Running Head: Supplements for Mixture Modeling.

Online supplements for:

A Gentle Introduction to Mixture Modeling Using Physical Fitness Performance Data

Order of Authorship and Contact Information

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Online supplements for Chapter 9.

A Gentle Introduction Mixture Modeling Using Singapore's National Physical Fitness Award Data

List of Appendices

Appendix 9.1.

Preliminary Confirmatory Factor Analyses

To obtain factor scores reflecting students' global levels of Physical Strength and Cardiovascular Fitness, we estimated a longitudinal confirmatory factor analytic (CFA) model (see chapter 5). In this model, two factors were estimated at each of the seven specific time points (for a total of 7 x 2 = 14 factors). These two factors reflected Physical Strength (using the results from the Sit-Ups, Broad Jumps, and Pull-Ups tests as the three factor indicators) and Cardiovascular Fitness (using the results from the Shuttle-Run, and Run-walk tests as factor indicators). This model was estimated as a multiple-group model separately in both gender groups. All models were specified as congeneric, with each item allowed to load on a single factor, and all factors freely allowed to correlate within time-points as well as across time-points. In these models, a priori correlated uniquenesses between matching indicators of the factors utilized at the different time-points should be included in longitudinal models to avoid converging on biased and inflated stability estimates (Jöreskog, 1979; Marsh, 2007). This inclusion reflects the fact that indicators' unique variance is known to emerge, in part, from shared sources of influences over time.

Tests of measurement invariance across combinations of groups and time points were then performed in sequence (Meredith,1993; Millsap, 2011; also see Chapter 6): (i) configural invariance (same measurement model), (ii) weak invariance (invariance of the factor loadings); (iii) strong invariance (invariance of the factor loadings and items' intercepts); (iv) strict invariance (invariance of the factor loadings, items' intercepts, and items' uniquenesses), (v) invariance of the variances and within-time covariances between the constructs, (vi) latent means invariance. However, relying on two indicators per construct (for the Cardiovascular Fitness factor) creates locally unidentified factors even though the model remains identified with more than two factors (Bollen, 1989). Thus, after the estimation of the model of configural invariance, a second model was estimated in which each factor was fully identified by using essentially tau-equivalent constraints (ETECs). Using ETECs involves placing equality constraints on both loadings to help locate the construct at the true centroid of the indicators (Little, Lindenberger, & Nesselroade, 1999). This procedure may results in a decrease in the fit of the models, which should not overly concern researchers if it is not dramatic (Little et al., 1999).

The fit results (see discussion presented in Chapter 5 and 6 for the interpretation of model fit; also see Chen, 2007; Cheung, & Rensvold, 2002; Hu & Bentler, 1999; Marsh, Hau, & Grayson, 2005) for these models are reported in the Table presented on the next page.

These results confirm the adequacy of both a priori longitudinal measurement models (with and without ETECs) with indices indicating excellent fit (RMSEA= .020 to .022; CFI = .990 to .992; TLI = .984 to .987). Across the full sequence of tests of measurement invariance, where invariance constraints were imposed across time periods, gender groups, and all time X gender combinations, observed changed in fit indices remained minimal and well under typical interpretation guidelines (e.g. Chen, 2007; Cheung & Rensvold, 2002: ΔRMSEA $\leq$ .015; ΔCFI and ΔTLI $\leq$ .010), supporting the complete measurement invariance of this model across time periods and gender. It is from this completely invariant model that factor scores were saved.

