Value Beliefs about Math: a Bifactor-ESEM Representation

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Abstract

This study proposed an improved representation of the factor structure of the Gaspard et al. (2015) value beliefs about math scale relying on bifactor exploratory structural equation modeling (B-ESEM). Using a convenience sample of 537 Italian students (327 males; M_{age}=18.2), our results supported the superiority of a B-ESEM solution including 9 specific factors (intrinsic, importance of achievement, personal importance, utility for school/job, utility for life, social utility, effort required, opportunity cost, emotional cost) and one global value factor. The results further revealed that the specific factors (with the exception of personal importance) retained meaning over and above participants' global levels of value. Finally, our results confirmed that global value beliefs predicted career aspirations, but expectancies of success remained the strongest predictor of math achievement.

Keywords: expectancy-value theory; math value; ESEM; bifactor analysis

Introduction

Despite the growing importance of math and science careers, there has been a worldwide decline of enrolments in science, technology, engineering, and math (OECD, 2006). To understand the motivation to pursue a math or science career, researchers have underscored the importance of considering competence-related and value beliefs in relation to these disciplines (Simpkins, Davis-Kean, & Eccles, 2006). Expectancy-value theory (EVT; Eccles, 2009; Eccles et al., 1983) posits that a person's expectancies of success in a given task in combination with that person's valuing of that task (i.e., task value) are key predictors of academic achievement, effort, engagement and career choices (see Appendix 1).

Along with measures of expectancies for success, Eccles and Wigfield (1995) developed and validated domain specific measures of valuing including four distinct dimensions: intrinsic value (subjective interest), attainment value (relevance of engaging in the task for confirming or disconfirming aspects of one's self-schema), utility value (extrinsic reasons), and costs (amount of time and energy lost for other activities). Despite the theoretical and empirical differentiation among these four components of value, many researchers have preferred to measure valuing as a general construct with a small number of items (Jacobs, Lanza, Osgood, Eccles, & Wigfield, 2002). In contrast, other researchers have developed scales focusing on a single value component, or on a subset of these components (Durik, Vida, & Eccles, 2006; Nagengast, Marsh, Scalas, Xu, Hau, & Trautwein, 2011).

The Math-Related Value Beliefs Scale

Based on Eccles et al.'s (1983) definition of these value components, Gaspard et al. (2015) proposed a comprehensive measure of math-related value beliefs. The authors adopted a multidimensional representation of math value beliefs, based on theory and empirical evidence suggesting that the various components could be characterized by multiple facets (Perez, Cromley, & Kaplan, 2014; Trautwein et al., 2013). Whereas the definition and nature of the intrinsic value component remained unidimensional due to its focus on positive feelings, other components were differentiated as follows. Two facets were proposed to underpin attainment value: importance of achievement (performance), and personal importance (mastering the content and its relation to one's identity). Three facets were proposed to underpin short-term utility: utility for school (for one's present and future education), utility for daily life (daily routines and leisure time activities), and social utility (utility of the knowledge for peer acceptation). Two facets were proposed to underpin long-term utility: utility for job (future career opportunities) and general utility for future life (unspecified future life activities). Finally, three facets were proposed to underpin costs: opportunity cost (time lost for other activities), effort required (perceived exhaustion), and emotional cost (negative emotions).

Gaspard et al. (2015) found support for the factor structure described above, and its invariance across genders. They also found girls to have lower intrinsic value, personal importance and utility for future life and job, but higher utility value for school, emotional costs and efforts. They also found that the first-order factor structure provided a better fit to the data than a higher-order model in which the facets were grouped into the a priori dimensions of intrinsic, attainment, utility, and cost. However, they noted the presence of high correlations among some of the facets, even facets theoretically associated with different components. For example, intrinsic value had very high correlations with facets of attainment value and cost, and personal importance had similarly high correlations with facets of utility value. Gaspard, Häfner, Parrisius, Trautwein and Nagengast (2017) investigated a refined version of this scale tapping value beliefs in five academic domains in a sample of students from Grades 5 to 12. Using confirmatory factor analyses (CFA), they found evidence of measurement invariance across academic domains and grade level, but still noted high correlations between some of the value facets.

Guo et al. (2016) found support for the a priori higher-order structure of the EVT value component through the incorporation of a global factor (G-factor) reflecting participants' global valuing of mathematics. However, possibly due to high correlations among some of the subscales, the authors incorporated a series of second-order factors to their models to reflect the covariance among various attainment, utility, and costs facets left unexplained by the global factor. On the basis of this complex hybrid representation, Guo and colleagues (2016) examined the proposed expectancy x value interaction among students' expectancy of success and the distinct value factors in the prediction of math-related outcomes. Their results showed that whereas self-concept was a relatively important predictor of achievement, the value components were more strongly associated with self-reported efforts, and the EVT interaction predicted the outcomes synergistically. Nonetheless, as for Gaspard et al. (2015, 2017), a key concern was the presence of subscale correlations high enough to call into question the discriminant validity of some subscales.

The Methodological Advantages of Exploratory Structural Equation Modelling (ESEM)

Gaspard et al. (2015) and Guo et al. (2016) both relied on CFA to examine the structure of the value beliefs scale. However, Morin et al. (Morin, Arens & Marsh, 2016a; Morin, Arens, Tran, & Caci, 2016b) noted that, whenever conceptually-related constructs are assessed, such as in a measure focusing on distinct facets of mathematics value, items are likely to present some degree of valid association with more than one construct. These associations, which would be expressed via cross-loadings in exploratory factor analytic (EFA), lead to inflated factors correlations in CFA. Interestingly, including unnecessary cross-loadings via EFA does not result in such biased estimates (Asparouhov, Muthén, & Morin, 2015). EFA models have now been integrated with CFA into ESEM, providing a way to advantageously rely on EFA measurement models for confirmatory (Marsh, Morin, Parker, & Kaur, 2014) and predictive (Mai, Zhang, & Wen, 2018) purposes. Importantly, target rotation (Asparouhov & Muthén, 2009) makes it possible to specify EFA/ESEM models in a confirmatory manner by "targeting" all freely estimated cross-loadings to be as close to 0 as possible. These developments make it possible to use ESEM for scale construction (like traditional EFA), refinement and validation (like traditional CFA), replication (via ESEM tests of measurement invariance), and predictive (like traditional SEM) models.

Bifactor Models and Hierarchical Multidimensional Constructs

Hierarchical representations of psychological constructs have traditionally been represented by higher-order models, which assume that the association between items and the higher-order factor is fully mediated by the first-order factors (McAbee, Oswald, & Connelly, 2014), so that the higher-order factor does not explain any unique variance over and above that already explained by the first-order factors. These first-order factors thus confound the variance explained by the higher-order factors and the variance uniquely attributable to each first-order factor (Morin, Myers, & Lee, 2018). Such models also rely on a restrictive implicit proportionality constraint according to which the ratio of item variance explained by the first- and higher-order factors is the same for all items associated with a single first-order factor (Gignac, 2016).

Bifactor models provide a more flexible alternative to the representation of hierarchical constructs, such as value beliefs (Gignac, 2016; Morin et al., 2016a). In a bifactor model, items simultaneously reflect a global (G-factor) construct underpinning responses to all items (global value), and specific (S-factors) components reflecting the variance shared among items forming a subscale but not explained by the G-factor (Morin et al., 2018). Due to their orthogonality, bifactor models are well-suited at solving issues related to high factor correlations compared to CFA, ESEM, or higher-order models (Litalien, Morin, Gagné, Vallerand, Losier, & Ryan, 2017) and provide a way to directly assess the distinct contribution of the S- and G- factors in terms of prediction (see Appendix 2).

The Present Study

This study further investigates the structure of Gaspard et al.'s (2015) value scale while relying on a bifactor-ESEM (Morin et al., 2016a, 2016b) representation of the data, and contrasting it with CFA, ESEM, and bifactor-CFA representations (see Figure 1). This approach is designed to provide a more accurate representation of the distinct nature of the various value dimensions incorporated in this model, while relying on a more precise disaggregation of the global extent to which students value math relative to the truly unique part of each value component. In addition, based on evidence suggesting that incorporating cross loadings is likely to result in a more precise, and reduced, estimate of factor correlations (e.g., Asparouhov et al., 2015), this approach has the possibility to help solve the high factor correlations issue raised by Gaspard et al. (2015, 2017), while providing a way to directly assess the added value of these specific value facets over and above the global value factor.

To achieve a better understanding of what might influence the motivation to pursue a math or science career, we tested the predictive effects of global and specific value beliefs and students'expectancies of success on scientific career interest and math achievement, as well as their interaction (see Figure 2). We hypothesize that the global and specific value components will positively predict career aspirations, whereas expectancy should be the strongest predictor of mathematics achievement (Eccles & Wigfield, 2002; Marsh et al., 2013; Simpkins et al., 2006); we also assumed that expectancy and value interact with one another in influencing mathematics achievement and career intentions (Guo, Marsh, Parker, Morin, & Dicke, 2017; Guo, Parker, Marsh, & Morin, 2015).

Method

Participants

This study relies on a convenience sample of 537 Italian high school students (Grades 12 and 13; 327 males and 210 females, *M*age=18.2, *SD*=.85). Each student received a parental consent form, with information about the study. On the testing date, active consent was sought from the students. The response rate was 98%. The participants completed the questionnaires in 30-minute group sessions, during school hours. The data were collected anonymously and confidentiality was guaranteed; the students had the opportunity to withdraw at any time without justification.

Measures

Participants completed an Italian version of the value scale (Gaspard et al., 2015) developed for the present study following translation-back translation procedures (Gudmundsson, 2009). Each of the 37 items (including 2 negatively-worded ones), were rated on a 6-point Likert scale (1=strongly disagree to 6=completely agree) rather than the original 4-point Likert scale to obtain a more accurate approximation of underlying continuity.

Expectancy was measured by five items (e.g., "*I find many mathematical problems interesting and challenging*"; Likert scale from 1=*false* to 6=*true*; ω=.993) of the mathematics scale from the Self-Description Questionnaire (SDQ-II; Marsh, 1992). Career aspirations were measured by three items (e.g., "*I expect to work in a job uses science*"; ω=.994; Likert scale from 1=*strongly disagree* to 6=*completely agree*), from the Program for International Student Assessment (OECD, 2007). A score of mathematics achievement (from 0 to 5) was computed by the sum of the responses to a logical-mathematical test composed of five items, including five responses options and scored as correct/incorrect.

