_t2 X3_t2;

Var-Cov

FX_T1 BY X1_t1 X2_t1 X3_t1	FX_T2 BY X1_t2 X2_t2 X3_t2
Y1_t1~0 Y2_t1~0 Y3_t1~0 (*t1 1);	Y1_t2~0 Y2_t2~0 Y3_t2~0 (*t2 1);
FY_T1 BY Y1_t1 Y2_t1 Y3_t1	FY_T2 BY Y1_t2 Y2_t2 Y3_t2
X1_t1~0 X2_t1~0 X3_t1~0 (*t1 1);	X1_t2~0 X2_t2~0 X3_t2~0 (*t2 1);
[X1_t1 X2_t1 X3_t1] (i1-i3);	[X1_t2 X2_t2 X3_t2] (i1-i3);
[Y1_t1 Y2_t1 Y3_t1] (i4-i6);	[Y1_t2 Y2_t2 Y3_t2] (i4-i6);
X1_t1 X2_t1 X3_t1 (u1-u3);	X1_t2 X2_t2 X3_t2 (u1-u3);
Y1_t1 Y2_t1 Y3_t1 (u4-u6);	Y1_t2 Y2_t2 Y3_t2 (u4-u6);
FX_T1@1; FY_T1@1;	FX_T2@1; FY_T2@1;
FX_T1 WITH FY_T1 (c1);	FX_T2 WITH FY_T2 (c1);
[FX_T1@0]; [FY_T1@0];	[FX_T2*]; [FY_T2*];
X1_t1 X2_t1 X3_t1 pwith X1_t2 X2_t2 X3_t2;	

Means

FX_T1 BY X1_t1 X2_t1 X3_t1 Y1_t1~0 Y2_t1~0 Y3_t1~0 (*t1 1); FY_T1 BY Y1_t1 Y2_t1 Y3_t1 X1_t1~0 X2_t1~0 X3_t1~0 (*t1 1); [X1_t1 X2_t1 X3_t1] (i1-i3); [Y1_t1 Y2_t1 Y3_t1] (i4-i6); X1_t1 X2_t1 X3_t1 (u1-u3); Y1_t1 Y2_t1 Y3_t1 (u4-u6); FX_T1@1; FY_T1@1; FX_T1 WITH FY_T1 (c1); [FX_T1@0]; [FY_T1@0];	FX_T2 BY X1_t2 X2_t2 X3_t2 Y1_t2~0 Y2_t2~0 Y3_t2~0 (*t2 1); FY_T2 BY Y1_t2 Y2_t2 Y3_t2 X1_t2~0 X2_t2~0 X3_t2~0 (*t2 1); [X1_t2 X2_t2 X3_t2] (i1-i3); [Y1_t2 Y2_t2 Y3_t2] (i4-i6); X1_t2 X2_t2 X3_t2 (u1-u3); Y1_t2 Y2_t2 Y3_t2 (u4-u6); FX_T1@1; FY_T1@1; FX_T2 WITH FY_T2 (c1); [FX_T2@0]; [FY_T2@0];
X1_t1 X2_t1 X3_t1 pwith X1_t2 X2_t2 X3_t2;	

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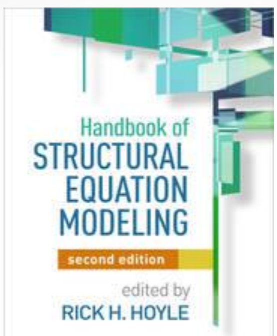
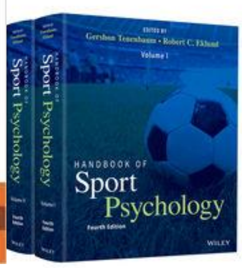
Chapter 10:
ESEM
Morin, Marsh, & Nagengast



Morin, A.J.S. (2023). Exploratory structural equation modeling. In R.H. Hoyle (Ed)., *Handbook of Structural Equation Modeling, Second Edition* (pp. 503-524). Guilford.