Fit Results from the Longitudinal Measurement Invariance Models

	χ^2	df	CFI	TLI	RMSEA	90% CI
Configural Invariance	2183.316*	728	0.992	0.987	0.020	0.019-0.021
Essent. Tau Equivalence	2544.312*	742	0.990	0.984	0.022	0.021-0.023
Weak Invariance (Time)	3163.920*	766	0.987	0.980	0.025	0.024-0.026
Weak Invariance (Gender)	2952.654*	756	0.988	0.981	0.024	0.023-0.025
Weak Invariance (Total)	3192.947*	768	0.987	0.979	0.025	0.024-0.026
Strong Invariance (Time)	3214.120*	816	0.987	0.981	0.024	0.023-0.025
Strong Invariance (Gender)	3197.946*	789	0.987	0.980	0.025	0.024-0.026
Strong Invariance (Total)	3215.380*	819	0.987	0.981	0.024	0.023-0.025
Strict Invariance (Time)	3684.103*	879	0.985	0.979	0.025	0.024-0.026
Strict Invariance (Gender)	3694.321*	854	0.984	0.978	0.026	0.025-0.027
Strict Invariance (Total)	3915.491*	884	0.983	0.978	0.026	0.025-0.027
Var.-Covar. Invariance (Time)	4149.397*	920	0.982	0.977	0.027	0.026-0.027
Var.-Covar. Invariance (Gender)	4036.416*	905	0.983	0.977	0.026	0.026-0.027
Var.-Covar. Invariance (Total)	4217.417*	923	0.982	0.977	0.027	0.026-0.028
Latent Means Invariance(Time)	4241.355*	935	0.982	0.977	0.027	0.026-0.027
Latent Means Invariance(Gender)	4218.890*	925	0.982	0.977	0.027	0.026-0.028
Latent Means Invariance (Total)	4242.439*	937	0.982	0.977	0.027	0.026-0.027

Note. *$p < .01$; χ^2: Chi-square; *df*: Degrees of freedom; CFI: Comparative fit index; TLI: Tucker-Lewis index; RMSEA: Root mean square error of approximation; 90% CI: 90% confidence interval of the RMSEA.

References used in this Appendix (but not in the main chapter)

Bollen, K.A. (1989). *Structural equations with latent variables*. New York, NY: Wiley.

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Cheung, G.W., & Rensvold, R.B. (2002). Evaluating goodness-of fit indexes for testing measurement invariance. *Structural Equation Modeling*, *9*, 233–255.

Hu, L.-T., & Bentler, P.M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling, 6,* 1–55.

Jöreskog, K.G. (1979). Statistical models and methods for the analysis of longitudinal data. In K.G. Jöreskog & D. Sörbom (Eds.), *Advances in Factor Analysis and Structural Equation Models*. Cambridge, MA: Abt Books.

Little, T.D., Lindenberger, U., & Nesselroade, J.R. (1999). On selecting indicators for multivariate measurement and modeling with latent variables: When "good" indicators are bad and "bad" indicators are good. *Psychological Methods*, *4*, 192-211.

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Millsap, R.E. (2011). *Statistical approaches to measurement invariance*. NewYork: Taylor & Francis.

Appendix 9.2.

How to ensure that the final solution is well replicated?

The best way to ensure that the final solution is well-replicated and represents a true maximum likelihood rather than a local solution is to increase the number of starts values. This is done in the ANALYSIS section of the Mplus input code. For instance:

```
ANALYSIS:
TYPE = MIXTURE COMPLEX;
ESTIMATOR = MLR;
process = 3;
STARTS = 5000 200;
STITERATIONS = 100;
```

This section of input request the estimation of a mixture model (TYPE = MIXTURE) including a correction for the nesting of students within schools (TYPE = COMPLEX) and using the robust maximum likelihood estimator (ESTIMATOR = MLR). The function PROCESS = 3 requires that the model be estimated using 3 of the available processors to speed up the estimation (this number can be increased or decreased depending on the availability of processors). The function STARTS = 5000 200 requests 5000 sets of random start values, and that the best 200 of these starts be kept for final stage optimization. The function STITERATIONS = 100 requests that all random starts be allowed a total of 100 iterations.

