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Chapter 9.

A Gentle Introduction to Mixture Modeling Using Physical Fitness Performance Data

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A Gentle Introduction to Mixture Modeling Using Physical Fitness Performance Data

Abstract

This chapter provides a non-technical introduction to mixture modeling for sport and exercise sciences researchers. Although this method has been around for quite some time, it is still underutilized in sport and exercise research. The data set used for this illustration consists of a sample of 10,000 students who annually completed physical fitness tests for 7 years in Singapore. First, we illustrate latent profile analyses (LPA). Next, we illustrate how to include covariates in LPA and how to test the invariance of LPA solutions across groups, as well as over time using latent transition analyses. Following that, we illustrate the estimation of mixture regression models to identify subgroups of participants differing from one another at the levels of the relations among constructs. Finally, a growth mixture modeling example is shown to identify subgroups of participants following distinct longitudinal trajectories.

Keywords: Person-Centered analyses, Mixture Modeling, Latent Profile Analyses, Latent Transition, Covariates, Mixture Regression, Growth Mixture.

General Introduction

Writing a gentle non-technical introduction to mixture models is a challenge, given the scope and complexity of this method. Years ago, multiple regression was presented as a generic framework for the analysis of relations between continuous or categorical predictors, their interactions, and a continuous outcome (Cohen, 1968). Multiple regression was later shown to be a special case of canonical correlation analysis, where multiple outcomes could be considered simultaneously (Knapp, 1978). Structural Equation Modeling (SEM) was then presented as an even broader framework (Bagozzi, Fornell, & Larker, 1981) allowing for the estimation of sequences of relations between continuous latent variables (i.e., factors) corrected for measurement error. The generalized SEM (GSEM) framework now integrates SEM and mixture models, allowing for the estimation of relations between any type of continuous and categorical observed and latent variables (Muthén, 2002; Skrondal & Rabe-Hesketh, 2004). The key difference between SEM and GSEM is that SEM is *variable-centered*, yielding results reflecting a synthesis of relations observed in the total sample and assuming that all individuals are drawn from a single population. GSEM relaxes this assumption by considering the possibility that the relations differ across subgroups of participants.

GSEM thus combines SEM with a mixture modeling *person-centered* framework aiming to identify relatively homogeneous subgroups of participants, also called latent classes or profiles, differing qualitatively and quantitatively from one another in relation to: (a) specific observed and/or latent variable(s) and/or (b) relations among observed and/or latent variables(Borsboom, Mellenbergh, & Van Heerden, 2003; Morin & Marsh, 2014). Person-centered analyses are thus *typological* in nature, resulting in a classification system designed to help categorize individuals more accurately into qualitatively and quantitatively distinct subpopulations (e.g., Bergman, 2000). The resulting profiles are also *prototypical* in nature, with all participants having a probability of membership in all profiles based on their degree of similarity with the profiles' specific configurations (McLachlan & Peel, 2010). More precisely, profile membership is not known *a priori* but rather inferred from the data and represented by a latent categorical variable where each category represents an inferred subpopulation. Because these profiles are latent, participants are not "forced" into a single profile, but are rather assigned a probability of membership in all profiles.

Mixture models have been around for decades (Gibson, 1959; Lazarsfeld & Henry, 1968), but it is their integration within GSEM, coupled with the development user-friendly statistical packages (e.g., Latent GOLD: Vermunt & Magidson, 2005; Mplus: Muthén & Muthén, 2014; GLLAMM: Rabe-Hesketh, Skrondal, & Pickles, 2004), that have given these models a very high level of flexibility and popularity. Indeed, GSEM makes possible the extraction of subgroups differing from one another on any part of any type of SEM model. For this reason, a complete coverage of all possibilities provided by mixture models and GSEM is clearly beyond the scope of an introductory chapter and some choices had to be made. Thus, we elected to focus on a series of models aiming to provide a broad overview of the possibilities offered by this framework, as implemented in the Mplus package. Although mixture models can be used indirectly (in a *variable-centered* manner) to obtain more exact estimates of non-linearity, non-normality, and interactions among variables (Bauer, 2005), we focus on direct (*person-centered*) applications of mixture models where the latent categorical variable is assumed to reflect substantively meaningful subpopulations (Borsboom et al., 2003), adopting the perspective that "*the real added value of person-centered approaches is heuristic: human beings naturally conceptualize things and persons in terms of categories and a person-centered approach is better suited to these natural mindsets than a variable-centered approach, for an equivalent predictive value*" (Morin, Morizot, Boudrias, & Madore, 2011, p. 76). We assume that readers have a good understanding of Structural Equation Models (Chapter 5), tests of measurement invariance (Chapter 6) and latent growth modeling (Chapter 7) covered elsewhere in this book.

The models illustrated in this chapter are represented in Figure 9.1, where octagons represent categorical latent variables (C1 to Ci), squares represent manifest variables (X1 to Xi for indicators, P for predictors, O for outcomes), and circles represent continuous latent factors (here the intercepts α and slopes β of latent growth models, see chapter 7), We start our illustration with latent profile analyses (LPA; Model 1 in Figure 9.1), a model-based approach to the classification of participants based on their scores on a set of indicators (e.g., Morin, Morizot et al., 2011; Wang, Biddle, Liu, & Lim, 2012). We then illustrate how to test the invariance of LPA solutions across observed subgroups of participants (Eid, Langeheine, & Diener, 2003; Kankaraš, Moors, & Vermunt, 2011), how to integrate predictors and outcomes to a LPA solution (Model 2 in Figure 9.1; Asparouhov & Muthén, 2014; Wang, Liu, Chatzisarantis, & Lim, 2012), and how to test for the longitudinal stability of LPA solutions using Latent Transition Analyses (LTA; Model 3 in Figure 9.1; e.g., Collins & Lanza, 2009; Kam, Morin, Meyer, & Topolnytsky, 2014). Next, we illustrate the estimation of mixture regressions models (MRM; Henson, Reise, & Kim, 2007; Morin, Scalas, & Marsh, 2014), allowing for the identification of subgroups differing from one another at the level of the relations among constructs, in addition to the levels of the constructs themselves (Model 4 in Figure 9.1). Finally, we illustrate growth mixture models (GMM), allowing for the identification of subgroups of participants following distinct longitudinal trajectories (Model 5 in Figure 9.1; e.g., Grimm, Ram, & Estabrook, 2010; Morin, Maïano et al., 2011).

Utility of the Method in Sport and Exercise Science

In sport and exercise science, most quantitative research uses *variable-centered* approaches as the main form of analysis. These *variable-centered* approaches describe relations among *variables* and address questions regarding the relative contributions that predictor variables can make to an outcome, or the way variables can be grouped together. The use of (M)ANOVAs, regressions, factor analysis, and SEM are examples of *variable-centered* approaches.

Person-centered analyses focus on relations among *individuals* and more specifically investigate the way individuals can be grouped together based on their similarity on a set of indicators, or the differential way variables relate to one another in these specific subgroups. Cluster analyses have often been used to achieve such classifications. However, cluster analyses: (a) are highly reactive to the clustering algorithm, measurement scales, and distributions, (b) do not provide clear guidelines to select the optimal number of profiles in the sample, (c) rely on rigid assumptions; (d) assume that participants correspond to a single profile (i.e., they are *typological* without being *prototypical*), (e) have not been integrated to the GSEM framework. In contrast, mixture models provide a model-based approach to classification that is: (a) integrated into the GSEM framework, (b) extract *prototypical* profiles, (c) rely on less stringent assumptions that can often be relaxed or empirically tested, (d) can easily accommodate covariates, (e) are associated with indices to help in the selection of the optimal model (Magidson & Vermunt, 2002, 2004; Vermunt & Madgidson, 2002).

For these reasons, LPA appear to be gaining popularity in sport and exercise science as providing a more robust, less subjective, and more generalizable typological approach (Wang et al., 2012). The desire to classify objects, or persons, to better make sense of our environments has always been present across the social, psychological, and health disciplines that form the core of sport and exercise science, and can be traced back to the Greek philosophers (e.g., Bergman, 2000; Bergman & Trost, 2006). The reasons why researchers would want to incorporate mixture models to their research are numerous: (a) to simply categorize individuals, (b) to investigate whether a specific latent construct should be represented as a categorical entity rather than as a continuum (e.g., Borsboom et al., 2003; Masyn, Henderson, & Greenbaum, 2010), or (c) to investigate the invariance of measurement models (see chapter 6) across the full range of unobserved latent subpopulations present in a sample (Tay, Newman, & Vermunt, 2011).