Analyses

Items distribution showed adequate values for univariate skewness (range from -0.013 to -1.298) and kurtosis (range from 0.052 to -1.300see also Gaspard et al., 2018). Models were estimated using Mplus 7.3 (Muthén & Muthén, 2014) robust maximum likelihood (MLR) estimator, and full information maximum likelihood (FIML; Enders, 2010) to handle missing data (0% to 1.7%, *M*=0.5%) (see Appendices 3 and 5 for model specification, input files, and fit indices). The best latent model[1] was retained for predictive analyses in which all value components, in combination with a factor representing students' expectancies of success, were used to predict career aspirations and mathematics achievement. The latent interaction between the global value and the expectancy factor was tested using the product of indicators approach (Marsh, Wen & Hau, 2004 - see Appendix 5). As recommended by Morin et al. (2016b), we also report model-based omega coefficients of composite reliability (McDonald, 1970): $\omega=(\Sigma|\lambda_i|)^2/([\Sigma|\lambda_i|]^2+\Sigma\delta_{ii})$, where λ_i are the factor loadings and δ_{ii} the error variances.

Results

The a priori CFA solution, corresponding to Gaspard et al.'s (2015) specifications, resulted in estimation problems related to a linear dependency due to strong correlations between utility for school and utility for job (*r*=.975, s.e.=.050) and utility for daily life and general utility for future life (*r*=.995, s.e.=.022). Therefore, items related to utility for school and for job were merged to reflect a single utility for school/job factor, and items related to utility for daily and future life were merged to reflect a single utility for life factor (see Appendix 4).

CFA vs. ESEM

The goodness-of-fit of all alternative models are reported in Table 1. The results showed that ESEM resulted in a higher level of fit to the data than CFA (lower information criteria and RMSEA, and changes in CFI/TLI≥.010) and lower factor correlations (see Table 2). The standardized factor loadings from the ESEM solution are reported in Table A6.1 (see Appendix 6). These results showed most factors to be well-defined by the presence of target loadings greater than .300, with the sole exception of the personal importance factor, which appeared to be weakly defined, in part due to the negatively-worded items (16 and 36). As expected, multiple non-target cross-loadings were also present, providing additional support for ESEM. These cross-loadings remained small, thus not interfering with the interpretation of the factors. Intercorrelations showed positive relations among the various facets of each component, as well as between facets of intrinsic, attainment, and utility value.

In line with theoretical expectations, correlations between facets of cost and other value components were negative (see Table 2).

Bifactor-ESEM

The bifactor-CFA[2] solution failed to achieve an acceptable level of fit to the data. Conversely, the bifactor-ESEM solution resulted in a slight increase in model fit when compared to ESEM, particularly in terms of Information Criteria, whereas the increment in CFI and TLI and the decrement in RMSEA was not substantial (ΔCFI/TLI<.010; ΔRMSEA<.015) (see Table 1). As noted by Morin et al. (2016a), this comparison should not be solely based on goodness-of-fit, but also consider parameter estimates given the ability of ESEM to absorb an unmodelled G-factor through inflated cross-loadings. The bifactor-ESEM parameter estimates and the omega coefficients of composite reliability are reported in Table 3. These results revealed a well-defined G-factor with moderate to strong target loadings from most of the value items (from .302 to .787, M=.580), with the exceptions of items 9 and 12 (λ<.10), which loaded more strongly on their target S-factors (respectively, opportunity cost and efforts). Most of the items (with few exceptions, such us the opportunity cost items) had higher factor loadings on the G-factor than the S-factors, thus contributing to the definition of the G-factor and supporting the need for a bifactor representation[3].

Over and above this G-factor, the S-factors (with the exception of personal importance) were well-defined by satisfactory target loadings and reliability (see Table 3). Despite being slightly lower than in the ESEM solution (as expected), these target loadings supported the idea that these S-factors tap into relevant specificity once the G-factor is taken into account. In contrast, the personal importance S-factor retained almost no specificity once the variance explained by the G-factor was taken into account, arguing against the added-value of this dimension. This solution also appeared to solve the previously noted issue related to items 16 and 36, which mainly served to reflect participants' global levels of math value. Cross-loadings also remained similar to those observed in ESEM.

Predictive Models

Because the bifactor-ESEM solution provided the best representation of the scale, this model was retained to test whether math expectancy, global value and specific value facets predicted scientific career interest and math achievement. An additional advantage of this model is that, due to the orthogonality of the bifactor model, multicollinearity is highly unlikely to play any role in the estimation of relations. Results (χ^{2}=1214.219; df=668; CFI=.958; TLI=.935; RMSEA=.039; CI=.036/.042) suggested that only global value predicted career aspirations in science (β=.468; s.e.=.104) while mathematics competence was predicted by expectancy (β=.462; s.e.=.159). A model adding the latent interaction effect of these variables showed that expectancy and value did not interact in the prediction of the outcomes.

Discussion

According to EVT (Eccles et al., 1983), value beliefs encompass four major components: attainment value, intrinsic value, utility value, and cost. However, the definition and operationalization of components suggests that most of them might incorporate multiple facets (Trautwein et al., 2013). In an effort to represent these distinct facets, Gaspard et al. (2015) developed a comprehensive math-related value beliefs questionnaire encompassing 11 distinct facets. The present study aimed to propose an improved representation of the inherent hierarchical and multidimensional nature of this instrument through the application of the bifactor-ESEM framework (Morin et al., 2016a).

First, our results clearly supported the need to incorporate cross-loadings to the model to reflect the imperfect nature of the value items in terms of indicating only one facet, and thus to achieve a clearer level of differentiation among the various value facets. This is consistent with recent observations showing that ESEM tends to provide more accurate estimates of factor correlations whenever cross-loadings are present in the population model, yet to remain unbiased otherwise (Asparouhov et al., 2015).

Second, our results also supported the benefits of incorporating a bifactor component to the ESEM solution, allowing each item to simultaneously reflect students' global levels of math value, as well as the specific facets of this construct. Most of the 37 items of the value scale show higher factor loadings on the G-factor than on their S-factors, with the exception of items 9 and 12, which presented non-statistically significant loadings on the G-factor, and the three opportunity cost items, which presented higher loadings on their S-factor. These results are in line with Revelle and Wilt (2013) conclusion that "When g has large saturations on each test, it is clearly useful to think in terms of g" and confirms the validity of the bifactor representation for the value scale. More precisely this G-factor seems to be interpretable as reflecting the global extent to which a student values mathematics, whereas eight of the nine facets seemed to reflect the residual quality of their value beliefs about math (pleasure, utility, etc.) once this global level is taken into account. Conversely, items associated with the personal importance facet mainly contributed to the global value factor, but did not retain specificity once the variance explained by the G-factor was taken into account. This suggests that this facet might represent value at a more global level when compared to the other facets. This is not uncommon for bifactor models, which should typically result in at least some well-defined S-factors (Morin et al., 2016a). Additional weakly defined S-factors should simply not be interpreted.

Third, analyses focusing on the prediction of mathematics competence and career aspirations confirmed the key role of value in the prediction of career aspirations, and of expectancy in the prediction of math competence (Eccles & Wigfield, 2002). However, no interaction was found between expectancy and value. Our results also revealed that the effects of math value on career aspirations was limited to the G-factor, with no residual effect found to be associated with the specific value facets. Should it be replicated in additional studies focusing on a greater variety of outcomes, this result suggests that predictive studies focusing on educational outcomes may only need to focus on global levels of mathematics value.

In summary, our study confirmed the validity of the value beliefs scale in math, initially developed by Gaspard et al. (2015), as providing a comprehensive reflection of the broad range of value facets proposed to play a role in math motivation according to EVT. However, our results go beyond that in supporting the idea that this new multidimensional representation is hierarchically organized around of a global value component co-existing with a series of specificity value facets. Importantly, the adoption of this new representation, as captured by the application of the bifactor-ESEM framework, made it possible to obtain a measure of math valuing that was untainted by multicollinearity among subscales, a critical limitation of Gaspard et al. (2015, 2017) studies. From a practical perspective, this contribution is important as this instrument provides a way to achieve a more comprehensive representation of EVT's value components than previous measures (Durik et al., 2006). Indeed, this measure appears able to capture, in addition to a global estimate of math valuing, a total of eight specific facets of math value beyond this global component.

Although further research is necessary to see how the current results generalize across genders, to additional samples representing a broader range of age and cultures, to a wider array of outcome variables, and to a greater variety of achievement domains, our results provide clear initial evidence for the multidimensional and hierarchical nature of this value beliefs scale. In relation to future research, even though large samples are always preferable, researchers relying on smaller samples should not refrain from using ESEM, bifactor-ESEM, or any other latent models. Indeed, statistical simulation studies have demonstrated that under certain circumstances these models can perform quite well with small samples (Mai et al., 2018; Marsh, Hau, Balla, & Grayson, 1998). Still, things get more complex for practical applications requiring individual-specific scores on these factors. Indeed, when the measure is known to follow a bifactor structure, typical scale scores (taking the mean or sum of a series of items) will result in a confused mix of global and specific variance, cross-loadings and measurement error, and therefore will not reflect the true structure of the instrument (Perreira, Morin, Hebert, Gillet, Houle, & Berta, 2018). In this situation, computerized scores, generated based on algorithms similar to those used to generate factor scores, should be used (Morin, Boudrias, et al., 2017; Perreira et al., 2018).

Endnotes

1. Additional analyses demonstrated the inadequacy of a higher-order alternative to the bifactor models tested here, which converged on improper parameter estimates (i.e., negative variance estimates), and resulted in a weak higher-order factor. Additional analyses also failed to support a hybrid bifactor-higher-order (Guo et al., 2016).

2. This bifactor-CFA model resulted in a negative residual variance estimate, which was fixed to a value of .1 to achieve an interpretable solution.

3. We tested also a bifactor-ESEM model with gender (male and female) as covariate. Our results showed no gender effect on expectancy and global value (see also Guo, Marsh, Parker, Morin, & Yeung, 2015). Anyhow, we found significant gender effects on utility for school/job (β=.159, s.e.=.076) and opportunity cost (β=-.138, s.e.=.076) S-factors.

References

Asparouhov, T., & Muthén, B.(2009). Exploratory structural equation modeling. *Structural Equation Modeling, 16,* 397-438.

Asparouhov, T., Muthén, B.O., & Morin, A.J.S.(2015). Bayesian Structural equation modeling with cross-loadings and residual covariances. *Journal of Management, 41*, 1561-1577.

Durik, A.M., Vida, M., & Eccles, J.S.(2006). Task values and ability beliefs as predictors of high school literacy choices: A developmental analysis. *Journal of Educational Psychology, 98*, 382-393.

Eccles, J.S.(2009). Who am I and what am I going to do with my life? Personal and collective identities as motivators of action. *Educational Psychologist, 44,* 78-89.

Eccles, J.S., Adler, T.F., Futterman, R., Goff, S.B., Kaczala, C.M., Meece, J.L., & Midgley, C.(1983). Expectancies, values and academic behaviors. In J.T. Spence (Ed.), *Achievement and achievement motives* (pp.75-146). San Francisco:Freeman.