Once the model is estimated, Mplus will provide (as part of the output) the loglikelihood values associated with all of the random starts retained for the final stage optimization. It will also indicate how many of the start value runs did not converge. This section of the output will appear like the following:

```
RANDOM STARTS RESULTS RANKED FROM THE BEST TO THE WORST LOGLIKELIHOOD VALUES

3323 perturbed starting value run(s) did not converge.
Final stage loglikelihood values at local maxima, seeds, and initial stage start numbers:

-32960.162        27690       2347
-32960.162        821740      4428
-32960.162        541128      4674
-32960.162        256363      2826
-32960.162        488581      688
-32960.162        476338      1418
-32960.162        579138      706
-32960.162        367683      4650
-32960.162        621055      4450
-32960.162        512820      1071
-32960.162        403892      2676
-32960.162        699337      4168
-32960.162        406734      2605
-32960.162        807339      4334
-32960.162        988537      1980
-32960.162        392717      4834
-32965.925        402699      604
-32965.925        960438      116
-32965.925        493718      2394
-32965.925        606094      2866
-32965.925        124661      2172
-32965.925        109524      4400
-32965.925        80226       3041
-32965.925        56630       2793

…
```

In this example, the best loglikelihood value was replicated 16 times (the number of times the value of -32960.162 appears in the first column), which is fully satisfactory. Although no clear-cut rule exists, we suggest that solutions should be replicated at least 5 times. Failing to do so, additional tests should be conducted while increasing the number of start values and/or iterations or using user-defined starts values (for instance, using the starts values from the best fitting solution provided when requesting SVALUES in the output section of the syntax and using these starts values in the model while keeping the random starts function active). The second column of the output section pasted above provides the model seed associated with each specific random start solution. Using the seed provides an easy way to replicate the final solution (or any other solution) while drastically decreasing computational time. To do so, the following ANALYSIS section can be used (here to replicate the best solution from the example above).

```
ANALYSIS:
TYPE = MIXTURE COMPLEX;
ESTIMATOR = MLR;
process = 3;
STARTS = 0;
OPTSEED = 27690;
STITERATIONS = 100;
```

As noted, we suggest that users systematically request SVALUES as part of the output. Here is a standard setup for requesting specific sections of output:

```
OUTPUT:
STDYX SAMPSTAT CINTERVAL SVALUES RESIDUAL TECH1 TECH7 TECH11 TECH14;
```

Each of these terms are defined in the Mplus manual. When SVALUES are requested, the exact values associated with the final model will be provided in the output as ready-to-use input command. For instance, here is a set up for the MODEL section of a 2-profile LPA model (see later sections of these supplements for more details):

```
MODEL:
 %OVERALL%
 %c#1%
 [ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run];
 ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run ;
 %c#2%
 [ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run];
 ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run ;
```

When SVALUES are requested, the output will include the following section, which can be cut-and-pasted and used as a replacement of the MODEL section represented above in conjunction with the STARTS function set to 0 (STARTS = 0) to exactly replicate the final solution. This fucntiin is particularly useful when one wants to include covariates in a model yet ensure that the final unconditional LPA solution remains unchanged.

```
MODEL COMMAND WITH FINAL ESTIMATES USED AS STARTING VALUES
    %OVERALL%
    [ c#1*0.23559 ];
    %C#1%
    [ zp5sit*0.42857 ];
    [ zp5flex*0.55041 ];
    [ zp5snr*0.28010 ];
    [ zp5shut*-0.47364 ];
    [ zp5sbj*0.50862 ];
    [ zp5run*-0.49124 ];
    zp5sit*0.71721;
    zp5flex*0.87515;
    zp5snr*1.04028;
    zp5shut*0.64504;
    zp5sbj*0.70734;
    zp5run*0.62410;
    %C#2%
    [ zp5sit*-0.54352 ];
    [ zp5flex*-0.69033 ];
    [ zp5snr*-0.35497 ];
    [ zp5shut*0.60033 ];
    [ zp5sbj*-0.64461 ];
    [ zp5run*0.62280 ];
    zp5sit*0.82980;
    zp5flex*0.29957;
    zp5snr*0.72302;
    zp5shut*0.80467;
    zp5sbj*0.62703;
    zp5run*0.78226;
```

Appendix 9.3.