There is a wide variety of applications of mixture models in sport and exercise science. At the most basic level, LPA aims to identity relatively homogenous subgroups of participants presenting similar patterns of scores on a set of categorical or continuous indicators. In this chapter, we illustrate LPA using fitness performance data, which are common in sport and exercise science. Wang et al. (2012) previously demonstrated the use of LPA in profiling physical and sedentary behavior patterns. However, LPA are not limited to physical data but can easily involve psychological (such as achievement goal profiles, or personality types) or social (such as types of team motivational climates). For instance, Wang et al. (2011) estimated profiles of students based on their perceptions of the motivational climate of their physical education classes, and then tested the association between these profiles and students achievement goals and affect. LPA can easily be extended to test the invariance of a profile solution across any meaningful subgroups of participants, such as different sports, levels of practice, or countries, or even across time points using LTA. Rather than classifying participants based on scores on a set of indicators, MRM can be used to extract subgroups of participants differing from one another in regards to the relations between sets of variables. For instance, Morin et al. (2014) used MRM to identify subgroups for which perceived actual and ideal levels of physical appearance were differentially related to more global self-conceptions. Finally, GMM is useful in studying dynamic patterns of change over time and in locating profiles presenting distinct longitudinal trajectories. For example, Morin et al. (2013) used GMM to extract subgroups of adolescents presenting distinct longitudinal trajectories of global self-esteem (GSE). They identified four distinct trajectories showing: (a) low and unstable levels of GSE, (b) moderate levels of GSE, (c) high and very stable levels of GSE, and (d) a switching pattern characterized by low and unstable levels of GSE at the start of secondary school, but high and stable levels a few years later.

The Substantive Example(s)

This illustration relies on a sample of 10,000 students (5000 boys; 5000 girls) who annually completed physical fitness tests for 7 years, starting in grade four of primary school (aged 9-10) until the fourth year of secondary school (aged 15-16). These students come from 217 primary and 167 secondary Singaporean schools. In Singapore, all healthy students are required to participate annually in the National Physical Fitness Award (NAPFA) test, starting in grade 4. The NAPFA is a high-stakes test conducted according to strict testing protocols: NAFPA testers must receive two days of training and be certified by the Singapore Sports Council, schools are required to inform the Ministry of Education (MOE) about test dates, and MOE performs random inspections of testing procedures.

The NAPFA involves six tests, recognized as reliable and valid indicators of physical fitness (Giam, 1981; Jackson, 2006): (a) Sit-Ups: The maximum number of bent-knee sit-ups in one minute; (b) Broad Jumps: The better of two standing broad jumps distance; (c) Sit-and-Reach: The better of two sit-and-reach forward distances; (d) Pull-Ups: The maximum number of overhand-grasp regular pull-ups (males over 14) or overhand-grasp inclined pull-ups (females and males younger than 14) in half a minute; (e) Shuttle-Run: The faster of two attempts to complete a 4 times 10 meters shuttle-run; (f) Run-walk: The minimum time taken for a 1.6 km (primary school) or 2.4 km (secondary school) run-walk. These tests are attempted on the same day with 2-5 minutes rest between them, apart from the Run-Walk test which may be attempted on a different day within a two-week period. As the norms for these tests differ as a function of grade and gender, scores were standardized within grade and gender. For details on time-related trends on these tests, see Wang, Pyun, Liu, Lim, and Li (2013).

Results on tests (a) (b) and (d) can be combined into single indicator of Physical Strength, results on tests (e) and (f) can be combined into a single indicator of Cardiovascular Fitness, whereas test (c) reflects Flexibility. For some analyses (MRM, GMM), we relied on these global indicators (Physical

Strength; Cardiovascular Fitness) computed as the factor scores from a fully invariant longitudinal confirmatory factor analytic model (see Appendix 9.1, also see chapters 5-6).

This data set comes from an official government testing program and has a restricted access. We are thus unable to share it with readers. However mixture models are computer- and time-intensive analyses for which users often struggle to get the models to converge properly on a well-replicated solution. Because of this, we strongly advocate users to use their own practice data sets.

In this illustration, we start by estimating LPA across separate samples of boys and girls in order to identify subgroups of participants with distinct profiles of physical fitness. To illustrate the inclusion of covariates, we use previous levels of Body-Mass Index (BMI; the only meaningful covariate available in this dataset) to predict profile membership, and use profile membership to predict later BMI. Then, we verify whether these LPA solutions are invariant across gender groups. Afterwards, we rely on LTA to test whether these cross sectional results remain the same across the combined pubertal and secondary school transition. To this end, we selected grade 5 (primary) as the first measurement point (at which all participants are unlikely to have experienced the onset of puberty) and the third year of secondary school (at which all participants are likely to be through major pubertal changes). For consistency, all preliminary analyses leading to these LTA are thus based on grade 5 students. To provide an alternative perspective of the estimation of relations between physical fitness and BMI, we also illustrate the use of MRM to extract profiles differing in the relations between predictors and later levels of BMI. Finally, we illustrate the use of GMM to extract profiles differing on their longitudinal trajectories of Physical Strength and Cardiovascular Fitness. Annotated input codes used to estimate the illustrated models are provided in Appendices 9.14 to 9.37. However, we first address two critical issues that are common to mixture models.

Class Enumeration in Mixture Models

Conventional goodness-of-fit indices (e.g., CFI, RMSEA) are not available for mixture models. Mixture models require that solutions including differing number of latent profiles be contrasted in order to select the final solution in a mainly exploratory manner (but see Finch & Bronk, 2011, for confirmatory applications). Typically, solutions including one latent profile up to a number of latent profiles that is higher than expectations are estimated and contrasted. Here, we contrasted models including one to eight profiles. To help in the selection of the optimal number of profiles, multiple sources of information can be considered. Clearly, the most important criteria in this decision are the substantive meaning and theoretical conformity of the solution (Marsh, Lüdtke, Trautwein, & Morin, 2009) as well as its statistical adequacy (e.g., absence of negative variance estimates). This last verification is important as mixture models frequently converge on improper solutions. Such improper solutions suggest that the model may have been overparameterized in terms of requesting too many latent profiles, or allowing too many parameters to differ across profiles (Bauer & Curran, 2003; Chen, Bollen, Paxton, Curran, & Kirby, 2001); more parsimonious models may thus be superior.

Several indicators also help in this decision: The Akaike Information Criterion (AIC), the Consistent AIC (CAIC), the Bayesian information criterion (BIC), and the sample-adjusted BIC (SABIC). A lower value on these indicators suggests a better-fitting model. Likelihood ratio tests (LRT) are inappropriate for comparisons of models including different number of profiles but may be used to compare models based on the same variables and number of profiles[1]. However, for purposes of class enumeration, LRT approximations are available: the standard and adjusted Lo, Mendel and Rubin’s (2001) LRTs (LMR/aLMR, typically yielding the same conclusions – here we only report the aLMR), and the Bootstrap LRT (BLRT; McLachlan & Peel, 2000). These tests compare a *k*-profile model with a *k-1*-profile model, and non-significant *p* values indicate that the *k*-1 profile model should be retained. Finally, the entropy indicates the precision with which the cases are classified into the profiles, with larger values (closer to 1) indicating fewer classification errors. The entropy should not be used to determine the optimal model, but nevertheless provides a useful summary of the classification accuracy of a model.

Simulation studies show that the BIC, SABIC, CAIC and BLRT are particularly effective in choosing the model which best recovers the sample’s true parameters (Henson et al., 2007; McLachlan & Peel, 2000; Nylund, Asparouhov, & Muthén, 2007; Peugh & Fan, 2013; Tein, Coxe, & Cham, 2013; Tofighi & Enders, 2008; Tolvanen, 2007; Yang, 2006). When these indicators fail to retain the optimal model, the AIC, ABIC, and BLRT tend to overestimate the number of profiles, whereas the BIC and CAIC tend to underestimate it. However, since these tests are all variations of tests of statistical significance, the class enumeration procedure can still be heavily influenced by sample size (Marsh, Lüdtke et al., 2009). More precisely, with sufficiently large sample sizes, these indicators may keep on suggesting the addition of profiles without ever reaching a minimum. In these cases, information criteria should be graphically presented through “elbow plots” illustrating the gains associated with additional profiles (Morin, Maïano, et al., 2011; Petras & Masyn, 2010). In these plots, the point after which the slope flattens out indicates the optimal number of profiles.