Eccles, J.S., & Wigfield, A.(1995). In the mind of the achiever: The structure of adolescents' academic achievement related beliefs and self-perceptions. *Personality and Social Psychology Bulletin, 21,* 215-225.

Eccles, J.S., & Wigfield, A.(2002). Motivational beliefs, values, and goals. *Annual Review of Psychology, 53,* 109-132.

Enders, C.(2010). Applied missing data analysis. New York:Guilford Press.

Gaspard, H., Dicke, A.L., Flunger, B., Schreier, B., Häfner, I., Trautwein, U., & Nagengast, B.(2015). More value through greater differentiation: Gender differences in value beliefs about math. *Journal of Educational Psychology, 107,* 663-677.

Gaspard, H., Häfner, I., Parrisius, C., Trautwein, U., & Nagengast, B.(2017). Assessing task values in five subjects during secondary school: Measurement structure and mean level differences across grade level, gender, and academic subject. *Contemporary Educational Psychology, 48,* 67-84.

Gaspard,H., Wigfield,A., Jiang,Y., Nagengast,B., Trautwein,U., & Marsh,H.W.(2018). Dimensional comparisons: How academic track students' achievements are related to their expectancy and value beliefs across multiple domains. *Contemporary Educational Psychology,52*,1-14.

Gignac, G.E.(2016). The higher-order model imposes a proportionality constraint: That is why the bifactor model tends to fit better. *Intelligence, 55,* 57-68.

Gudmundsson, E.(2009). Guidelines for translating and adapting psychological instruments. *Nordic Psychology, 61,* 29-45.

Guo, J., Marsh, H.W., Parker, P.D., Morin, A.J.S, & Dicke, T. (2017). Extending expectancy-value theory predictions of achievement and aspirations in science: Dimensional comparison processes and expectancy-by-value interactions. *Learning and Instruction, 49,* 81-91.

Guo, J., Marsh, H.W., Parker, P.D., Morin, A.J.S, & Yeung, A.S. (2015). Expectancy-value in mathematics, gender and socioeconomic background as predictors of achievement and aspirations: A multi-cohort study. *Learning and Individual Differences 37,* 161-168.

Guo, J., Nagengast, B., Marsh, H.W., Kelava, A., Gaspard, H., Brandt, H.,…Trautwein, U.(2016). Probing the unique contributions of self-concept, task values, and their interactions using multiple value facets and multiple academic outcomes. *AERA Open, 2,* 1-20.

Guo, J., Parker, P.D., Marsh, H.W., & Morin, A.J.S.(2015). Achievement, motivation, and educational choices: A longitudinal study of expectancy and value using a multiplicative perspective. *Developmental Psychology,51,*1163-1176.

Jacobs, J.E., Lanza, S., Osgood, D.W., Eccles, J.S., & Wigfield, A.(2002). Changes in children's self-competence and values: Gender and domain differences across grades on through twelve. *Child Development, 73*, 509-527.

Litalien, D., Morin, A.J.S., Gagné, M., Vallerand, R.J., Losier, G.F., & Ryan, R.M.(2017). Evidence of a continuum structure of academic self-determination: a two-study test using a bifactor-ESEM representation of academic motivation. *Contemporary Educational Psychology, 51,* 67-82.

Mai, Y., Zhang, Z., & Wen, Z.(2018). Comparing Exploratory Structural Equation Modeling and Existing Approaches for Multiple Regression with Latent Variables, *Structural Equation Modeling, 25*, 737-749.

Marsh, H.W.(1992). Self-Description Questionnaire III: A theoretical and empirical basis for the measurement of multiple dimensions of adolescent self-concept: A test manual and research monograph. Macarthur, Australia.

Marsh, H.W., Hau, K.T., Balla, J.R., & Grayson, D.(1998). Is more ever too much? The number of indicators per factor in confirmatory factor analysis. *Multivariate Behavioral Research, 33,* 181-220.

Marsh, H.W., Abduljabbar, A.S., Abu-Hilal, M., Morin, A.J.S., Abdelfattah, F., Leung, K.C. et al.(2013). Factorial, convergent, and discriminant validity of TIMSS math and science motivation measures: A comparison of Arab and Anglo-Saxon countries. *Journal of Educational Psychology,105*,108-128.

Marsh, H.W., Morin, A.J.S., Parker, P.D., & Kaur, G.(2014). Exploratory structural equation modeling: An integration of the best features of exploratory and confirmatory factor analysis. *Annual review of clinical psychology, 10*, 85-110.

Marsh, H.W., Wen, Z., & Hau, K.T.(2004). Structural equation models of latent interactions: Evaluation of alternative estimation strategies and indicator construction. *Psychological Methods, 9,* 275-300.

McAbee, S., Oswald, F., & Connelly, B.(2014). Bifactor models of personality and college student performance: A broad versus narrow view. *European Journal of Personality, 26,* 604-619.

McDonald, R.P.(1970). Theoretical foundations of principal factor analysis, canonical factor analysis, and alpha factor analysis. *British Journal of Mathematical & Statistical Psychology, 23,* 1-21.

Morin, A.J.S., Arens, A.K., & Marsh, H.W.(2016a). A bifactor exploratory structural equation modeling framework for the identification of distinct sources of construct-relevant psychometric multidimensionality. *Structural Equation Modeling, 23,* 116-139.

Morin, A.J.S., Arens, A.K., Tran, A., & Caci, H.(2016b). Exploring Sources of Construct- Relevant Multidimensionality in Psychiatric Measurement. *International Journal of Methods in Psychiatric Research, 25*, 277-288.

Morin, A.J.S., Boudrias, J.S., Marsh, H.W., McInerney, D.M., Dagenais-Desmarais, V., Madore, I., &Litalien, D.(2017). Complementary variable and person-centered approaches to the dimensionality of psychometric constructs: Application to psychological wellbeing at work. *Journal of Business and Psychology, 32*, 395-419.

Morin, A.J.S., Myers, N.D., & Lee, S.(2018). Modern factor analytic techniques: Bifactor models, exploratory structural equation modeling (ESEM) and bifactor-ESEM. In G. Tenenbaum, & R.C. Eklund, (Eds.), *Handbook of Sport Psychology* (4th ed.). New York: Wiley.

Muthén, L.K., & Muthén, B.O.(2014). *Mplus user's guide*. Los Angeles: Muthén & Muthén.

Nagengast, B., Marsh, H.W., Scalas, L.F., Xu, M., Hau, K.T., & Trautwein, U.(2011). Who took the "x" out of expectancy-value theory? *Psychological Science, 22*, 1058-1066.

Organization-for-Economic-Co-operation-and-Development–OECD (2006). *Evolution of Student Interest in Science and Technology Studies*. Paris: Author.

Organization-for-Economic-Co-operation-and-Development–OECD (2007). *PISA-2006 science competencies for tomorrow's world.* Paris: Author.

Perez, T., Cromley, J.G., & Kaplan, A.(2014). The role of identity development, values, and costs in college STEM retention. *Journal of Educational Psychology, 106,* 315-329.

Perreira, T.A., Morin, A.J.S., Hebert, M., Gillet, N., Houle, S.A., & Berta, W.(2018). The short form of the Workplace Affective Commitment Multidimensional Questionnaire (WACMQ-S): A bifactor-ESEM approach among healthcare professionals. *Journal of Vocational Behavior, 106,* 62-83.

Revelle, W., & Wilt, J.(2013). The general factor of personality: A general critique. *Journal of Research in Personality,47,*493-504.

Simpkins, S.D., Davis-Kean, P.E., & Eccles, J.S.(2006). Math and science motivation: A longitudinal examination of the links between choices and beliefs. *Developmental Psychology, 42,* 70-83.

Trautwein, U., Nagengast, B., Marsh, H.W., Gaspard, H., Dicke, A.L., Lüdtke, O., & Jonkmann, K.(2013). Expectancy-value theory revisited. In D.M. McInerney, H.W. Marsh, R.G. Craven, & F. Guay (Eds.), *Theory driving research: New wave perspectives on self-processes and human development* (pp. 233-249). Charlotte: Information Age.

Table 1. Fit Indices of Model Tested

Model	χ^2	*df*	CFI	TLI	RMSEA	RMSEA CI	AIC	CAIC	BIC	Adj-BIC
CFA	1332.014	592	.920	.910	.048	.045/.052	44647.626	44795.626	45281.954	44812.152
Bifactor-CFA	1907.811	592	.858	.840	.064	.061/.068	45396.304	45544.304	46030.632	45560.830
ESEM	635.827	368	.971	.948	.037	.032/.042	44035.867	44407.867	45630.258	44449.405
Bifactor-ESEM	552.706	340	.977	.955	.034	.029/.039	43989.807	44389.807	45704.206	44434.471

Note. CFA= confirmatory factor analysis; ESEM= exploratory structural equation model; χ^2= robust chi-square test of exact fit; CFI= comparative fit index; TLI= Tucker–Lewis index; RMSEA= root mean square error of approximation; CI= 90% confidence interval; AIC= Akaike information criterion; CAIC= consistent AIC; BIC= bayesian information criterion; Adj-BIC= sample-size adjusted BIC.