Correction for Nesting

It is possible to control for the non-independence of the observations due to students' nesting within schools using Mplus design-based correction (Asparouhov, 2005). However, when this correction is used, the BLRT cannot be computed. Fortunately, ignoring nesting is unlikely to affect the class enumeration process of LPA and GMM models, although it does affect standard errors and classification accuracy (Chen, Kwok, Luo, & Willson, 2010). Given the possible impact of nesting on estimates of regression coefficients which define MRM solutions, failure to control for nesting may result in a biased MRM class enumeration. Class enumeration was thus conducted without controlling for nesting in LPAs. However, the final LPA solutions, and all MRM solutions, were estimated while controlling for nesting. Finally, nesting was not controlled for in LTA and GMM because nesting changes over time and mixture models currently only accommodate one type of nesting structure.

This correction is implemented by adding "CLUSTER = *clustid*;" to the "VARIABLE:" section of the input (where *clustid* is the name of the clustering variable present in the dataset, for instance the unique identifier of the school the student attends), and "TYPE = COMPLEX;" in the "ANALYSIS" section of the input.

References used in this Appendix (but not in the main chapter)

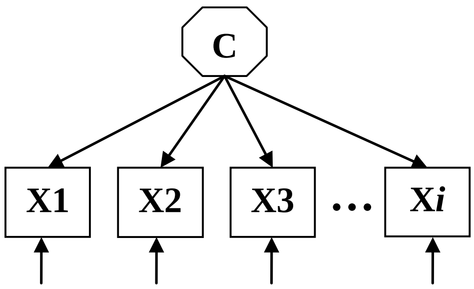
Asparouhov, T. (2005). Sampling weights in latent variable modeling. *Structural Equation Modeling*, *12*, 411-434.

Chen, Q., Kwok, O.-M., Luo, W., & Willson, V.L. (2010). The impact of ignoring a level of nesting structure in multilevel growth mixture models: A Monte Carlo study. *Structural Equation Modeling*, *17*, 570-589.

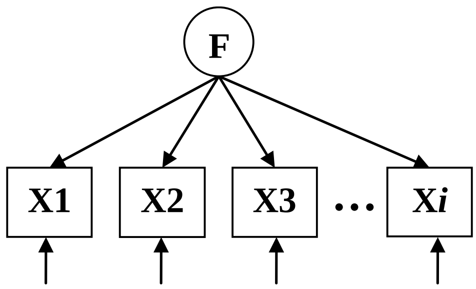
Appendix 9.4.

Introduction to Latent Profile Analyses and Alternative Specifications

In preparing this chapter, we have to make some assumptions of basic knowledge on the part of the readers. In particular, we have to assume that the reader is reasonably familiar with CFA (see Chapter 5), SEM (see chapter 5), and tests of measurement invariance (see Chapter 6) as these provide a critical pre-requisite backbone to the understanding of the models presented here which simply rely on the addition of categorical latent variables to the global CFA/SEM framework in order to extract unobserved subpopulations. However, these unobserved subpopulations are akin to the observed subgroups of participants typically considered in multiple groups CFA/SEM models apart from being estimated as part of the model rather than a priori specified. In other words, any kind of comparison that can be conducted across observed groups of participants, as well as any constraint that can be included across observed groups of participants, can likewise be realized across unobserved subpopulations. Furthermore, our later illustration of GMM also assumes reasonable familiarity with latent curve models (see Chapter 7). Without this a priori knowledge, we would strongly advise against the use of mixture modeling.



Model 1: Latent Profile Analysis



Model 2: Confirmatory Factor Analysis

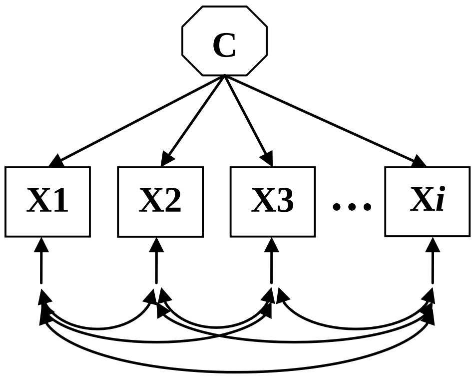
Note. Squares represent observed variables; Xs represent the observed indicators of the latent variables, ovals represent continuous latent variables; octagons represent categorical latent variables; C represent the categorical latent variables (the profiles); F represent the continuous latent variables (the factors), the arrows appearing under the indicators represent their uniquenesses.