The Estimation of Mixture Models

In this study, all models were estimated using the robust maximum likelihood estimator available in Mplus 7.2 (Muthén, & Muthén, 2014). Although we aim to provide a non-technical introduction to mixture models, technically-oriented readers may consult McLachlan and Peel (2000), Muthén and Shedden (1999), and Skrondal and Rabe-Hesketh (2004). An important challenge in mixture models is to avoid converging on a local solution (i.e., a false maximum likelihood). This problem often stems from inadequate start values. It is thus recommended to estimate the model with as many random sets of start values as possible (Hipp & Bauer, 2006; McLachlan & Peel, 2000), keeping in mind that more random starts requires more computational time. Here, due to the availability of powerful computers, we used 5000 random sets of start values for LPA (10,000 for more complex LTA, MRM, and GMM), 100 iterations for each random start (1000 for complex models), and retained the 200 best starts for final optimization (500 for complex models). In practice, we recommend using at least 3000 sets of random starts, 100 iterations, and to retain at least 100 for final stage optimisation. These values can be increased when the final solution is not sufficiently replicated (Appendix 9.2). Finally, it is possible to control for the non-independence of the observations due to nesting within larger units (e.g., schools) using Mplus design-based correction (Appendix 9.3).

The Synergy

Latent Profile Analyses of Grade 5 Students, and Tests of Invariance across Gender Groups.

The results of the class enumeration procedure used to determine the optimal number of latent profiles to describe NAPFA scores for boys and girls attending grade 5 are reported in Table 9.1. In these models, the six NAPFA tests were used as profile indicators and their means and variances were freely estimated in all profiles[2]. Although we relied on classical LPAs assuming conditional independence, we discuss alternative specifications in Appendix 9.4. As mentioned previously, it is typical for the indicators used in the class enumeration process, due to their sensitivity to sample size, to keep on suggesting the addition of profiles without ever reaching a minimum. This is what happened here, potentially due to the large sample size. We thus graphed, in Figure 9.2 for boys and Appendix 9.5 for girls, elbow plots representing the gains associated with additional profiles. These figures suggest that the improvement in fit reaches a plateau at 5 profiles and becomes negligible thereafter. Based on a detailed examination of the 5-profile solution, together with neighboring 4- and 6-profile solutions, the decision was made to retain 5 profiles for both genders based on clear qualitative differences between profiles, theoretical conformity, lack of small profiles (≤1-5% of cases), non-redundancies between profiles, and convergence across gender.

Once the 5-profile solution has been retained for both gender groups (see Appendices 9.14 and 9.15 for basic input files), it becomes possible to conduct more formal tests of invariance of this solution across genders. The sequence of invariance tests proposed here can easily be extended to the comparison of any subgroups of participants. This sequence is based on, and extends, a similar sequence previously proposed for latent class solutions (where the profile indicators are categorical rather than continuous, as in LPA; Eid et al., 2003; Kankaraš et al., 2011). This sequence starts with the verification of whether the same number of profiles can be identified in all subgroups. Once this has been ascertained, a multiple-group model of configural invariance where the same model, with the same number of profiles, is simultaneously estimated in all groups without added constraints (Appendix 9.17). Then it becomes possible to test whether the profiles themselves are similar across samples in terms of being characterized by similar levels on the profile indicators (i.e., structural invariance; Appendix 9.18) – in this example, the NAPFA tests. Evidence of configural and structural invariance (see chapter 6) is a prerequisite to subsequent tests. When the number and/or structure of the profiles differ across samples, all analyses must be conducted separately and further tests of invariance are neither possible nor relevant.

The third step tests whether the within-profile variability on the indicators is similar across samples (i.e., dispersion invariance; Appendix 9.19). LPAs do not assume that all individuals share the exact same configuration of indicators within each profile, but rather allow for within-profile variability. Testing for dispersion invariance involves testing whether the profiles are more or less internally consistent across samples. Regardless of whether dispersion invariance is supported, the fourth step assesses whether the size of the profiles is similar across samples (i.e., distributional invariance; Appendix 9.20). Predictors and outcomes can then be added to the most invariant model, starting minimally with a model of structural invariance. The fifth step tests whether the relations between predictors (i.e., Grade 4 BMI) and profile membership are invariant or moderated across samples (i.e., deterministic invariance; Appendices 9.21 and 9.22). Finally, the sixth step assesses whether the relations between profiles and outcomes (i.e., Grade 6 BMI) replicate across samples (i.e., predictive invariance; Appendices 9.23 and 9.24).

The results from these tests of invariance are reported in the lower section of Table 9.1, and support the structural and distributional invariance of the profiles (i.e., lower or equivalent values on the information criteria, non-significant LRTs), but not their dispersion invariance (i.e., higher values on the information criteria, significant LRT). We thus retained a model in which the within-profile means and relative sizes where invariant across gender. The results from this model are reported in Appendix 9.6 and depicted in Figure 9.3. The first profile describes 10.05% of the sample presenting well-below average scores on Physical Strength indicators (Sit-Ups, Pull-Ups, Broad Jumps), slightly below average scores on the indicator of Flexibility (Sit-and-Reach), and well above average scores on Cardiovascular Fitness indicators (Shuttle-Run, Run-Walk). We chose the label "Fit-Cardio" to describe this profile. The second profile describes 35.60% of the sample presenting slightly below average scores on indicators of Physical Strength and Flexibility, and slightly above average scores on Cardiovascular Fitness indicators. We chose the label "Average Fitness with Low Flexibility" to describe this profile.

The third profile (26.25%) is characterized by below average scores on indicators of Cardiovascular Fitness and Flexibility, and above average scores on indicators of Physical Strength. We chose the label "Moderately Strong" to describe this profile. The fourth profile (10.34%) presents below average scores on indicators of Cardiovascular Fitness but well above average scores on indicators of Physical Strength and Flexibility. We chose the label "Strong and Flexible" to describe this profile. Finally, the fifth profile (17.77%) presents scores that are close to average on indicators of Physical Strength and Cardiovascular Fitness, but well above average scores of Flexibility. We chose the label “Flexible” to describe this profile. As discussed previously, the profiles means and sizes were found to be invariant across gender, but not the variance estimates. Gender-specific estimates of variability, which can easily be compared using the confidence intervals (CI) routinely provided with Mplus outputs, are reported in Appendix 9.6. The estimates in bold are those that differ across gender. Here, these results mainly show higher levels of variability for girls (relative to boys) in Profile 2 (Average Fitness with Low Flexibility) on most indicators, and in Profile 4 (Strong and Flexible) on the run-walk test.

Inclusion of Covariates in LPA Solutions.

A critical advantage of mixture models over alternative procedures (e.g., cluster analyses) is the ability to include covariates (predictors and outcomes) directly in the model rather than to rely on two-steps procedures where profile membership information is saved to an external file and used in a new series of analyses. Although covariates should not qualitatively change the profiles per se (Marsh et al., 2009), this helps to limit Type 1 errors by combining analyses and have been shown to systematically reduce biases in the estimation of the model parameters, especially those describing the relations between the covariates and the profiles (which otherwise tend to be underestimated; Bolck, Croon, & Hagenaars, 2004; Clark & Muthén, 2009; Lubke & Muthén, 2007).

Before including covariates to the model, a critical question that needs to be answered is whether these covariates are logically and theoretically conceptualized as having an impact on profile membership (predictors) or as being impacted by profile membership (outcomes). In both cases, we recommend that covariates be included to the model after the class enumeration procedure has been completed. This method allows for the verification of the stability of the model following covariates inclusion (Marsh et al., 2009; Tofighi & Enders, 2008). But, more importantly, the inclusion or exclusion of covariates should not change the substantive interpretation of the profiles. Observing such a change would indicate a violation of the assumption that covariates predict membership into profiles or are predicted by it, and would rather show that the nature of the profiles is dependent on the choice of the predictors (Marsh et al., 2009; Morin, Morizot et al., 2011). Using the procedure described in Appendix 9.2, involving the use of the starts values from the final unconditional solution, typically prevents running into such interpretation problems, especially when there are few covariates.

However, should such problems be encountered in the estimation process, Mplus has recently implemented a series of AUXILLIARY procedures to include covariates without allowing them to influence the nature of the profiles. We refer the reader to Asparouhov and Muthén (2014) for advice on the implementation of these alternative procedures.

The predictor (Grade 4 BMI) was directly incorporated to the model by way of a multinomial logistic regression[3], whereas the outcome (Grade 6 BMI) was directly incorporated as a distal outcome. At the bottom of Table 1, we contrast the results of the models in which the relations between these covariates and the profiles were freely estimated across gender, with models in which these relations where constrained to invariance. These results suggest that neither the deterministic nor the predictive invariance of the LPA solution was supported, showing that all of these relations are moderated by gender. The results from these analyses (using the fifth profile "Flexible" as comparison profile; see Appendix 9.7) globally show that higher levels of BMI in Grade 4 increase the likelihood of membership into Profile 1 (Fit-Cardio) – particularly among girls – but decrease the likelihood of membership into the third (Moderately Strong) and fourth (Strong and Flexible) profiles – particularly among boys. Regarding the outcomes, BMI levels in Grade 6 appeared to be higher in Profile 1 (Fit-Cardio) than in Profiles 2 (Average Fitness with Low Flexibility) and 5 (Flexible), and higher in these profiles than in Profiles 3 (Moderately Strong) and 4 (Strong and Flexible). Furthermore, BMI levels in Grade 6 are higher for girls corresponding to Profile 5 (Flexible) than to Profile 2 (Average Fitness with Low Flexibility), whereas no such difference could be observed for boys.