Morin, A.J.S., Myers, N.D., & Lee, S. (2020). Modern factor analytic techniques: Bifactor models, exploratory structural equation modeling (ESEM) and bifactor-ESEM. In G. Tenenbaum & R.C. Eklund (Eds.), *Handbook of Sport Psychology, 4th Edition* (pp. 1044-1073). London, UK: Wiley

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Limitations of ESEM

<u>Identification is automatically UVI (Unit Variance Identification) for the variance-covariance matrix</u>: For comparison purposes, it is useful to also use UVI with other models.

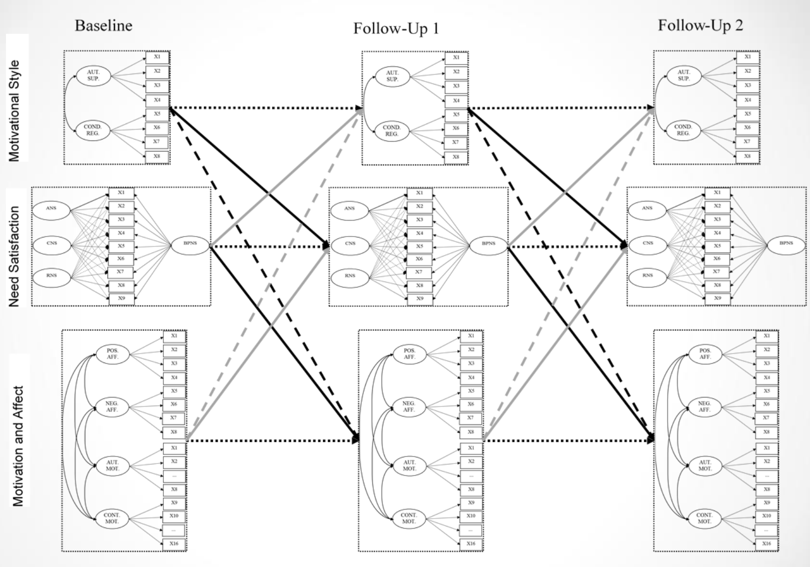
The full latent variance-covariance matrix "moves" together:

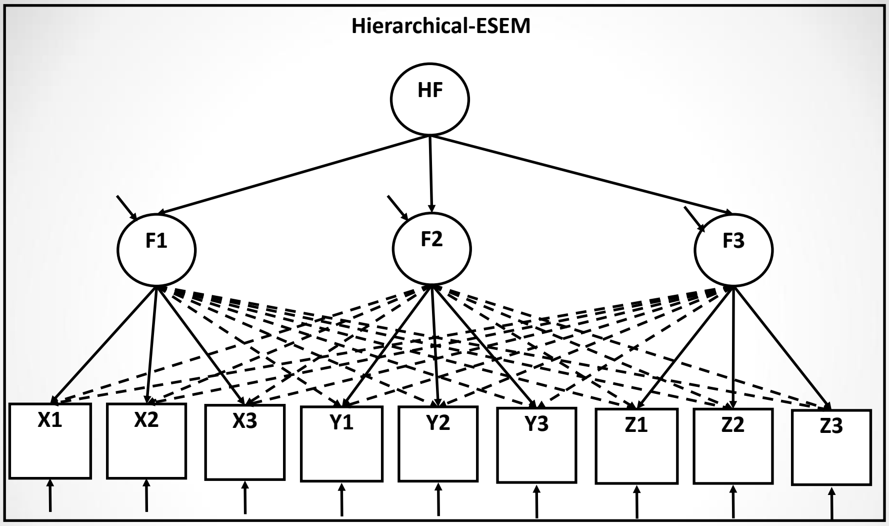
1) All factors forming a set need to be simultaneously related to the same variables outside of this set (predictors, outcomes, correlates), in the same manner.
2) Constraints need to be imposed on the full latent-variance covariance matrix simultaneously (no separate tests of the invariance of factor variances versus covariances).
3) No partial test of weak invariance (factor loadings) are possible.

In is not possible to impose constraints on factor loadings.

Higher-order ESEM models cannot be estimated, **BUT**…

Multilevel or mixture applications of EFA factors are limited.