In its most simple expression, LPA (see Model 1) present a high level of similarities with CFA (see Model 2). The key difference between these models is that LPA relies on a categorical latent variable (i.e., the profiles) to regroup persons, whereas CFA relies on continuous latent variables (i.e., the factors) to regroup variables (Cattel, 1952; Lubke & Muthén, 2005). Thus, "the common factor model decomposes the covariances to highlight relationships among the variables, whereas the latent profile model decomposes the covariances to highlight relationships among individuals" (Bauer & Curran, 2004, p. 6). Choosing between these representations is not easy since a *k*-profile LPA has identical covariance implications than a *k-1*-factor CFA and thus represents an equivalent model (Bauer & Curran, 2004; Steinley & McDonald, 2007). Simulation studies also showed that spurious latent classes may emerge when none exist as a way to account for violations of the model distributional assumptions (e.g., Bauer, 2007). Although many attempts to provide a solution to this issue have been proposed, none is fully satisfactory (Lubke & Neale, 2006, 2008; Muthén, & Asparouhov, 2009; Steinley & McDonald, 2007). Indeed, the existence of equivalent statistical models providing radically different pictures of the reality is almost universal (Cudeck & Henly, 2003; Hershberger, 2006; Muthén, 2003), and prioritizing one over the other typically remains a theoretical decision related in part to the theoretical underpinnings of the research question being asked (Borsboom et al., 2003). Hence our perspective that relying on variable, versus person, centered analyses involves a paradigmatic shift (Morin, Morizot et al., 2011). In the end, the best way to support a substantive interpretation of the profiles as reflecting significant subgroups of participants (or factors as reflecting meaningful underlying dimensions) requires a process of construct validation taking into account the heuristic value of the profiles, their conformity to theoretical expectations, their differential associations to meaningful covariates, and their generalizability to new samples (Cudeck & Henly, 2003; Marsh et al., 2009; Morin, Morizot et al., 2011; Muthén, 2003).

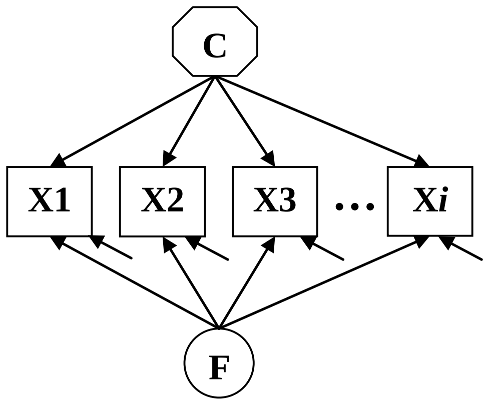
The generic LPA model is expressed as (e.g., Lazafeld & Henry, 1978; Peugh & Fan, 2013):

$$\sigma_y^2 = \sum_{k=1}^{K} \pi_k (\mu_{yk} - \mu_y)^2 + \sum_{k-1}^{K} \pi_k \sigma_{yk}^2$$

For y observed indicators and k latent profiles, the LPA model decomposes the variance into and between-profile (the first term) and within-profile (the second term) components. In this expression, the profile-specific means (μ_{yk}) and variances (σ_{yk}^2) of the observed indicators are expressed as a function of the density function π_k which reflects the proportion of participants in each profile. Implicit in this expression and in the previous figures is a conditional independence assumption (that is also shared with CFA) that, conditional on the latent profiles, all observed indicators are uncorrelated with one another.



Model 3: Latent Profile Analysis with Correlated Uniquenesses



Model 4: Factor Mixture Analysis

Note. Squares represent observed variables; Xs represent the observed indicators of the latent variables, ovals represent continuous latent variables; octagons represent categorical latent variables; C represent the categorical latent variables (the profiles); F represent the continuous latent variables (the factors), the arrows appearing under the indicators represent their uniquenesses.