Latent Transition Analyses.

Although LTA (e.g., Collins & Lanza, 2009; Kam, Morin, Meyer, & Topolnytsky, 2014) may appear complex at first sight, they mainly involve the estimation of LPA solutions at two separate time points, and provide a way to estimate the connections between these two solutions (i.e., the transitions between profiles membership over time). In their most simple expression, LTA involve the estimation of LPA solutions based on the same set of indicators and including the same number of profiles at both time points, and provide a way to test the longitudinal invariance of LPA solutions. However, LTA can be extended to tests of the longitudinal connections between any type of mixture models, whether or not they are based on the same set of indicators (see, for example, Nylund-Gibson, Grimm, Quirk, & Furlong, 2014). To illustrate these models, we started with the estimation of separate LPA solutions, using all six NAPFA tests, in Grade 5 and the third year of secondary school. The results from these analyses are reported in Appendix 9.8, and support the superiority of a 5-profile model at both time points. From this evidence of configural invariance, these two LPA models were combined into a single LTA model (Appendix 9.25) through which it is possible to test the structural (Appendix 9.26), dispersion (Appendix 9.27), and distributional (Appendix 9.28) invariance of the model across time. These results show that the structural invariance of the model was not supported, precluding further tests of invariance.

Grade 5 profiles are already illustrated in Figure 9.3, whereas Secondary 3 profiles are illustrated in Figure 9.4. The relative sizes of all profiles and transition probabilities are presented in Table 9.2. These results show that Profile 1 (Fit-Cardio) remains unchanged across time points, and that most members of Profile 1 in Grade 5 remain within the same profile in Secondary 3 (63%) or switch to profiles showing an average level of fitness on most indicators (14.5% switch to Profile 2, 21.1% switch to Profile 5). Profile 2, which was globally average in Grade 5, remains similarly average in Secondary 3, while presenting lower levels of upper body strength (Pull-Ups) and higher levels of Flexibility. Over time, the relative size of this profile decreased from 36.51% to 19.94%. Most members of Profile 2 in Grade 5 transitioned either to Profile 2 (23.8%) or to the new average Profile 5 (53.9%) in Secondary 3. Profile 3 remains of the same relative size and Moderately Strong across time, but presents higher levels of Flexibility in Secondary 3. Most members of this profile in Grade 5 remain in the same profile in Secondary 3 (70%). Conversely, Profile 4 presents lower levels of Flexibility, as well as of middle and lower body strength (Sit-Ups, and Broad Jumps) and was relabeled “Strong (Upper Body)” rather than “Strong and Flexible”. This profile remains relatively small in size across time points, and relatively stable in terms of membership (76.9%). Finally, Profile 5 is the most changed over time, substantially increases in size (from 16.81% to 31.21%) and now mainly presents an average level on all indicators. Globally members of Profile 5 in Grade 5 tend to remain members of “average” profiles in Secondary 3 (36% in Profile 2; 38.3% in Profile 5).

Mixture Regression Analyses of Grade 5 Students.

There are relatively few examples of MRM in the literature (e.g., Morin, Scalas et al., 2014;

Van Horn et al., 2009), which is surprising given the potential of MRM to identify subgroups of participants differing at the levels of estimated relations between constructs. Here, we use MRM to estimate subgroups of participants differing from one another on the relations between Grade 5 BMI, Physical Strength, Cardiovascular Fitness and Flexibility, and Grade 6 BMI. Controlling for previous levels of BMI provides a direct test of the relations between the physical fitness indicators, and later increases or decreases in BMI levels (for a discussion of the similarity between this operationalization and models involving change scores, see Morin, Scalas, et al., 2014). In addition the mean and variance of all predictors and outcomes were freely estimated in all profiles. The free estimation of the outcomes' means and variance is typical in MRM. These means and variances respectively reflect the intercepts and residuals of the regressions of the outcomes on the predictors, making them essential to the estimation of profile-specific regression equations (Henson et al., 2007; Wedel, 2002). The free estimation of the predictors' means and variances provides additional flexibility and practical utility for the classification of current and later cases with incomplete information (Ingrassia, Minotti, & Vittadini, 2012; Wedel, 2002). Such models also reveal potential interactions among predictors, resulting in profiles in which the relations among constructs may differ as a function of predictors' levels (Bauer, 2005; Bauer & Shanahan, 2007). As for LPA (see endnote 2), we recommend to start with a model where all of these parameters are freely estimated across profiles and, failing to obtain converging or proper solutions, to fall back on simpler models where the variances are specified as equal between profiles. If this is not sufficient, then it is possible to move on to models where the predictors' means are also constrained to equality across profiles.

The results from MRM conducted separately for Grade 5 boys and girls, are reported in Appendix 9.9. For boys and girls, the information criteria kept on decreasing up to a 6-Profile solution, but this decrease reached a plateau around 3-Profiles and became negligible afterward. Similarly, the aLMR supported the 3-profile solution in both groups (see Appendix 9.29 for input). An examination of alternative solutions further supported the decision to retain three profiles based on the observation of greater theoretical meaningfulness and convergence between boys and girls, whereas solutions with more profiles tended to included very small (≤1-5% of cases), redundant, or meaningless profiles. We then tested the invariance of this MRM solution across genders (Appendix

9.30), using a sequence of invariance tests similar to the one presented for LPA but starting by an additional test of invariance of the regression coefficients (Appendix 9.31), followed by tests of structural (Appendix 9.32), dispersion (Appendix 9.33), and distributional (Appendix 9.34) invariance. Although some LRT proved significant, information criteria kept on decreasing – or presenting only negligible levels of increases – across the full sequence, supporting the complete invariance of the model. This conclusion is also supported by the examination of the parameter estimates from the various models in the sequence. The detailed results from this invariant 3-Profile solution are reported in Table 9.3.

Looking at the mean differences across profiles provides valuable information. Profile 2 (20.74% of the sample) presents the highest BMI of all profiles in Grade 5, the lowest level of Physical Strength, a midrange level of Flexibility, but the highest level of Cardiovascular Fitness. In contrast, Profile 3 (37.90%) presents average levels on all indicators. Finally, Profile 1 already presents moderately low levels of BMI in Grade 5, the highest levels of Physical Strength and Flexibility of all profiles, but the lowest levels of Cardiovascular Fitness. Comparing the regression coefficients across profiles is even more informative. For instance, BMI appears to be most stable in Profile 1, showing that low levels of BMI in Grade 5 tend to persist over time, and least stable in Profile 2, showing that kids with high levels of BMI can expect improvements over time. Even more interesting is the observation that higher levels of Physical Strength and Cardiovascular Fitness both predict decreases in BMI over time in Profile 2, whereas none of the NAPFA indicators predicts changes in BMI level in Profile 1. In Profile 3, only a small negative relation between Physical Strength and later decreases in BMI can be observed. These results clearly support the efficacy of improvements in Cardiovascular Fitness as a way to help kids with high BMI to regain more normative BMI levels.

Latent Basis Growth Mixture Analyses: Cardiovascular Fitness.

No introduction to mixture modeling would be complete without a presentation of GMM, which allows for the estimation of subgroups of participants presenting distinct longitudinal trajectories on one – or many – outcome(s) of interest over time. However, GMMs are complex and can easily deserve a complete chapter in their own right. For this reason, we present a longer description of these models in Appendix 9.10 (also see Grimm et al., 2010; Morin, Maïano et al., 2011; Ram & Grimm, 2009), but here we present two short illustrations based on non-linear specifications that we find to be particularly useful. Using the global indicator of Cardiovascular Fitness across the seven time waves, we estimated a latent basis GMM (Model 5 in figure 9.1; for details, see Appendices 9.10 and 9.35; for an illustration, see Morin et al., 2013), which provides a flexible way to estimate longitudinal trajectories differing in shape across multiple profiles of participants. In this illustration, all parameters from the model were freely estimated across profiles.