Higher-Order ESEM

It is not directly possible to estimate higher-order CFA factor(s) from first-order ESEM factors. However, it is possible to reproduce the first-order ESEM model using the CFA framework, and then to use this model to estimate a higher-order CFA factor from the first-order ESEM factors.

It is not directly possible to estimate higher-order ESEM factor(s) from first-order ESEM or CFA factors. However, it is possible to use the latent variance-covariance matrix of the ESEM or CFA solution as the input (rather than using the raw data) for the higher-order analysis.

EFA-within-CFA

Jöreskog (1969) proposed EFA-within-CFA to estimate EFA-like factors (with all possible cross-loadings) in CFA/SEM.

The idea is to build in sufficient constraints for identification, that is m^2 restrictions where m=number of factors.

1. Constrain all factor variances to 1 (UVI) = m restrictions.
2. Select one referent indicator per factor and constrain all cross-loadings to 0 for this indicator = $m^2 – m$ restrictions.

e.g., 3 factors (4 factors)

1. 3 variances = 3 restrictions (4)
2. 2 cross-loadings * 3 factors = 6 restrictions (4*3 = 12)

ESEM-within-CFA

From this idea, Marsh, Nagengast, and Morin (2012) and Morin, Marsh, & Nagengast (2013) proposed ESEM-within-CFA as a way to **circumvent many limitations of ESEM**.

The SVALUES command of the output section will provide the exact starts values from the final selected ESEM solution.

These starts values then need to be pasted in the model section of the new ESEM-within-CFA input, and relevant constraints are then added to reach m^2 restrictions.

1. Constrain all factor variances to 1 = m restrictions.
2. Select one referent indicator per factor and constrain (@) all of its cross-loadings to their exact ESEM values = $m^2 – m$ restrictions.
3. Freely estimate all other parameters (*) using their exact start value from the ESEM solution.

OUTPUT:
sampstat standardized modindices cinterval residual svalues tech1 tech3 tech4;

CFA MODEL COMMAND WITH FINAL ROTATED ESTIMATES USED AS STARTING VALUES

```
F1 BY x1_1*0.73658;
F1 BY x2_1*0.63553;
F1 BY x3_1*0.65835;
F1 BY x4_1*-0.08283;
F1 BY x5_1*-0.20022;
F1 BY  x6_1*0.03567;
F1 BY x7_1*-0.03889;
F1 BY x8_1*-0.15079;
F1 BY x9_1*-0.02702;

F2 BY x4_1*0.69678;
F2 BY x5_1*0.64070;
F2 BY x6_1*0.89045;
F2 BY x1_1*0.01108;
F2 BY x2_1*0.12496;
F2 BY x3_1*0.06918;
F2 BY x7_1*0.09678;
F2 BY x8_1*0.04070;
F2 BY x9_1*0.19045;

F3 BY x7_1*0.81108;
F3 BY x8_1*0.82496;
F3 BY x9_1*0.66918;
F3 BY x1_1*0.01108;
F3 BY x2_1*0.12496;
F3 BY x3_1*0.06918;
F3 BY x4_1*-0.08283;
F3 BY x5_1*-0.20022;
F3 BY  x6_1*0.03567;

•••
```

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```
F1 BY x1_1*0.73658;
F1 BY x2_1*0.63553;
F1 BY x3_1*0.65835;
F1 BY x4_1*-0.08283;
F1 BY x5_1*-0.20022;
F1 BY  x6_1@0.03567;
F1 BY x7_1*-0.03889;
F1 BY x8_1@-0.15079;
F1 BY x9_1*-0.02702;

F2 BY x4_1*0.69678;
F2 BY x5_1*0.64070;
F2 BY x6_1*0.89045;
F2 BY x1_1@0.01108;
F2 BY x2_1*0.12496;
F2 BY x3_1*0.06918;
F2 BY x7_1*0.09678;
F2 BY x8_1@0.04070;
F2 BY x9_1*0.19045;

F3 BY x7_1*0.81108;
F3 BY x8_1*0.82496;
F3 BY x9_1*0.66918;
F3 BY x1_1@0.01108;
F3 BY x2_1*0.12496;
F3 BY x3_1*0.06918;
F3 BY x4_1*-0.08283;
F3 BY x5_1*-0.20022;
F3 BY  x6_1@0.03567;