As it is the case with CFA models, it is possible to relax this assumption through the inclusion of correlations among the uniquenesses of the observed indicators (see Model 3). Although some simulation studies have shown, under highly specific conditions, that relaxing this assumption may help to recover true population parameters in the class enumeration process (e.g., Uebersax, 1999; Peugh & Fan, 2013) , we argue that the decision to relax this assumption should be made with caution, and based on strong theoretical assumptions of expected relations among the indicators that exist over and above the expected profiles (to reflect wording, or informant, effects for example). As noted by Marsh et al. (2009, p. 199): “By analogy, in CFA, correlations among indicators are assumed to be explained in terms of latent factors. Although it is possible to relax this assumption of conditional independence by the inclusion of correlated uniquenesses (correlations among indicators not explained by factors), best practice […] is not to do so except in special circumstances that are posited a priori”. Importantly, the inclusion of these correlated uniquenesses completely changes the meaning of the extracted latent profiles, and thus their ex post facto inclusion in an atheoretical manner been labeled as a “disaster” for psychological research (Schweizer, 2012, p.1). In fact, when legitimate a priori controls are required in CFA applications, method factors should generally be preferred to correlated uniquenesses because they provide a more explicit estimate construct-irrelevant sources of variance (Schweizer, 2012). Similarly, this type of control is also possible the context of LPA conducted within the GSEM frameworks which makes it possible to combine continuous and categorical latent variables into the same model. Such a model (see Model 4, also see Appendix 9.16) is called a factor mixture model (e.g., Lubke & Muthén, 2005).

In the simplest expression of factor mixture models, the continuous latent factor component of the model is specified as completely invariant across profiles and simply used to control global tendencies that are shared among all observed indicators in order to extract cleaner profiles presenting clearer qualitative differences. As discussed extensively by Morin and Marsh (2014), the inclusion such a global factors underlying the observed indicators may be particularly useful for applications of LPA when there is a reason to expect that their exist a global construct underlying responses to the observed indicators (e.g., global competencies, global commitment) that needs to be controlled in order to extract cleaner profiles. Indeed, in the person-centered literature, one common assumption (e.g., Bauer, 2007; De Boek, Wilson, & Acton, 2005) is the need to observe clear qualitative differences between the profiles to support their meaningfulness. Conversely, the extraction of profiles showing only quantitative differences (i.e., with profile simply presenting a higher or lower levels on all variables considered), would be better represented by a continuous latent factor. However, there are some areas of research where there are reasons to expect both qualitative and quantitative differences between profiles due to the expectation that there exist a global underlying dimension to the observed indicators. In Morin and Marsh (2014), this global dimension is the global level of competencies of University teachers, over and above which the authors wanted to extract specific subgroups of teachers presenting differentiated profiles of strength and weaknesses. In these cases, failure to control for the global level of competencies may preclude the extraction of clearly defined profiles due to the conditional independence assumption of traditional LPA. In the present study, we could likewise have argued that there was a global level of physical fitness to be controlled. However, examination of the extracted profiles, of the estimated CFA models (see Appendix 9.1.), and even the estimation of preliminary factor mixture models (see Morin and Marsh for extensive discussions of these models, together with annotated Mplus input syntax) rather showed that these models would have been inappropriate in regards to the relative orthogonal nature of the indicators of Flexiblity, Physical Strength, and Cardiovascular Fitness. Indeed, when attempts where made to estimate factor mixture analyses, the factors loadings on this global factor were negative for the indicators of Physical Strength, and Positive for the indicators of Cardiovascular Fitness (or the opposite), providing a hard-to-interpret control for global levels of physical fitness.

However, factor mixtures are much more flexible than this and provide an integrative framework than can be used to investigate the underlying continuous or categorical nature of various phenomenon, as well as for the investigation of measurement invariance of psychometric measures across unobserved subpopulations. Although a presentation of the full range of possibilities provided by factor mixture models is beyond the scope of the present study, we invite the interested readers to consult the following references: (a) Lubke and Muthén (2005) for a global introduction to these models; (b) Masyn, Henderson, and Greenbaum (2010) for the application of factor mixture models to the investigation of the dimensional-categorical nature of psychological constructs; (c) Clark, Muthén, Kaprio, D’Onofrio, Viken, and Rose (2013) for a pedagogical illustration of a framework similar to the one presented in Masyn et al. (2010); (d) Tay, Newman, and Vermunt (2011) for an illustration of the use of mixture models to investigate the possible non-invariance of psychometric measures across unobserved subgroups of participants.