Table 2. Standardized Factor Correlations for the Confirmatory Factor Analysis (Above the Diagonal) and Exploratory Structural Equation Model (Below the Diagonal)

	FS1	FS2	FS3	FS4	FS5	FS6	FS7	FS8	FS9
FS1	1	.692**	.816**	.473**	.449**	.335**	-.657**	-.776**	-.405**
FS2	0.571**	1	.921**	.756**	.611**	.531**	-.311**	-.498**	-.171**
FS3	0.478*	0.553**	1	.797**	.732**	.472**	-.479**	-.657**	-.278**
FS4	0.260**	0.323**	0.463*	1	.786**	.436**	-.150**	-.365**	-.015
FS5	0.390**	0.441**	0.516**	0.557**	1	.403**	-.209**	-.361**	-.104
FS6	0.335**	0.428**	0.397**	0.305**	0.407**	1	-.101	-.180**	.117*
FS7	-0.466**	-0.079	-0.065	-0.104	-0.092	-0.051	1	.904**	.717**
FS8	-0.438**	-0.256**	-0.305**	-0.029	-0.209	-0.089	0.357**	1	.703**
FS9	-0.303**	-0.076	-0.166	-0.076	-0.111	0.082	0.530**	0.363**	1

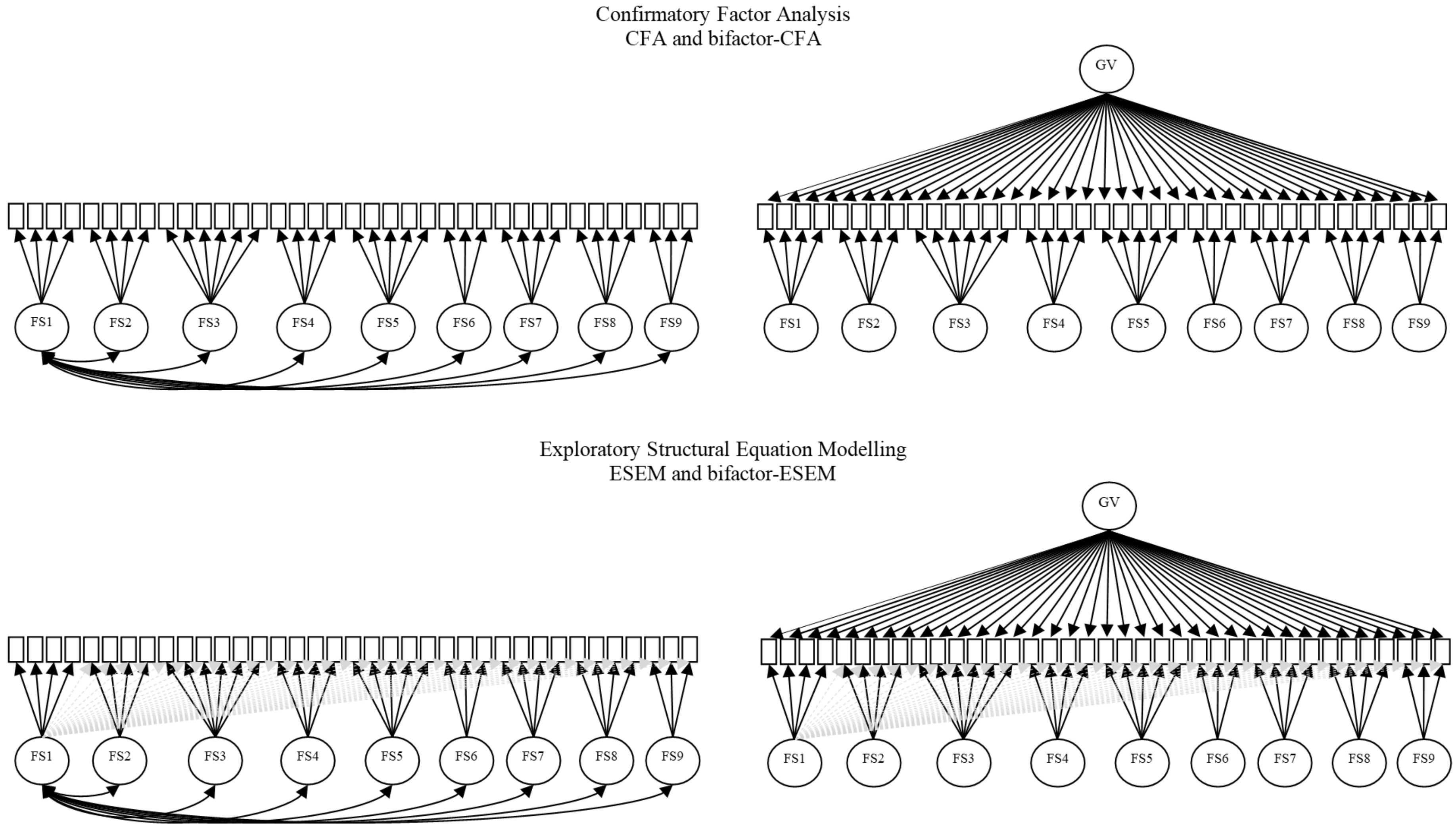
Note. FS1= intrinsic, FS2= importance of achievement, FS3= personal importance, FS4= utility for school/job, FS5= utility for life, FS6= social utility, FS7= effort required, FS8= emotional cost, FS9= opportunity cost.
** p<.01; *p<.05

Table 3. Standardized Parameter Estimates from the Bifactor-ESEM Solution and Omega Coefficient

Item	GV(λ)	FS1(λ)	FS2(λ)	FS3(λ)	FS4(λ)	FS5(λ)	FS6(λ)	FS7(λ)	FS8(λ)	FS9(λ)	δ
1. Math is fun to me.	**0.701**	**0.408**	0.024	-0.012	0.135	-0.083	0.029	-0.068	0.013	-0.045	0.309
10. I like doing math.	**0.771**	**0.520**	-0.066	-0.071	-0.056	-0.088	-0.012	-0.063	-0.080	-0.022	0.103
19. I simply like math.	**0.740**	**0.456**	0.024	-0.028	-0.029	-0.067	-0.034	-0.09	-0.125	-0.018	0.212
28. I enjoy dealing with mathematical topics.	**0.758**	**0.336**	-0.049	0.161	-0.067	-0.069	0.035	-0.047	-0.050	0.032	0.269
4. It is important to me to be good at math.	**0.645**	-0.053	**0.328**	-0.027	0.015	-0.040	0.025	0.161	0.086	0.017	0.437
13. Being good at math means a lot to me.	**0.709**	0.034	**0.280**	-0.018	0.036	-0.047	0.117	0.171	0.116	0.097	0.348
22. Performing well in math is important to me.	**0.739**	-0.016	**0.442**	0.057	0.093	0.011	0.05	0.075	0.062	0.032	0.234
31. Good grades in math are very important to me.	**0.674**	-0.063	**0.239**	0.205	0.148	0.002	0.105	0.089	0.082	0.033	0.394
7. I care a lot about remembering the things we learn in math.	**0.657**	0.094	0.163	**0.101**	-0.010	-0.005	0.078	0.074	0.169	0.051	0.480
16. Math is not meaningful to me. (R)	**0.679**	0.104	0.071	**0.029**	-0.116	0.038	-0.044	-0.042	-0.208	0.040	0.459
25. I'm really keen on learning a lot in math.	**0.736**	0.006	0.150	**0.248**	0.173	0.001	0.000	0.070	0.092	0.087	0.323
34. Math is very important to me personally.	**0.734**	-0.003	-0.002	**0.190**	0.170	0.250	0.052	0.081	-0.043	-0.063	0.318
36. To be honest, I don't care about math. (R)	**0.715**	0.089	0.019	**-0.024**	-0.078	0.001	-0.137	-0.043	-0.140	0.022	0.432
37. It is important to me to know a lot of math.	**0.787**	-0.039	0.018	**0.315**	-0.025	0.017	0.016	0.014	0.144	0.056	0.254
2. It is worth making an effort in math, because it will save me a lot of trouble at school in the next years.	**0.521**	-0.046	0.143	-0.040	**0.345**	0.068	0.005	0.111	0.069	0.054	0.561
11. Good grades in math can be of great value to me later on.	**0.544**	-0.109	-0.001	-0.040	**0.363**	0.102	0.038	0.165	-0.006	0.125	0.504
17. Being good at math pays off, because it is simply needed at school.	**0.420**	0.024	0.123	0.124	**0.538**	0.224	0.052	0.042	0.024	0.075	0.442
26. Learning math is worthwhile, because it improves my job and career chances.	**0.585**	-0.177	-0.054	0.06	**0.436**	0.228	0.005	0.114	0.088	0.087	0.350
5. Understanding math has many benefits in my daily life.	**0.577**	-0.09	0.002	-0.125	-0.05	**0.491**	-0.019	0.149	0.161	0.051	0.349
14. Math contents will help me in my life.	**0.657**	-0.189	-0.098	-0.134	0.097	**0.491**	-0.015	0.098	0.175	0.084	0.207
20. Math comes in handy in everyday life and leisure time.	**0.515**	-0.025	0.068	-0.027	-0.014	**0.553**	0.026	0.038	0.003	0.020	0.420
29. I will often need math in my life.	**0.563**	-0.087	-0.141	0.158	0.215	**0.582**	0.051	0.079	-0.034	0.049	0.233
32. Math is directly applicable in everyday life.	**0.355**	0.085	0.089	0.131	0.177	**0.794**	0.077	-0.035	-0.056	-0.034	0.168
8. Being well versed in math will go down well with my classmates.	**0.302**	0.009	0.131	-0.084	-0.04	0.010	**0.606**	0.031	0.117	0.127	0.484
23. I can impress others with intimate knowledge in math.	**0.422**	0.077	0.015	0.088	0.056	0.085	**0.440**	0.089	0.029	0.098	0.586
35. If I know a lot in math, I will leave a good impression on my classmates.	**0.336**	-0.046	0.003	0.044	0.054	0.054	**0.832**	0.044	0.011	0.110	0.170
3. Doing math is exhausting to me.	**-0.483**	-0.192	-0.005	0.042	0.173	0.084	0.054	**0.508**	0.207	0.091	0.379
12. I often feel completely drained after doing math.	**-0.032**	-0.028	0.070	-0.16	0.004	0.094	0.074	**0.295**	0.184	0.328	0.725
21. Dealing with math drains a lot of my energy.	**-0.399**	0.074	0.174	0.037	0.023	0.061	0.05	**0.513**	0.169	0.279	0.427
30. Learning math exhausts me.	**-0.509**	-0.082	0.079	0.085	0.115	0.097	0.033	**0.604**	0.206	0.131	0.272
6. I'd rather not do math, because it only worries me.	**-0.605**	0.008	0.123	-0.035	-0.058	0.006	0.092	0.081	**0.305**	0.174	0.475
15. When I deal with math, I get annoyed.	**-0.587**	-0.121	0.043	0.037	0.080	0.059	0.034	0.177	**0.579**	0.134	0.242
24. Math is a real burden to me.	**-0.641**	-0.084	0.032	0.106	0.110	0.110	0.062	0.341	**0.289**	0.223	0.292
33. Doing math makes me really nervous.	**-0.579**	-0.045	0.088	0.081	0.032	0.088	0.068	0.358	**0.404**	0.122	0.328
9. I have to give up other activities that I like to be successful at math.	**-0.066**	-0.023	-0.012	-0.048	0.022	-0.009	0.157	0.108	0.051	**0.574**	0.624
18. I have to give up a lot to do well in math.	**-0.324**	-0.009	0.112	0.001	0.058	0.072	0.107	0.157	0.153	**0.752**	0.249
27. I'd have to sacrifice a lot of free time to be good at math.	**-0.319**	-0.014	-0.043	0.112	0.164	0.055	0.077	0.350	0.189	**0.528**	0.411
Omega coefficient	.989	.944	.841	.440	.902	.970	.947	.918	.881	.950	