The results of the class enumeration procedure for models including 1 to 5 classes (models with more than 5 classes systematically converged on improper solutions and non-replicated local maximum, suggesting that these models were inappropriate due to overparameterization) are reported in Appendix 9.11. These results generally support the 3-Profile solution, which is illustrated in Figure 9.5. Detailed parameter estimates from this model are presented in Appendix 9.12. The first profile corresponds to 26.5% of the sample presenting levels of Cardiovascular Fitness that remain high over the course of the follow-up, while showing a U-Shaped trajectory characterized by an initial decrease (i.e., negative loadings on the slope factor) followed by an increase (positive loadings on the slope factor). The overall level of change over time (the mean of the slope factor, reflecting the level of change between the first and last time point respectively fixed to be 0 and 1) is significant, and the loadings on this slope factor reflect the proportion of the total change occurring at each time point (e.g., the decrease between Time 1 and 2 corresponds to 23.9% of the total change, as represented by a significant loading of -.239 on the slope factor). The second profile (41.7%) presents moderate levels of Cardiovascular Fitness that tend to increase over the course of the study, particularly around the time of the transition to secondary school (as shown by an important increase in the factor loadings on the slope factor at this time point). Finally, the third profile (31.9%) presents low and stable (as shown by a non-significant slope factor mean) levels of Cardiovascular Fitness. Although differences in the average levels of the trajectories may appear minimal, we note that there is a substantial level of between-person variability within each of the profiles showing that within-profiles differences may be even more pronounced than between-profile differences around trajectories showing on the average either a U-Shaped (Profile 1), Increasing (Profile 2) or Stable (profile 3) longitudinal profile. We finally note that the time-specific residuals remain globally similar across profiles and of a typical magnitude. It is important to carefully examine these residuals as they may something considerably enrich the interpretation of the profiles (for examples, see Morin et al., 2012, 2013), although not in this specific application.

Piecewise Growth Mixture Analyses: Physical Strength.

As a final illustration, we estimated Piecewise GMM using the global indicator of Physical Strength (see Appendices 9.10 and 9.36). Piecewise models allow for the estimation of a change in the direction of longitudinal trajectories before and after a transition point here represented by the entry into secondary school. To keep this model as simple as possible, we estimated models involving linear trajectories before and after the transition point, although more complex non-linear models can also be estimated. Here, all parameters from the model were freely estimated across profiles. However, most models converged on improper solutions, mainly due to the presence of negative estimates of the time-specific residuals – something that frequently occur in GMM when the time-specific residuals are allowed to be freely estimated across time waves and profiles. However, before moving to more parsimonious representations involving the inclusion of unrealistic invariance constraints on the time-specific residuals or other model parameters, it is possible to constrain the time-specific residuals to take a value ≥ 0 as part of the estimation process in order to help the model to converge on proper solutions (see Appendix 9.37; for examples of this procedure in the estimation of similarly complex models, see Marsh et al., 2012; Morin, Marsh, Nagengast, & Scalas, 2014; Morin et al., 2013). A limitation of this procedure is that the LMR/aLMR is not available with constrained estimation.

In Appendix 9.11, we report the results of the class enumeration process for the unconstrained (noting that the solutions are improper, but provide access to the aLMR) and constrained models including 1 to 5 classes. These results generally support the 4-Profile solution, which is illustrated in Figure 9.6. Parameter estimates from this model are presented in Appendix 9.13. Profile 1 corresponds to 24% of the population presenting low and decreasing levels of Physical Strength, a decrease that is unaffected by the transition to secondary school as illustrated by almost identical average values on the first (describing growth in primary school) and second (describing growth in secondary school) slope factors. The second profile is encouraging and describes 14.5% of the population who start the study with an average level of Physical Strength which increases over time over the course of Primary school and stabilizes at high levels afterwards, as illustrated by a significant and positive first slope followed by a non-significant second slope. The third profile describes the majority of the sample (59.1%) for whom levels of Physical Strength remain high and stable (as illustrated by non-significant slope factors) over the course of the study. Finally, the fourth latent profile describes a small number of participants (2.4%) who present moderate levels of Physical Strength that remain stable over the course of the study. The latent correlations estimated between the growth factors also reveal another interesting difference between the profiles. In Profiles 1, 2 and 3, the correlation between the intercept and both slope factors is significant and negative, suggesting that students with lower initial levels of Physical Strength present more pronounced levels of growth over time across both the primary and secondary school years. In contrast, these correlations are significant and positive in Profile 4, showing that for individuals with mid-range levels of Psychical Strength, higher initial levels that are associated with more pronounced increases over time. Finally, although the two slope factors appear to be uncorrelated in Profiles 1 and 3, they are negatively related in Profile 2 (showing that greater increases in primary school are associated with lower increases in secondary school), and positively related in Profile 4. Again, we note that the time-specific residuals remain globally similar across profiles and of a typical magnitude.

Summary

Mixture modeling is a person-centered approach to data analysis that is both typological and prototypical in nature, seeking to classify individuals according to the unique characteristics they possess. This introductory chapter presented an illustration of the use of LPA using a set of fitness performance data. In this illustration, LPA was used to identify subgroups of participants with distinct profiles of physical fitness performance. Prior and subsequent BMI levels were used as predictors and outcomes to illustrate the inclusion of covariates into the LPA model. We showed that the covariates can be used to verify the stability of the LPA model. In addition, we demonstrated how to testing the invariance of LPA solutions across genders, and across time points using LTA. We then illustrated the use of MRM to test how the relations between a set of predictors (Grade 5 BMI, Physical Strength, Cardiovascular Fitness, and Flexibility) on a single outcome (Grade 6 BMI) differed across subgroups of participants. Finally, we demonstrated the GMM method to extract profiles in terms of the longitudinal trajectories of Physical Strength and Cardiovascular Fitness. In doing so, we provided guidance stemming from both the statistical literature and extensive practical experience in the implementation of these models in order to help potential users to get the most out of these models.

While it is neither possible, nor realistic, to present all the possibilities provided by mixture modeling within a single introductory chapter targeting applied researchers, we aimed to provide interested readers with a non-technical introduction to mixture models that was as broad a possible and highlighted the possible usage of these models in sport and exercise science. As we have noted all along, the methods presented here only remain the tip of the iceberg of the possibilities provided by mixture models. We hope our presentation may have motivated readers to pursue their exploration.

One important point to highlight is that the methods of analysis need to match with the scientific problem under investigation. Although mixture models are by nature exploratory, a theory-based approach is typically required to guide the selection of the optimal solution and converge on meaningful interpretation of the results. In addition, because LPA, MRM, and GMM focus on individual-level variables, it is possible that contextual factors (such as class, teachers, schools, etc.) be present and need to be taken into account. In the current application, clustering of students into schools was taken into account whenever possible. However, an interesting extension of the models presented here is the estimation of multilevel mixture models allowing for the estimation of profiles at multiple levels of analyses (e.g., Asparouhov & Muthén, 2008).

Finally, it is also important to reinforce the point that person- and variable-centered approaches should be viewed as complementary, rather than opposite. When combined, these approaches have the potential to provide incredibly rich mutually-reinforcing yet complementary view of the same reality (Bergman & Trost, 2006). Combining a focus on variables and average relations with a focus on persons and similarities can be seen as opening different windows on the same world.

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Footnotes

[1] LRTs are computed as minus two times the difference in the log likelihood of the nested models and interpreted as chi-squares with degrees of freedom equal to the difference in the number of free parameters between models. With MLR estimation, LRTs needs to be divided by a scaling correction composite, *cd*, where: (i) *cd*=(*p0***co* - *p1***c1*)/(*p0*-*p1*); (ii) *p0* and *p1* are the number of free parameters in the nested and comparison models; and (iii) *c0* and *c1* are the scaling correction factors for the nested and comparison models (Satorra, & Bentler, 1999).

[2] Freely estimating the variances of the LPA indicators within profiles requires relaxing Mplus defaults which constrains them to be invariant across profiles. This more flexible parameterization does not involve reliance on the untested and unrealistic implicit assumption that all profile groups will present the same level of within-profile variability (see e.g., Morin, Maïano, et al., 2011), and has been shown to help recover the population true parameters (Peugh & Fan, 2013). However, this specification may not always be possible to implement (e.g., for more complex models or smaller sample sizes.) and often tends to result in convergence problems – suggesting that more parsimonious models with invariant variances should be estimated. Our recommendation would be to always start with these more flexible models, and reduce model complexity when necessary.

[3] In multinomial logistic regressions, each predictor has *k*-1 (with *k* being the number of profiles) effects for each possible pairwise comparison of profiles. Each regression coefficient reflects the

expected increase, for each one-unit increase in the predictor, in the Log odds of the outcome (i.e., the probability of membership in one profile versus another). For greater simplicity, odds ratios (OR) are typically reported and reflect the change in the likelihood of membership in a target profile versus a comparison profile associated for each unit of increase in the predictor. For example, an OR of 3 suggests that each unit of increase in the value of the predictor is associated with participants being three-times more likely to be member of the target profile (versus the comparison profile). ORs under 1 corresponds to negative logistic regression coefficients and suggest that the likelihood of membership in the target profile is reduced (e.g., an OR of .5 shows that a one unit increase in the predictor reduces by 50% the likelihood of being a member of the target, versus comparison, profile).