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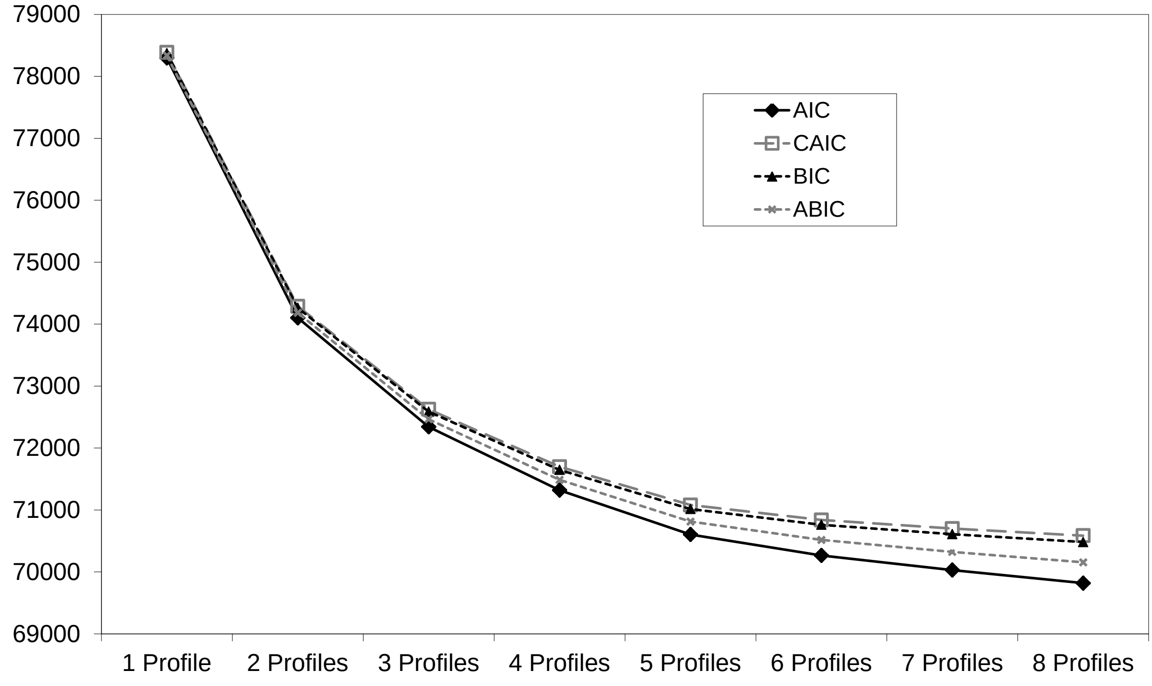
Appendix 9.5.

Elbow Plot of the Information Criteria for the LPA, Girls, Grade 5



Elbow Plot of the Information Criteria for the LPA, Girls, Grade 5

Appendix 9.6.

Detailed results from the final LPA solution, Grade 5.