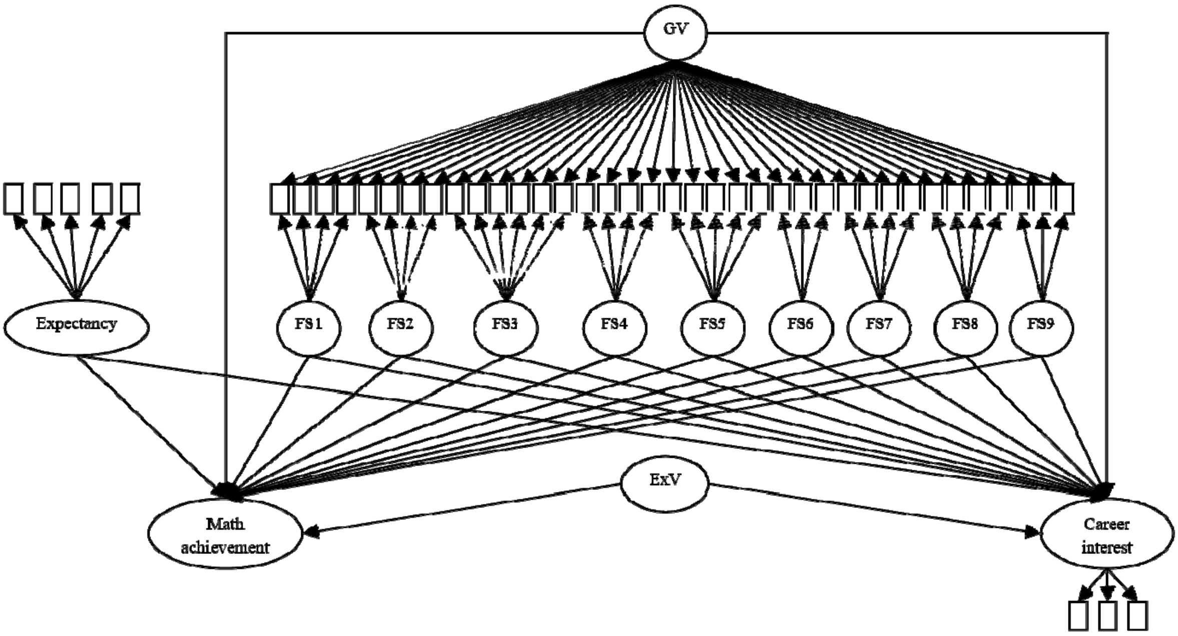
Note. λ= standardized factor loading; δ=standardized item uniqueness; bold=target factor loadings; GV=global value factor, FS1=intrinsic, FS2= importance of achievement, FS3= personal importance, FS4= utility for school/job, FS5= utility for life, FS6= social utility, FS7= effort required, FS8= emotional cost, FS9= opportunity cost.

Figure 1. Simplified Conceptual Representations of the Estimated Models



Note. Factors intercorrelations and ESEM cross-loadings (in the ESEM models all items are allowed to cross-load on all of the specific-factors) are not included in this figure to avoid cluttering, but simply illustrated for Factor 1.

Figure 2. Simplified Conceptual Representation of the Predictive Model



Note. Correlations between expectancy and value predictors (Global value factor and Specific value facets) are not included in this figure to avoid cluttering. ExV is a representation of the latent interaction effects between expectancy and global value (the product of indicators approach; see Appendix 5).

Electronic Supplementary Material for:

Value Beliefs about Math: a Bifactor-ESEM Representation

Appendix 1
EVT Theory

Eccles and colleagues defined expectancies of success as individuals' beliefs about how well they will do on upcoming tasks, either in the immediate or long-term future (Eccles & Wigfield, 2002). The broad definition of task value assumes that the extent to which a person values a specific task is determined by the value that a person attaches to that task, and is influenced by characteristics of the task itself, as well as by the needs, goals, motivational orientations and affective memories of the person in relation to similar tasks (Eccles et al., 1983). Even though a global task value is well aligned to EVT assumptions, Eccles et al. (Eccles et al., 1983; Eccles & Wigfield, 2002; Wigfield, Rosenzweig, & Eccles, 2017) defined four major components contributing to the value of a task: Research (e.g., Eccles, 2009; Guo, Marsh, Morin, Parker, & Kaur, 2015a) has supported the key EVT proposition that these expectancies and value components reflect core motivational processes with a notable impact on achievement outcomes (e.g., performance, effort, engagement and future aspirations) in a variety of domains, including mathematics. When expectancy and value are considered simultaneously, expectancy typically comes out as the strongest predictor of mathematics achievement, whereas value tends to emerge as the strongest predictor of career aspirations (Eccles & Wigfield, 2002; Marsh, Hau, Wen, Nagengast, & Morin, 2013; Simpkins, Davis-Kean, & Eccles, 2006). Moreover, expectancy and value are assumed to interact with one another in influencing mathematics achievement and career intentions (Guo, Parker, Marsh, & Morin, 2015b; Nagengast, Marsh, Scalas, Xu, Hau, & Trautwein, 2011).

Appendix 2
Bifactor Models

The G- and S- factor from a bifactor model logically correspond, respectively, to the higher-order factor and to the disturbances associated with the first-order factors in a higher-order model, but without unrealistic proportionality constraints. In fact, when these proportionality constraints are met in the population model, a bifactor model can easily be converted to a higher-order equivalent (Jennrich & Bentler, 2011).

Since bifactor models are orthogonal, they are also better suited at solving issues related to high correlations among specific factors compared to CFA, ESEM, or higher-order models (Litalien, Morin, Gagné, Vallerand, Losier, & Ryan, 2017) and provide a way to directly assess the distinct contribution of the S- and G- factors in terms of prediction. For instance, considering the similarity between bifactor S-factors and the disturbances of the first-order factors in a hierarchical model, it is easy to see that both are specified as orthogonal to one another and to the G- or higher-order factor. However, this is not the case for the first-order factor themselves whose correlations in fact only reflect the effect of the higher-order factor, thus creating multicollinearity in any predictive model where they would be included along with the higher-order factor (Morin, Myers, & Lee, 2018). This limitation does not apply to bifactor models. For all these reasons, bifactor models should be preferred over higher-order models unless there are strong theoretical reasons suggesting the need for indirect relations (and related proportionality constraints) between the indicators, the G-factor, and the disturbances of the first-order factors (Gignac, 2016).

It is interesting to note that the incorporation of one global orthogonal factor to an otherwise multidimensional factor model could be used to reflect the presence of shared response tendencies (i.e., shared method factor) across items (e.g., Podsakoff, MacKenzie, Lee, & Podsakoff, 2003). However, it is important to note that this "method factor" approach, contrary to the bifactor approach, does not rely on the orthogonality of the specific factors, thus resulting in a highly different representation of the data (for a demonstration see Morin et al., 2018). In addition, research has shown this global "method factor" to retain some meaningful information that could not be explained away as a simple methodological artefact (Richardson, Simmering, & Sturman, 2009).

Appendix 3
CFA, ESEM and Bifactor-ESEM Solutions

In the CFA model, items were only allowed to load on their a priori factor, without cross-loadings. In the bifactor-CFA model, the specific factors (S-factors) were specified as in CFA, and all items were also allowed to load simultaneously on one global factor (G-factor). In accordance with typical bifactor assumptions, the global factor and all specific factors were specified as orthogonal (e.g., Morin, Arens, & Marsh, 2016).

The ESEM solution was specified while relying on oblique target rotation (Asparouhov & Muthén, 2009), in which all cross-loadings were “targeted” to be close to zero, whereas all of the main loadings were freely estimated as in the CFA model. Finally, the bifactor-ESEM solution was estimated as the ESEM model using a bifactor orthogonal target rotation (Reise, 2012), allowing for the estimation of a G-factor (see Figure 1).

One a priori correlated uniqueness was incorporated to all models to take into account the methodological artifact known to be associated with the two (16 and 36) negatively-worded items (Gaspard et al., 2015; Morin et al., 2016).

The following goodness-of-fit indexes were examined: the chi-square (χ^2) test of exact fit, the comparative fit index (CFI), the Tucker–Lewis index (TLI), and the root mean square error of approximation (RMSEA) with its 90% confidence interval. For model comparisons, the following guidelines were used (Chen, 2007; Cheung & Rensvold, 2002): a change in CFI or TLI of .01 or less and a change in RMSEA of .015 or less between two models indicate that the most parsimonious model should be retained. We also considered the Akaike information criterion (AIC), the consistent AIC (CAIC), the bayesian information criterion (BIC) and the sample-size adjusted BIC (adj-BIC). Lower values on these indexes reflect better fit.

Appendix 4
Preliminary Analysis

The a priori CFA solution, corresponding to Gaspard et al.’ (2015) model, resulted in estimation problems related to the presence of a linear dependency among some of the factors. Indeed, strong correlations were found between the following factors: (1) utility for school and utility for job (r =.975, s.e. =.050); (2) utility for daily life and general utility for future life (r =.995, s.e. =.022). Gaspard et al. (2015) intercorrelations were respectively: r = .66 and r = .86. The corresponding ESEM solution with 11 factors and target rotation did not converge due to a negative residual variance that could not be fixed within this framework. Moreover, similar results, albeit not as extreme, were also observed for all alternative models, supporting the idea that this overlap was not limited to a specific measurement model (CFA), and suggesting the presence of conceptual overlap between these scales, which is coherent from a statistical (Bagozzi & Kimmel, 1995) and theoretical standpoint (Eccles & Wigfield, 2002). The definition of utility value in EVT relates directly to internalized short- as well as long-term goals (Eccles & Wigfield, 2002), therefore, Gaspard et al.’s (2015) value scale consisted of items assessing the utility of different life domains from a short- term (school, daily life, social life; Eccles et al., 1983; Hulleman & Harackiewicz, 2009) as well as from a long-term perspective (job, future life in general; Conley, 2012; Hulleman, Durik, Schweigert, & Harackiewicz, 2008). This decision is supported by the nature of our sample that included mainly students attending the last year of high school, so the distinction between utility for school and job, for daily or future life could be not strongly defined. Therefore, in all subsequent analyses, items related to utility for school and for job were merged to reflect a single utility for school/job factor, and items related to utility for daily and future life were merged to reflect a single utility for life factor. This 9-factor model (intrinsic, importance of achievement, personal importance, utility for school/job, utility for life, social utility, effort required, opportunity cost, emotional cost) did not lead to any further estimation problems.