Table 9.1. Results from the Latent Profile Analyses (Grade 5)

Model	LL	#fp	Scaling	AIC	CAIC	BIC	ABIC	Entropy	aLMR	BLRT
Class Enumeration, Boys										
1 Profile	-39009.296	12	1.1352	78042.591	78131.795	78119.795	78081.663	Na	Na	Na
2 Profile	-35991.611	25	1.4041	72033.222	72219.061	72194.061	72114.621	0.768	≤ 0.001	≤ 0.001
3 Profile	-34941.597	38	1.4072	69959.194	70241.671	70203.671	70082.921	0.780	≤ 0.001	≤ 0.001
4 Profile	-34128.854	51	1.4119	68359.707	68738.820	68687.820	68525.762	0.819	≤ 0.001	≤ 0.001
5 Profile	-33659.337	64	1.4003	67446.674	67922.424	67858.424	67655.057	0.827	≤ 0.001	≤ 0.001
6 Profile	-33409.163	77	1.3617	66972.325	67544.712	67467.712	67223.035	0.805	≤ 0.001	≤ 0.001
7 Profile	-33218.378	90	1.3384	66616.755	67285.778	67195.778	66909.793	0.796	≤ 0.001	≤ 0.001
8 Profile	-33054.893	103	1.3359	66315.787	67081.447	66978.447	66651.152	0.790	0.004	≤ 0.001
Class Enumeration, Girls										
1 Profile	-39137.000	12	1.1317	78298.000	78387.230	78375.230	78337.098	Na	Na	Na
2 Profile	-37026.139	25	1.3237	74102.279	74288.173	74263.173	74183.732	0.698	≤ 0.001	≤ 0.001
3 Profile	-36132.889	38	1.3303	72341.779	72624.338	72586.338	72465.588	0.789	≤ 0.001	≤ 0.001
4 Profile	-35608.657	51	1.4121	71319.315	71698.539	71647.539	71485.480	0.774	≤ 0.001	≤ 0.001
5 Profile	-35237.664	64	1.3754	70603.329	71079.218	71015.218	70811.850	0.791	≤ 0.001	≤ 0.001
6 Profile	-35055.796	77	1.3995	70265.591	70838.145	70761.145	70516.469	0.771	0.031	≤ 0.001
7 Profile	-34924.944	90	1.4579	70029.888	70699.107	70609.107	70323.121	0.760	0.329	≤ 0.001
8 Profile	-34806.684	103	1.5504	69819.369	70585.253	70482.253	70154.958	0.739	0.652	≤ 0.001
Model	LL	#fp	Scaling	AIC	CAIC	BIC	ABIC	Entropy	LRT	df
Final 5-Profile Model, Including Correction for Nesting										
Boys	-33659.337	64	2.9958	67446.674	67922.424	67858.424	67655.057	0.827	Na	Na
Girls	-35237.664	64	3.0487	70603.329	71079.218	71015.218	70811.850	0.791	Na	Na
Invariance of the LPA solution										
Configural	-75279.495	129	3.2672	150816.991	151865.480	151736.480	151326.539	0.866	Na	Na
Structural (M)	-75305.956	99	3.6807	150809.912	151614.567	151515.567	151200.961	0.866	27.815	30
Dispersion (M, V)	-75460.796	69	4.6163	151059.592	151620.412	151551.412	151332.142	0.864	202.561*	30
Distributional (M, P)	-75326.302	95	3.7025	150842.603	151614.747	151519.747	151217.853	0.865	12.865	4
Deterministic Invariance										
Free	-72635.877	103	3.5199	145477.754	146312.624	146209.624	145882.307	0.867	Na	Na
Invariant	-72662.917	99	3.5986	145523.835	146326.283	146227.283	145912.678	0.867	34.400*	4
Predictive Invariance										
Free	-87691.334	106	3.6233	175594.667	176459.838	176353.838	176016.987	0.856	Na	Na
Invariant	-87720.652	101	3.7506	175643.305	176467.666	176366.666	176045.704	0.855	55.746*	5

Note. *: p ≤ .01; LL: Model LogLikelihood; #fp: Number of free parameters; Scaling = scaling factor associated with MLR loglikelihood estimates; AIC: Akaïke Information Criteria; CAIC: Constant AIC; BIC: Bayesian Information Criteria; ABIC: Sample-Size adjusted BIC; aLMR: Adjusted Lo-Mendell-Rubin likelihood ratio test; BLRT: Bootstrap Likelihood ratio test; LRT: Likelihood Ratio Test; df: Degrees of freedom associated with the LRT; M: Means; V: Variances; P: Class probabilities.

Table 9.2. Profile Proportions and Transitions Probabilities for the Latent Transition Analyses

	Transition Probabilities to Secondary 3 Profiles					
	Pr.1 (Fit-Cardio)	Pr.2 (Average, Low Upper Body)	Pr.3 (Moderately Strong)	Pr.4 (Strong-Upper Body)	Pr.5 (Average)	Relative Size
Grade 5 Profiles						
Pr.1 (Fit-Cardio)	0.630	0.145	0.003	0.011	0.211	11.06%
Pr.2 (Average Fitness, Low Flexibility)	0.074	0.238	0.102	0.047	0.539	36.51%
Pr.3 (Moderately Strong)	0.007	0.061	0.700	0.105	0.127	25.26%
Pr.4 (Strong and Flexible)	0.005	0.072	0.769	0.064	0.090	10.34%
Pr.5 (Flexible)	0.069	0.360	0.150	0.038	0.383	16.81%
Relative Size	10.93%	19.94%	31.80%	6.11%	31.21%	

Table 9.3. Results From the Final 3 Class MRM Solution

	Profile 1			Profile 2			Profile 3			
Means	Mean	Confidence Interval (CI)		Mean	Confidence Interval (CI)		Mean	Confidence Interval (CI)		Comparaisons
BMI (Gr.5)	-0.415	[-0.491; -0.340]		0.972	[0.820; 1.124]		-0.350	[-0.444; -0.256]		2 > 1 = 3
Strength (Gr.5)	0.429	[0.353; 0.506]		-0.770	[-0.901; -0.638]		0.231	[0.132; 0.331]		1 > 3 > 2
Cardio (Gr.5)	-0.436	[-0.515; -0.356]		0.751	[0.622; 0.880]		-0.222	[-0.323; -0.122]		2 > 3 > 1
Flexibility (Gr.5)	1.301	[1.241; 1.361]		-0.115	[-0.266; 0.037]		-0.222	[-0.627; -0.532]		1 > 2 > 3
Variances	Variance	CI		Variance	CI		Variance	CI		Comparaisons
BMI (Gr.5)	0.427	[0.361; 0.493]		1.067	[0.924; 1.210]		0.456	[0.386; 0.526]		2 > 1 = 3
Strength (Gr.5)	0.667	[0.631; 0.702]		0.711	[0.671; 0.751]		0.676	[0.641; 0.711]		1 = 2 = 3
Cardio (Gr.5)	0.699	[0.662; 0.737]		0.660	[0.625; 0.695]		0.691	[0.654; 0.728]		1 = 2 = 3
Flexibility (Gr.5)	0.298	[0.264; 0.332]		0.919	[0.774; 1.064]		0.211	[0.188; 0.233]		2 > 1 = 3
Regressions	b (s.e.)	CI	β	b (s.e.)	CI	β	b (s.e.)	CI	β	
Intercept	-0.042 (0.021)*	[-0.084; 0.000]		0.069 (0.037)	[-0.003; 0.142]		-0.042 (0.022)	[-0.085; 0.001]		
BMI (Gr.5)	0.906 (0.011)**	[0.885; 0.928]	0.918	0.831 (0.019)**	[0.793; 0.869]	0.826	0.887 (0.012)**	[0.864; 0.911]	0.900	1 > 2; 1 = 3; 2 = 3
Strength (Gr.5)	-0.063 (0.046)	[-0.153; 0.026]	-0.080	-0.367 (0.107)**	[-0.576; -0.158]	-0.298	-0.103 (0.044)*	[-0.190; -0.016]	-0.127	2 > 1; 1 = 3; 2 = 3
Cardio (Gr.5)	-0.058 (0.048)	[-0.152; 0.036]	-0.075	-0.257 (0.115)*	[-0.481; -0.032]	-0.201	-0.080 (0.044)	[-0.167; 0.006]	-0.100	1 = 2 =3
Flexibility (Gr.5)	0.011 (0.013)	[-0.014; 0.036]	0.010	0.007 (0.018)	[-0.028; 0.042]	0.006	-0.006 (0.014)	[-0.033; 0.022]	-0.004	1 = 2 =3
R^2 (boys)	0.847 (0.014)**			0.717 (0.015)**			0.824 (0.016)**			
R^2 (girls)	0.831 (0.016)**			0.667 (0.031)**			0.824 (0.014)**			