F1-F3@1;
```

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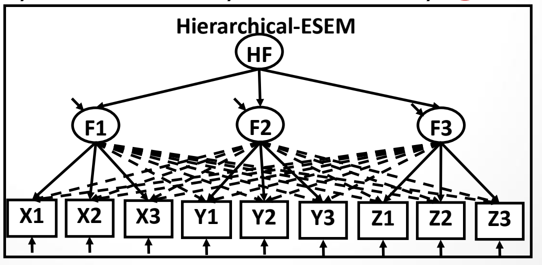
- Marsh, H.W., Nagengast, B., & Morin, A.J.S. (2013). Measurement invariance of big-five factors over the life span: ESEM tests of gender, age, plasticity, maturity, and La Dolce Vita effects. *Developmental Psychology*, *49*, 1194-1218.
- Morin, A. J. S., Marsh, H.W., & Nagengast, B. (2013). Exploratory structural equation modeling. In Hancock, G. R., & Mueller, R. O. (Eds.). *Structural equation modeling: A second course* (2nd ed., pp. 395-436). Charlotte, NC: Information Age Publishing, Inc.

Higher-Order ESEM with ESEM-Within-CFA

- Also need to fix the **main loading** of the referent indicator to its ESEM value, allowing for the free estimation of first-order factor variances.
- See: Morin, A.J.S., & Asparouhov, T. (2018). Estimation of a hierarchical Exploratory Structural Equation Model (ESEM) using ESEM-within-CFA. Montreal, QC: Substantive Methodological Synergy Research Laboratory.

https://smslabstats.weebly.com/uploads/1/0/0/6/100647486/webnote_-_hierarchical_exploratory_structural_equation_model.pdf

```
Model: ! ESEM Within CFA
f1 BY x1*0.74674; f1 BY x2*0.80372; f1 BY x3@0.80739; f1 BY x4*0.83759;
f1 BY y1*-0.05015; f1 BY y2*0.20610; f1 BY y3*-0.09183; f1 BY y4@0.03835;
f1 BY z1*0.22881; f1 BY z2*0.02457; f1 BY z3*0.01376; f1 BY z4@-0.13088;
f2 BY y1*0.79513; f2 BY y2*0.80701; f2 BY y3*0.95053; f2 BY y4@0.90008;
f2 BY x1*-0.12434; f2 BY x2*0.15514; f2 BY x3@-0.07168; f2 BY x4*0.08193;
f2 BY z1*0.06430; f2 BY z2*0.31927; f2 BY z3*-0.14645; f2 BY z4@-0.00922;
f3 BY z1*0.71349; f3 BY z2*0.66022; f3 BY z3*0.96202; f3 BY z4@0.95145;
f3 BY x1*0.11211; f3 BY x2*-0.13255; f3 BY x3@0.15235; f3 BY x4*-0.02669;
f3 BY y1*0.16258; f3 BY y2*-0.02858; f3 BY y3*0.07700; f3 BY y4@-0.01649;
f1-f3*1;
HF BY F1* F2 F3;
HF@1;
```



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Factor Covariance Matrix

At the end of the input for the final first-order ESEM (or CFA) model, indicate:

SAVEDATA:

TECH4 IS tech4.dat;

Then use this file as the data set for further analyses.

DATA:

File is tech4.dat;

TYPE is MEANS COVARIANCE;

NOBSERVATIONS = 1000; *! Your sample size*

With the order of the USEVARIABLE list corresponding to the order of appearance of the latent variables (see TECH4 for details).

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OR

Simply save the factor scores from the model and use them as indicators of the higher-order factor:

SAVEDATA:

FILE IS FSCORES.dat;

SAVE = Fscores;

This will create a new data file including the items used in the analysis (and listed as auxiliary), followed by the ID variable and then the factor scores. Mplus provides this list at the end of the output:

SAVEDATA INFORMATION

Save file

FSCORESLTA.dat

Order and format of variables

MISSING ARE ALL *

alexandre.morin@concordia.ca

https://smslabstats.weebly.com/