Appendix 5
Mplus Input

CFA

```
ANALYSIS:
   estimator = MLR;
MODEL:
   FS1 BY V1 V10 V19 V28;
   FS2 BY V4 V13 V22 V31;
   FS3 BY V7 V16 V25 V34 V36 V37;
   FS4 BY V8 V23 V35;
```

```
   FS5 BY V11 V26 V2 V17;
   FS6 BY V14 V29 V5 V20 V32;
   FS7 BY V3 V21 V30 V12;
   FS8 BY V6 V15 V24 V33;
   FS9 BY V9 V18 V27;
   V16 WITH V36; ! negatively-worded items
```

Bifactor-CFA

```
! The @ symbol is used to fix parameter estimates to a specific value
! All factors are set to be orthogonal (correlations @0)
ANALYSIS:
   estimator = MLR;
   ITERATIONS = 100000;
MODEL:
   GV BY V1-V37;
   FS1 BY V1 V10 V19 V28;
   FS2 BY V4 V13 V22 V31;
   FS3 BY V7 V16 V25 V34 V36 V37;
   FS4 BY V8 V23 V35;
   FS5 BY V11 V26 V2 V17;
   FS6 BY V14 V29 V5 V20 V32;
   FS7 BY V3 V21 V30 V12;
   FS8  BY V6 V15 V24 V33;
   FS9  BY V9 V18 V27;
   V16 WITH V36;
   GV WITH FS1@0 FS2@0 FS3@0 FS4@ FS5@0 FS6@0 FS7@0 FS8@0 FS9@0;
   FS1 WITH FS2@0 FS3@0 FS4@ FS5@0 FS6@0 FS7@0 FS8@0 FS9@0;
   FS2 WITH FS3@0 FS4@ FS5@0 FS6@0 FS7@0 FS8@0 FS9@0;
   FS3 WITH FS4@ FS5@0 FS6@0 FS7@0 FS8@0 FS9@0;
   FS4 WITH FS5@0 FS6@0 FS7@0 FS8@0 FS9@0;
   FS5 WITH FS6@0 FS7@0 FS8@0 FS9@0;
   FS6 WITH FS7@0 FS8@0 FS9@0;
   FS7 WITH FS8@0 FS9@0;
   FS8 WITH FS9@0;
   FS3@.1; ! the residual variance was fixed to.1 to achieve an interpretable solution
```

ESEM

```
! An ESEM model is specified with target oblique rotation
ANALYSIS:
  estimator = MLR;
  ROTATION = TARGET;
! The factors (FS1 to FS9) are defined with main loadings from their respective items
! In addition to these main loadings, all other cross-loadings are estimated but targeted
! to be as close to 0 as possible (~0)
! Factors forming a single set of ESEM factors (with cross-loadings between factors)
! are indicated by using the same label in parenthesis after * (*1)
MODEL:
FS1 BY
  V1 V2~0 V3~0 V4~0 V5~0 V6~0 V7~0 V8~0 V9~0
  V10 V11~0 V12~0 V13~0 V14~0 V15~0 V16~0 V17~0 V18~0
  V19 V20~0 V21~0 V22~0 V23~0 V24~0 V25~0 V26~0
  V27~0 V28 V29~0 V30~0
  V31~0 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS2 BY V1~0 V2~0 V3~0 V4 V5~0 V6~0 V7~0 V8~0 V9~0
  V10~0 V11~0 V12~0 V13 V14~0 V15~0 V16~0 V17~0 V18~0
```

```
  V19~0 V20~0 V21~0 V22 V23~0 V24~0 V25~0 V26~0
  V27~0 V28~0 V29~0 V30~0
  V31 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS3 BY V1~0 V2~0 V3~0 V4~0 V5~0 V6~0 V7 V8~0 V9~0
  V10~0 V11~0 V12~0 V13~0 V14~0 V15~0 V16 V17~0 V18~0
  V19~0 V20~0 V21~0 V22~0 V23~0 V24~0 V25 V26~0
  V27~0 V28~0 V29~0 V30~0
  V31~0 V32~0 V33~0 V34 V35~0 V36 V37(*1);
FS4 BY V1~0 V2 V3~0 V4~0 V5~0 V6~0 V7~0 V8~0 V9~0
  V10~0 V11 V12~0 V13~0 V14~0 V15~0 V16~0 V17 V18~0
  V19~0 V20~0 V21~0 V22~0 V23~0 V24~0 V25~0 V26
  V27~0 V28~0 V29~0 V30~0
  V31~0 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS5 BY V1~0 V2~0 V3~0 V4~0 V5 V6~0 V7~0 V8~0 V9~0
  V10~0 V11~0 V12~0 V13~0 V14 V15~0 V16~0 V17~0 V18~0
  V19~0 V20 V21~0 V22~0 V23~0 V24~0 V25~0 V26~0
  V27~0 V28~0 V29 V30~0
  V31~0 V32 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS6 BY V1~0 V2~0 V3~0 V4~0 V5~0 V6~0 V7~0 V8 V9~0
  V10~0 V11~0 V12~0 V13~0 V14~0 V15~0 V16~0 V17~0 V18~0
  V19~0 V20~0 V21~0 V22~0 V23 V24~0 V25~0 V26~0
  V27~0 V28~0 V29~0 V30~0
  V31~0 V32~0 V33~0 V34~0 V35 V36~0 V37~0(*1);
FS7 BY
  V1~0 V2~0 V3 V4~0 V5~0 V6~0 V7~0 V8~0 V9~0
  V10~0 V11~0 V12 V13~0 V14~0 V15~0 V16~0 V17~0 V18~0
  V19~0 V20~0 V21 V22~0 V23~0 V24~0 V25~0 V26~0
  V27~0 V28~0 V29~0 V30
  V31~0 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS8 BY
  V1~0 V2~0 V3~0 V4~0 V5~0 V6 V7~0 V8~0 V9~0
  V10~0 V11~0 V12~0 V13~0 V14~0 V15 V16~0 V17~0 V18~0
  V19~0 V20~0 V21~0 V22~0 V23~0 V24 V25~0 V26~0
  V27~0 V28~0 V29~0 V30~0
  V31~0 V32~0 V33 V34~0 V35~0 V36~0 V37~0(*1);
FS9 BY
  V1~0 V2~0 V3~0 V4~0 V5~0 V6~0 V7~0 V8~0 V9
  V10~0 V11~0 V12~0 V13~0 V14~0 V15~0 V16~0 V17~0 V18
  V19~0 V20~0 V21~0 V22~0 V23~0 V24~0 V25~0 V26~0
  V27 V28~0 V29~0 V30~0
  V31~0 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
  V16 WITH V36;
```

Bifactor-ESEM

```
! A Bifactor-ESEM model is specified with orthogonal target rotation
ANALYSIS:
   estimator = MLR;
   ROTATION = TARGET (orthogonal);
! The specific factors (FS1 to FS9) are defined with main loadings from their respective items
! All other cross-loadings are estimated but targeted to be as close to 0 as possible (~0)
! The global factor (GV) is defined through main loadings from all items, and is included in
! the same set of ESEM factors as FS1-FS9 (*1)
MODEL:
GV BY V1-V37(*1);
FS1 BY
```

```
   V1 V2~0 V3~0 V4~0 V5~0 V6~0 V7~0 V8~0 V9~0
   V10 V11~0 V12~0 V13~0 V14~0 V15~0 V16~0 V17~0 V18~0
   V19 V20~0 V21~0 V22~0 V23~0 V24~0 V25~0 V26~0
   V27~0 V28 V29~0 V30~0
   V31~0 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS2 BY V1~0 V2~0 V3~0 V4 V5~0 V6~0 V7~0 V8~0 V9~0
   V10~0 V11~0 V12~0 V13 V14~0 V15~0 V16~0 V17~0 V18~0
   V19~0 V20~0 V21~0 V22 V23~0 V24~0 V25~0 V26~0
   V27~0 V28~0 V29~0 V30~0
   V31 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS3 BY V1~0 V2~0 V3~0 V4~0 V5~0 V6~0 V7 V8~0 V9~0
   V10~0 V11~0 V12~0 V13~0 V14~0 V15~0 V16 V17~0 V18~0
   V19~0 V20~0 V21~0 V22~0 V23~0 V24~0 V25 V26~0
   V27~0 V28~0 V29~0 V30~0
   V31~0 V32~0 V33~0 V34 V35~0 V36 V37(*1);
FS4 BY V1~0 V2 V3~0 V4~0 V5~0 V6~0 V7~0 V8~0 V9~0
   V10~0 V11 V12~0 V13~0 V14~0 V15~0 V16~0 V17 V18~0
   V19~0 V20~0 V21~0 V22~0 V23~0 V24~0 V25~0 V26
   V27~0 V28~0 V29~0 V30~0
   V31~0 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS5 BY V1~0 V2~0 V3~0 V4~0 V5 V6~0 V7~0 V8~0 V9~0
   V10~0 V11~0 V12~0 V13~0 V14 V15~0 V16~0 V17~0 V18~0
   V19~0 V20 V21~0 V22~0 V23~0 V24~0 V25~0 V26~0
   V27~0 V28~0 V29 V30~0
   V31~0 V32 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS6 BY V1~0 V2~0 V3~0 V4~0 V5~0 V6~0 V7~0 V8 V9~0
   V10~0 V11~0 V12~0 V13~0 V14~0 V15~0 V16~0 V17~0 V18~0
   V19~0 V20~0 V21~0 V22~0 V23 V24~0 V25~0 V26~0
   V27~0 V28~0 V29~0 V30~0
   V31~0 V32~0 V33~0 V34~0 V35 V36~0 V37~0(*1);
FS7 BY
   V1~0 V2~0 V3 V4~0 V5~0 V6~0 V7~0 V8~0 V9~0
   V10~0 V11~0 V12 V13~0 V14~0 V15~0 V16~0 V17~0 V18~0
   V19~0 V20~0 V21 V22~0 V23~0 V24~0 V25~0 V26~0
   V27~0 V28~0 V29~0 V30
   V31~0 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS8 BY
   V1~0 V2~0 V3~0 V4~0 V5~0 V6 V7~0 V8~0 V9~0
   V10~0 V11~0 V12~0 V13~0 V14~0 V15 V16~0 V17~0 V18~0
   V19~0 V20~0 V21~0 V22~0 V23~0 V24 V25~0 V26~0
   V27~0 V28~0 V29~0 V30~0
   V31~0 V32~0 V33 V34~0 V35~0 V36~0 V37~0(*1);
FS9 BY
   V1~0 V2~0 V3~0 V4~0 V5~0 V6~0 V7~0 V8~0 V9
   V10~0 V11~0 V12~0 V13~0 V14~0 V15~0 V16~0 V17~0 V18
   V19~0 V20~0 V21~0 V22~0 V23~0 V24~0 V25~0 V26~0
   V27 V28~0 V29~0 V30~0
   V31~0 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
   V16 WITH V36;
```