Note. *: p ≤ .05; **: p ≤ .01; b = unstandardized regression coefficient; s.e.: Standard error of the estimate; β: standardized regression coefficient

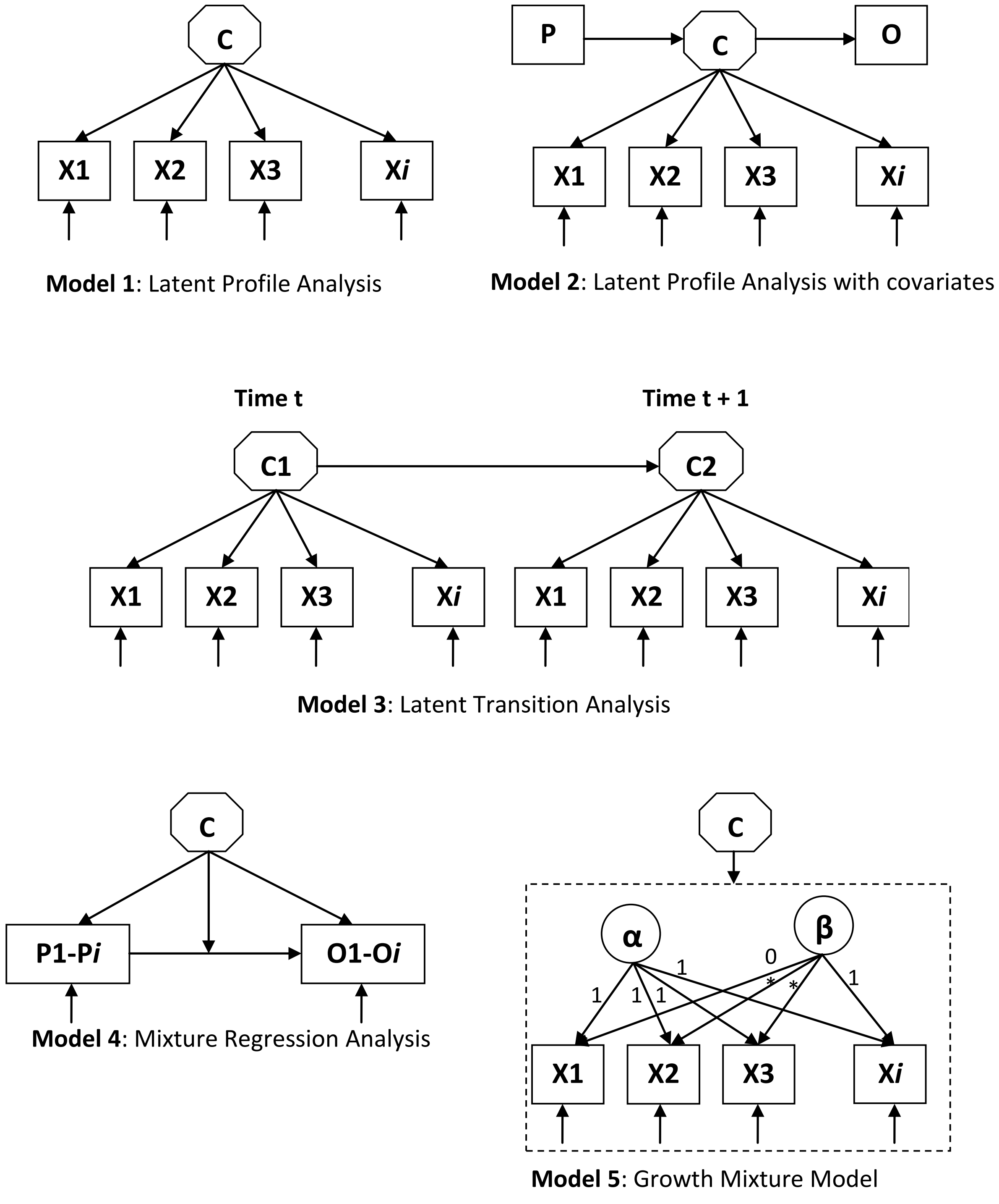

Figure 9.1. *Representation of Key Models Illustrated in this Chapter*

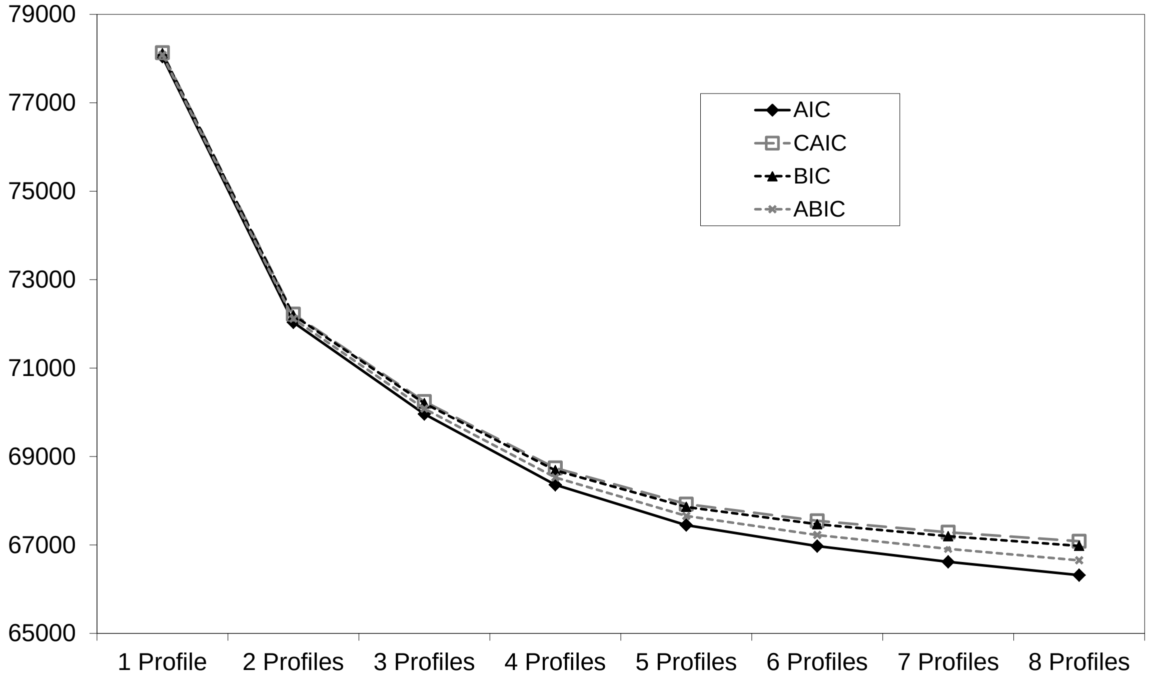

Figure 9.2. Elbow Plot of the Information Criteria for the LPA, Boys, Grade 5

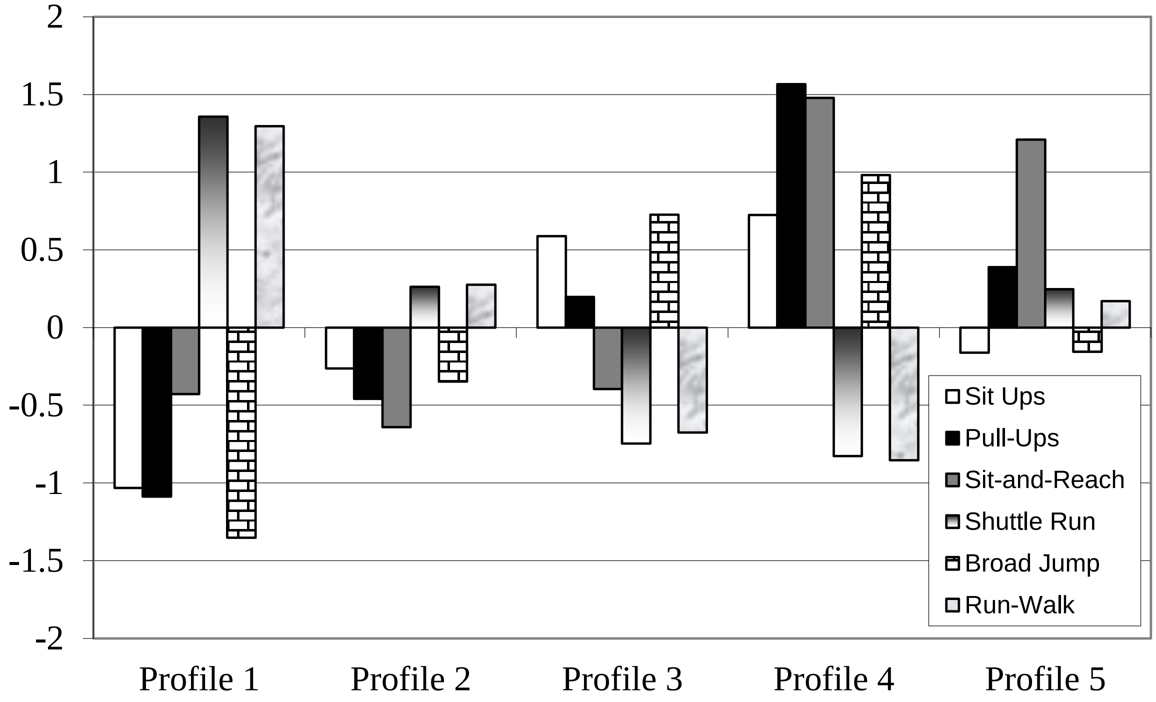