Predictive model

```
USEVARIABLES ARE V1-V37 C1-C3 SDQ1 SDQ2 SDQ3 SDQ4 SDQ5 MAT;
ANALYSIS:
   estimator = MLR;
   ROTATION = TARGET (orthogonal);
```

```
MODEL:
GV BY V1-V37(*1);
FS1 BY
   V1 V2~0 V3~0 V4~0 V5~0 V6~0 V7~0 V8~0 V9~0
   V10 V11~0 V12~0 V13~0 V14~0 V15~0 V16~0 V17~0 V18~0
   V19 V20~0 V21~0 V22~0 V23~0 V24~0 V25~0 V26~0
   V27~0 V28 V29~0 V30~0
   V31~0 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS2 BY V1~0 V2~0 V3~0 V4 V5~0 V6~0 V7~0 V8~0 V9~0
   V10~0 V11~0 V12~0 V13 V14~0 V15~0 V16~0 V17~0 V18~0
   V19~0 V20~0 V21~0 V22 V23~0 V24~0 V25~0 V26~0
   V27~0 V28~0 V29~0 V30~0
   V31 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS3 BY V1~0 V2~0 V3~0 V4~0 V5~0 V6~0 V7 V8~0 V9~0
   V10~0 V11~0 V12~0 V13~0 V14~0 V15~0 V16 V17~0 V18~0
   V19~0 V20~0 V21~0 V22~0 V23~0 V24~0 V25 V26~0
   V27~0 V28~0 V29~0 V30~0
   V31~0 V32~0 V33~0 V34 V35~0 V36 V37(*1);
FS4 BY V1~0 V2 V3~0 V4~0 V5~0 V6~0 V7~0 V8~0 V9~0
   V10~0 V11 V12~0 V13~0 V14~0 V15~0 V16~0 V17 V18~0
   V19~0 V20~0 V21~0 V22~0 V23~0 V24~0 V25~0 V26
   V27~0 V28~0 V29~0 V30~0
   V31~0 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS5 BY V1~0 V2~0 V3~0 V4~0 V5 V6~0 V7~0 V8~0 V9~0
   V10~0 V11~0 V12~0 V13~0 V14 V15~0 V16~0 V17~0 V18~0
   V19~0 V20 V21~0 V22~0 V23~0 V24~0 V25~0 V26~0
   V27~0 V28~0 V29 V30~0
   V31~0 V32 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS6 BY V1~0 V2~0 V3~0 V4~0 V5~0 V6~0 V7~0 V8 V9~0
   V10~0 V11~0 V12~0 V13~0 V14~0 V15~0 V16~0 V17~0 V18~0
   V19~0 V20~0 V21~0 V22~0 V23 V24~0 V25~0 V26~0
   V27~0 V28~0 V29~0 V30~0
   V31~0 V32~0 V33~0 V34~0 V35 V36~0 V37~0(*1);
FS7 BY
   V1~0 V2~0 V3 V4~0 V5~0 V6~0 V7~0 V8~0 V9~0
   V10~0 V11~0 V12 V13~0 V14~0 V15~0 V16~0 V17~0 V18~0
   V19~0 V20~0 V21 V22~0 V23~0 V24~0 V25~0 V26~0
   V27~0 V28~0 V29~0 V30
   V31~0 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS8 BY
   V1~0 V2~0 V3~0 V4~0 V5~0 V6 V7~0 V8~0 V9~0
   V10~0 V11~0 V12~0 V13~0 V14~0 V15 V16~0 V17~0 V18~0
   V19~0 V20~0 V21~0 V22~0 V23~0 V24 V25~0 V26~0
   V27~0 V28~0 V29~0 V30~0
   V31~0 V32~0 V33 V34~0 V35~0 V36~0 V37~0(*1);
FS9 BY
   V1~0 V2~0 V3~0 V4~0 V5~0 V6~0 V7~0 V8~0 V9
   V10~0 V11~0 V12~0 V13~0 V14~0 V15~0 V16~0 V17~0 V18
   V19~0 V20~0 V21~0 V22~0 V23~0 V24~0 V25~0 V26~0
   V27 V28~0 V29~0 V30~0
   V31~0 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
   V16 WITH V36;
CAR BY C1-C3;
SDQ BY SDQ1 SDQ2 SDQ3 SDQ4 SDQ5;
CAR ON SDQ GV FS1 FS2 FS3 FS4 FS5 FS6 FS7 FS8 FS9;
```

MAT ON SDQ GV FS1 FS2 FS3 FS4 FS5 FS6 FS7 FS8 FS9;

Predictive model with the ExV interaction

```
! The product of indicators approach was used to test latent interactions (Marsh, Wen, & Hau, 2004)
USEVARIABLES ARE V1-V37 C1-C3 SDQ1 SDQ2 SDQ3 SDQ4 SDQ5 MAT E1XV1 E2XV2
E3XV3 E4XV4 E5XV5;
 CENTERING = GRANDMEAN (E1XV1 E2XV2 E3XV3 E4XV4 E5XV5);
 DEFINE:
   E1XV1 = SDQ3* V37;
   E2XV2 = SDQ4* V10;
   E3XV3 = SDQ2* V28;
   E4XV4 = SDQ5* V19;
   E5XV5 = SDQ1* V34;
ANALYSIS:
  estimator = MLR;
  iterations = 10000;
  ROTATION = TARGET (orthogonal);
MODEL:
  INTERAC BY E1XV1 E2XV2 E3XV3 E4XV4 E5XV5;
  GV BY V1-V37(*1);
FS1 BY
   V1 V2~0 V3~0 V4~0 V5~0 V6~0 V7~0 V8~0 V9~0
   V10 V11~0 V12~0 V13~0 V14~0 V15~0 V16~0 V17~0 V18~0
   V19 V20~0 V21~0 V22~0 V23~0 V24~0 V25~0 V26~0
   V27~0 V28 V29~0 V30~0
   V31~0 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS2 BY V1~0 V2~0 V3~0 V4 V5~0 V6~0 V7~0 V8~0 V9~0
   V10~0 V11~0 V12~0 V13 V14~0 V15~0 V16~0 V17~0 V18~0
   V19~0 V20~0 V21~0 V22 V23~0 V24~0 V25~0 V26~0
   V27~0 V28~0 V29~0 V30~0
   V31 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS3 BY V1~0 V2~0 V3~0 V4~0 V5~0 V6~0 V7 V8~0 V9~0
   V10~0 V11~0 V12~0 V13~0 V14~0 V15~0 V16 V17~0 V18~0
   V19~0 V20~0 V21~0 V22~0 V23~0 V24~0 V25 V26~0
   V27~0 V28~0 V29~0 V30~0
   V31~0 V32~0 V33~0 V34 V35~0 V36 V37(*1);
FS4 BY V1~0 V2 V3~0 V4~0 V5~0 V6~0 V7~0 V8~0 V9~0
   V10~0 V11 V12~0 V13~0 V14~0 V15~0 V16~0 V17 V18~0
   V19~0 V20~0 V21~0 V22~0 V23~0 V24~0 V25~0 V26
   V27~0 V28~0 V29~0 V30~0
   V31~0 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS5 BY V1~0 V2~0 V3~0 V4~0 V5 V6~0 V7~0 V8~0 V9~0
   V10~0 V11~0 V12~0 V13~0 V14 V15~0 V16~0 V17~0 V18~0
   V19~0 V20 V21~0 V22~0 V23~0 V24~0 V25~0 V26~0
   V27~0 V28~0 V29 V30~0
   V31~0 V32 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS6 BY V1~0 V2~0 V3~0 V4~0 V5~0 V6~0 V7~0 V8 V9~0
   V10~0 V11~0 V12~0 V13~0 V14~0 V15~0 V16~0 V17~0 V18~0
   V19~0 V20~0 V21~0 V22~0 V23 V24~0 V25~0 V26~0
   V27~0 V28~0 V29~0 V30~0
   V31~0 V32~0 V33~0 V34~0 V35 V36~0 V37~0(*1);
FS7 BY
   V1~0 V2~0 V3 V4~0 V5~0 V6~0 V7~0 V8~0 V9~0
   V10~0 V11~0 V12 V13~0 V14~0 V15~0 V16~0 V17~0 V18~0
   V19~0 V20~0 V21 V22~0 V23~0 V24~0 V25~0 V26~0
```

```
    V27~0 V28~0 V29~0 V30
    V31~0 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
FS8 BY
    V1~0 V2~0 V3~0 V4~0 V5~0 V6 V7~0 V8~0 V9~0
    V10~0 V11~0 V12~0 V13~0 V14~0 V15 V16~0 V17~0 V18~0
    V19~0 V20~0 V21~0 V22~0 V23~0 V24 V25~0 V26~0
    V27~0 V28~0 V29~0 V30~0
    V31~0 V32~0 V33 V34~0 V35~0 V36~0 V37~0(*1);
FS9 BY
    V1~0 V2~0 V3~0 V4~0 V5~0 V6~0 V7~0 V8~0 V9
    V10~0 V11~0 V12~0 V13~0 V14~0 V15~0 V16~0 V17~0 V18
    V19~0 V20~0 V21~0 V22~0 V23~0 V24~0 V25~0 V26~0
    V27 V28~0 V29~0 V30~0
    V31~0 V32~0 V33~0 V34~0 V35~0 V36~0 V37~0(*1);
    V16 WITH V36;
CAR BY CARR1-CARR3;
SDQ BY SDQ1 SDQ2 SDQ3 SDQ4 SDQ5;
CAR ON INTERAC SDQ GV FS1 FS2 FS3 FS4 FS5 FS6 FS7 FS8 FS9;
MAT ON INTERAC SDQ GV FS1 FS2 FS3 FS4 FS5 FS6 FS7 FS8 FS9;
    E1XV1 WITH SDQ3 V37;
    E2XV2 WITH SDQ4 V10;
    E3XV3 WITH SDQ2 V28;
    E4XV4 WITH SDQ5 V19;
    E5XV5 WITH SDQ1 V34;
```