Figure 9.3. Within-Profile Means for the LPA solution, Grade 5

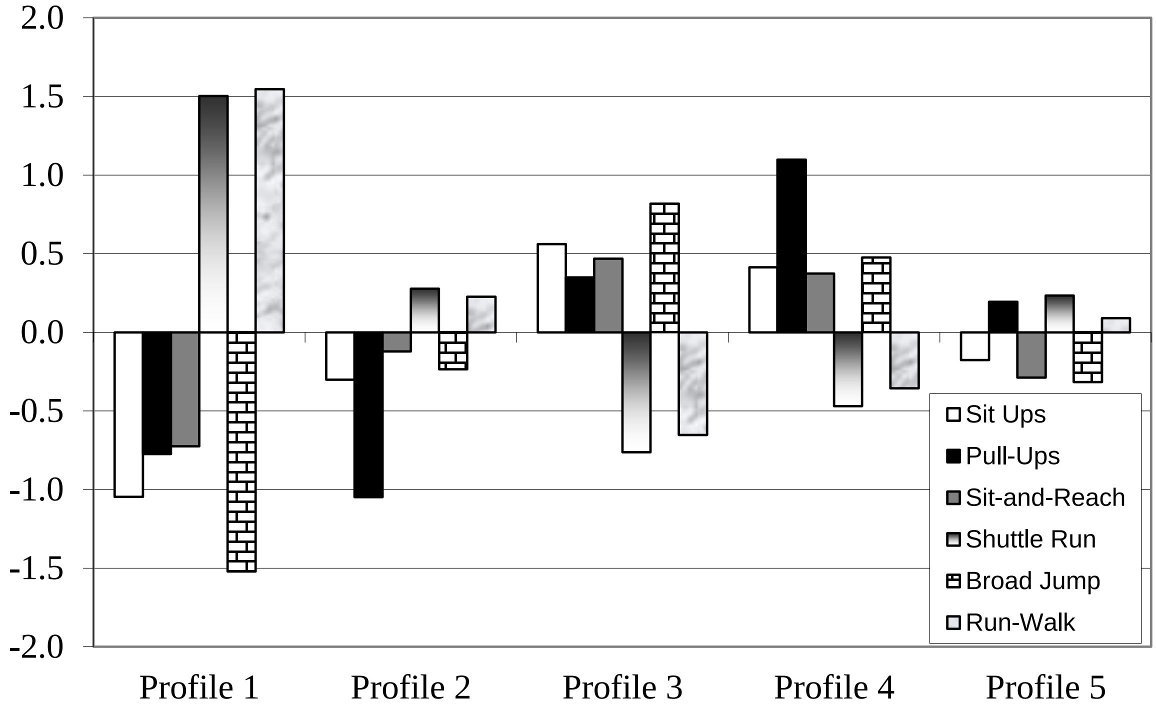

Figure 9.4. Within-Profile Means for the LPA solution, Secondary 3

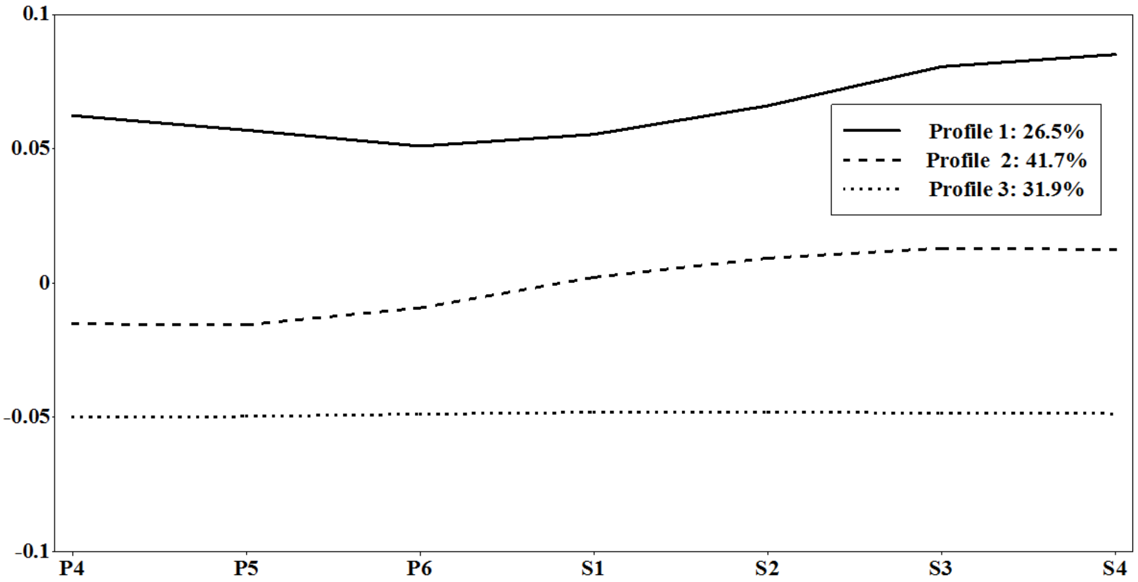

Figure 9.5. Graph of the Final Latent Basis GMM of Cardiovascular Fitness Trajectories.

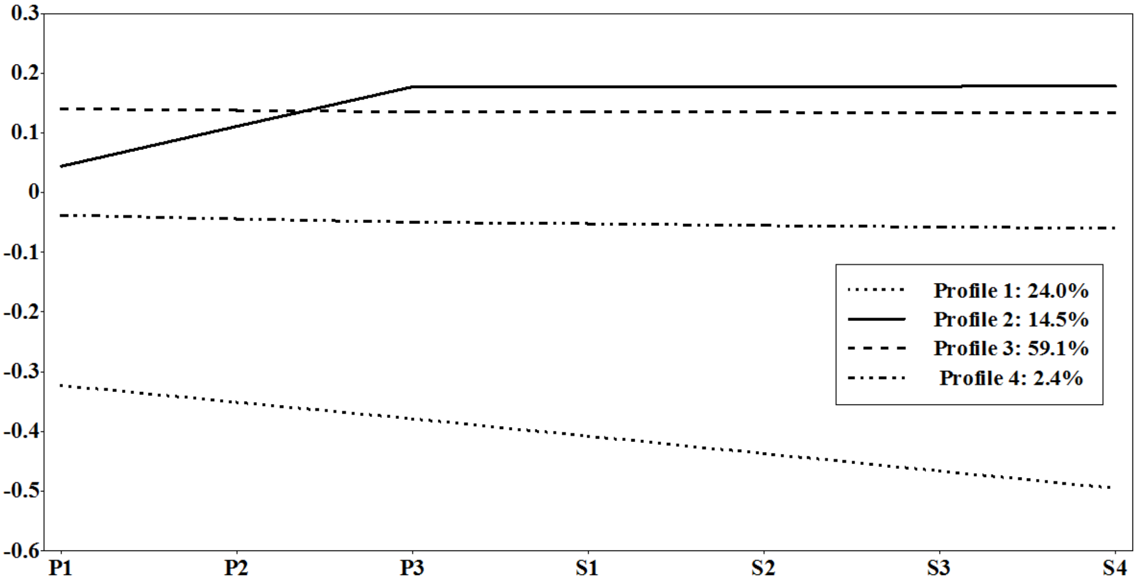

Figure 9.6. Graph of the Final Piecewise GMM of Physical Strength Trajectories.