Appendix 6

Table A6.1. Standardized Parameter Estimates from the ESEM Solution

Item	FS1(λ)	FS2(λ)	FS3(λ)	FS4(λ)	FS5(λ)	FS6(λ)	FS7(λ)	FS8(λ)	FS9(λ)	δ
1. Math is fun to me.	**0.692**	0.071	0.119	-0.128	0.037	0.071	0.028	-0.011	0.077	0.308
10. I like doing math.	**0.893**	-0.041	0.023	0.018	0.017	0.038	0.077	-0.114	-0.043	0.115
19. I simply like math.	**0.765**	0.068	-0.007	0.067	0.017	-0.001	-0.022	-0.085	-0.025	0.209
28. I enjoy dealing with mathematical topics.	**0.629**	-0.038	0.299	0.011	-0.005	0.081	-0.094	0.015	0.023	0.269
4. It is important to me to be good at math.	0.002	**0.582**	0.120	0.068	0.028	0.048	0.099	-0.071	-0.073	0.440
13. Being good at math means a lot to me.	0.158	**0.494**	0.133	0.079	0.005	0.151	0.137	-0.053	-0.005	0.347
22. Performing well in math is important to me.	-0.034	**0.709**	0.113	0.131	0.058	0.050	-0.082	0.032	-0.038	0.229
31. Good grades in math are very important to me.	-0.039	**0.399**	0.323	0.195	-0.014	0.131	-0.085	0.085	-0.034	0.392
7. I care a lot about remembering the things we learn in math.	0.217	0.283	**0.317**	-0.055	0.077	0.098	0.023	0.082	-0.009	0.476
16. Math is not meaningful to me. (R)	0.261	0.186	**0.104**	0.027	0.151	-0.008	-0.174	-0.158	0.031	0.494
25. I'm really keen on learning a lot in math.	0.114	0.276	**0.435**	0.223	0.002	0.012	-0.093	0.077	0.048	0.321
34. Math is very important to me personally.	0.132	0.037	**0.243**	0.250	0.299	0.094	-0.030	0.034	-0.130	0.324
36. To be honest, I don't care about math. (R)	0.288	0.166	**0.161**	0.050	0.137	-0.098	-0.083	-0.239	0.009	0.445
37. It is important to me to know a lot of math.	0.097	0.126	**0.641**	-0.011	0.100	0.057	-0.151	0.074	0.019	0.268
2. It is worth making an effort in math, because it will save me a lot of trouble at school in the next years.	0.017	0.287	0.027	**0.396**	0.050	0.022	0.135	-0.124	-0.007	0.578
11. Good grades in math can be of great value to me later on.	0.018	0.108	0.060	**0.487**	0.072	0.081	0.176	-0.240	0.058	0.498
17. Being good at math pays off, because it is simply needed at school.	0.023	0.127	-0.009	**0.554**	0.130	0.033	-0.020	0.071	0.053	0.501
26. Learning math is worthwhile, because it improves my job and career chances.	-0.093	0.027	0.230	**0.469**	0.229	0.036	0.103	-0.142	0.028	0.372
5. Understanding math has many benefits in my daily life.	0.011	0.081	0.097	-0.156	**0.769**	-0.016	0.188	-0.090	-0.022	0.343
14. Math contents will help me in my life.	-0.087	-0.002	0.179	0.010	**0.739**	0.013	0.165	-0.217	0.016	0.218
20. Math comes in handy in everyday life and leisure time.	-0.007	0.093	-0.048	-0.069	**0.772**	0.008	-0.048	0.018	-0.010	0.423
29. I will often need math in my life.	-0.004	-0.210	0.149	0.238	**0.690**	0.057	-0.057	0.031	0.022	0.238
32. Math is directly applicable in everyday life.	0.016	-0.050	-0.220	0.102	**0.930**	0.010	-0.197	0.299	-0.034	0.167
8. Being well versed in math will go down well with my classmates.	-0.023	0.153	-0.073	-0.144	0.013	**0.693**	0.029	-0.014	0.043	0.491
23. I can impress others with intimate knowledge in math.	0.166	-0.028	0.050	0.064	0.054	**0.512**	0.021	0.057	0.027	0.587
35. If I know a lot in math, I will leave a good impression on my classmates.	-0.070	-0.072	-0.055	0.031	-0.043	**0.986**	-0.024	-0.001	0.000	0.162
3. Doing math is exhausting to me.	-0.193	-0.082	-0.061	0.184	-0.064	0.015	**0.514**	0.213	-0.004	0.386
12. I often feel completely drained after doing math.	0.052	0.098	-0.099	-0.030	0.134	0.041	**0.328**	-0.061	0.289	0.723
21. Dealing with math drains a lot of my energy.	0.132	0.092	-0.165	0.027	-0.044	-0.047	**0.416**	0.324	0.228	0.455
30. Learning math exhausts me.	-0.066	-0.015	-0.091	0.124	-0.047	-0.045	**0.517**	0.345	0.055	0.325
6. I'd rather not do math, because it only worries me.	-0.144	0.034	-0.129	-0.250	-0.038	0.012	0.131	**0.268**	0.192	0.500
15. When I deal with math, I get annoyed.	-0.278	-0.029	0.116	-0.188	0.012	-0.046	0.294	**0.330**	0.125	0.359
24. Math is a real burden to me.	-0.174	-0.115	-0.013	-0.011	-0.013	-0.035	0.282	**0.372**	0.214	0.287
33. Doing math makes me really nervous.	-0.110	-0.031	0.016	-0.148	0.008	-0.018	0.392	**0.440**	0.067	0.321
9. I have to give up other activities that I like to be successful at math.	0.013	-0.033	0.022	0.019	-0.029	0.119	-0.038	-0.127	**0.640**	0.630
18. I have to give up a lot to do well in math.	-0.071	0.053	-0.029	-0.013	0.029	-0.016	-0.091	0.047	**0.880**	0.239
27. I'd have to sacrifice a lot of free time to be good at math.	0.051	-0.167	0.097	0.151	-0.066	-0.009	0.199	0.164	**0.560**	0.412

Note. λ= standardized factor loading; δ= standardized item uniqueness; bold= target factor loadings; FS1= intrinsic, FS2= importance of achievement, FS3= personal importance, FS4= utility for school/job, FS5= utility for life, FS6= social utility, FS7= effort required, FS8= emotional cost, FS9= opportunity cost

References for Supplementary Material

Asparouhov, T., & Muthén, B.O. (2009). Exploratory structural equation modeling. *Structural Equation Modeling,16,*397-438.

Bagozzi, R.P., & Kimmel, S.K. (1995). A comparison of leading theories for the prediction of goal-directed behaviours. *British Journal of Social Psychology,34*,437-461.

Chen, F.F.(2007). Sensitivity of goodness of fit indexes to lack of measurement. *Structural Equation Modeling,14,*464-504.

Cheung, G.W., & Rensvold, R.B.(2002). Evaluating goodness-of-fit indexes for testing measurement invariance. *Structural Equation Modeling,9,*233-255.

Conley, A.M. (2012). Patterns of motivation beliefs: Combining achievement goal and expectancy-value perspectives. *Journal of Educational Psychology,104,*32-47.

Eccles, J.S. (2009). Who am I and what am I going to do with my life? Personal and collective identities as motivators of action. *Educational Psychologist,44,*78-89.

Eccles, J.S., Adler, T.F., Futterman, R., Goff, S.B., Kaczala, C.M., Meece, J.L., & Midgley, C. (1983). Expectancies, values and academic behaviors. In J.T. Spence (Ed.), *Achievement and achievement motives* (pp.75-146). San Francisco: Freeman.

Eccles, J.S., & Wigfield, A. (2002). Motivational beliefs, values, and goals. *Annual Review of Psychology,53,*109-132.

Gaspard, H., Dicke, A.L., Flunger, B., Schreier, B., Häfner, I., Trautwein, U., & Nagengast, B. (2015). More value through greater differentiation: Gender differences in value beliefs about math. *Journal of Educational Psychology,107,*663-677.

Gignac, G.E. (2016). The higher-order model imposes a proportionality constraint: That is why the bifactor model tends to fit better. *Intelligence,55,*57-68.

Guo, J., Marsh, H.W., Morin, A.J.S., Parker, P.D., & Kaur, G. (2015a). Directionality of the associations of high school expectancy-value, aspirations, and attainment: a longitudinal study. *American Educational Research Journal,52,*371-402.

Guo, J., Parker, P.D., Marsh, H.W., & Morin, A.J.S. (2015b). Achievement, motivation, and educational choices: A longitudinal study of expectancy and value using a multiplicative perspective. *Developmental Psychology,51,*1163-1176.

Hulleman, C.S., Durik, A.M., Schweigert, S.B., & Harackiewicz, J.M. (2008). Task values, achievement goals, and interest: An integrative analysis. *Journal of Educational Psychology,100,*398-416.

Hulleman, C.S., & Harackiewicz, J.M. (2009). Promoting interest and performance in high school science classes. *Science,326,*1410-1412.

Jennrich, R.I., & Bentler, P.M. (2011). Exploratory bi-factor analysis. *Psychometrika,76*,537-549.

Litalien, D., Morin, A.J.S., Gagné, M., Vallerand, R.J., Losier, G.F., & Ryan, R.M. (2017). Evidence of a continuum structure of academic self-determination: a two-study test using a bifactor-ESEM representation of academic motivation. *Contemporary Educational Psychology,51,*67-82.

Marsh, H.W., Hau, K.T., Wen, Z., Nagengast, B., & Morin, A.J.S. (2013). Moderation. In T.D. Little (Ed.), *The Oxford handbook of quantitative methods* (pp.361-386). Oxford University Press.

Marsh, H.W., Wen, Z., & Hau, K.T. (2004). Structural equation models of latent interactions: Evaluation of alternative estimation strategies and indicator construction. *Psychological Methods,9,*275-300.

Morin, A.J.S., Arens, A.K., & Marsh, H.W. (2016). A bifactor exploratory structural equation modeling framework for the identification of distinct sources of construct-relevant psychometric multidimensionality. *Structural Equation Modeling,23,*116-139.

Morin, A.J.S., Myers, N.D., & Lee, S. (2018). Modern factor analytic techniques: Bifactor models, exploratory structural equation modeling (ESEM) and bifactor-ESEM. In G. Tenenbaum, & Eklund, R.C. (Eds.), *Handbook of Sport Psychology* (4th ed.). New York, NY: Wiley.

Nagengast, B., Marsh, H.W., Scalas, L.F., Xu, M., Hau, K.T., & Trautwein, U. (2011). Who took the “x” out of expectancy-value theory? A psychological mystery, a substantive-methodological synergy, and a cross-national generalization. *Psychological Science,22*,1058-1066.

Podsakoff, P., MacKenzie, S., Lee, J., & Podsakoff, N. (2003). Common method biases in behavioral research: A critical review of the literature and recommended remedies. *Journal of Applied Psychology,88,*879-903.

Reise, S.P. (2012). The rediscovery of bifactor measurement models. *Multivariate Behavioral Research,47,*667-696.

Richardson, H. A., Simmering, M. J., & Sturman, M. C. (2009). A tale of three perspectives: Examining post hoc statistical techniques for detection and correction of common method variance. *Organizational Research Methods,12,*762-800.

Simpkins, S.D., Davis-Kean, P.E., & Eccles, J.S. (2006). Math and science motivation: A longitudinal examination of the links between choices and beliefs. *Developmental Psychology,42,*70-83.

Wigfield, A., Rosenzweig, E.Q., & Eccles, J.S. (2017). Achievement values: Interactions, interventions, and future directions. In A.J. Elliot, C.S. Dweck & D.S. Yeager (Hrsg.), *Handbook of competence and motivation: Theory and application* (2nd ed., S. 116–134). New York, NY: Guilford Press.