		Profile 1		Profile 2		Profile 3		Profile 4		Profile 5	
Task	Sample	Mean	CI	Mean	CI	Mean	CI	Mean	CI	Mean	CI
Sit Ups	Invariant	-1.033	-1.143; -0.923	-0.263	-0.352; -0.174	0.589	0.504; 0.674	0.725	0.636; 0.814	-0.161	-0.233; -0.088
Pull-Ups	Invariant	-1.087	-1.152; -1.022	-0.458	-0.592; -0.324	0.197	0.118; 0.276	1.566	1.434; 1.699	0.388	0.297; 0.479
Sit-and-Reach	Invariant	-0.429	-0.591; -0.267	-0.641	-0.682; -0.601	-0.396	-0.440; -0.351	1.478	1.399; 1.558	1.209	1.140; 1.277
Shuttle Run	Invariant	1.358	1.216; 1.500	0.263	0.158; 0.368	-0.747	-0.850; -0.643	-0.827	-0.925; -0.729	0.247	0.173; 0.321
Broad Jump	Invariant	-1.352	-1.451; -1.252	-0.347	-0.403; -0.291	0.727	0.652; 0.801	0.982	0.880; 1.085	-0.155	-0.229; -0.081
Run-Walk	Invariant	1.296	1.197; 1.395	0.276	0.196; 0.356	-0.676	-0.754; -0.598	-0.854	-0.941; -0.768	0.170	0.084; 0.256
		Variance	CI	Variance	CI	Variance	CI	Variance	CI	Variance	CI
Sit Ups	Boys	0.932	0.790; 1.074	**0.591**	**0.526; 0.656**	0.684	0.604; 0.765	0.548	0.458; 0.638	0.697	0.608; 0.786
	Girls	1.038	0.835; 1.242	**0.780**	**0.697; 0.863**	0.566	0.471; 0.661	0.784	0.583; 0.985	0.788	0.681; 0.895
Pull-Ups	Boys	0.147	0.111; 0.183	0.364	0.275; 0.452	0.429	0.387; 0.472	0.374	0.150; 0.599	1.040	0.946; 1.133
	Girls	0.095	0.053; 0.138	0.392	0.288; 0.496	0.493	0.404; 0.581	0.729	0.504; 0.955	1.144	1.030; 1.258
Sit-and-Reach	Boys	0.796	0.569; 1.023	0.217	0.191; 0.243	0.220	0.173; 0.268	0.267	0.220; 0.313	0.279	0.214; 0.343
	Girls	0.867	0.621; 1.113	0.210	0.185; 0.235	0.241	0.186; 0.295	0.292	0.180; 0.404	0.315	0.239; 0.391
Shuttle Run	Boys	1.091	0.840; 1.342	**0.351**	**0.306; 0.396**	0.432	0.367; 0.497	0.602	0.385; 0.819	0.438	0.367; 0.510
	Girls	1.142	0.867; 1.418	**0.498**	**0.450; 0.546**	0.511	0.408; 0.614	0.536	0.382; 0.690	0.581	0.469; 0.694
Broad Jump	Boys	0.741	0.531; 0.952	**0.248**	**0.215; 0.282**	0.588	0.487; 0.688	0.734	0.618; 0.851	0.414	0.318; 0.510
	Girls	0.837	0.634; 1.039	**0.426**	**0.360; 0.491**	0.603	0.529; 0.677	0.635	0.499; 0.771	0.500	0.429; 0.570
Run-Walk	Boys	0.684	0.571; 0.796	0.579	0.527; 0.631	0.468	0.394; 0.543	**0.366**	**0.303; 0.429**	0.553	0.481; 0.625
	Girls	1.019	0.771; 1.266	0.604	0.535; 0.673	0.575	0.506; 0.644	**0.627**	**0.524; 0.730**	0.685	0.598; 0.773

Appendix 9.7.

Relations Between the Covariates and the Final LPA Solution, Grade 5

Results from the Multinomial Logistic Regressions for the Effects of Grade 4 BMI on Grade 5 Profile Membership.

	Latent profile 1 Vs 5		Latent profile 2 Vs 5		Latent profile 3 Vs 5		Latent profile 4 Vs 5	
	Coef. (SE)	OR	Coef. (SE)	OR	Coef. (SE)	OR	Coef. (SE)	OR
Boys BMI	1.050 (0.083)**	2.859	-0.048 (0.080)	0.953	-0.976 (0.099)**	0.377	-1.017 (0.114)**	0.362
Girls BMI	0.784 (0.104)**	2.191	-0.143 (0.084)	0.867	-0.623 (0.085)**	0.536	-0.690 (0.094)**	0.502

Note. SE: standard error of the coefficient; OR: Odds Ratio

Associations between Grade 5 Profile Membership and Grade 6 BMI.

	Profile 1	Profile 2	Profile 3	Profile 4	Profile 5	Differences between profiles
Boys BMI	1.305	0.022	-0.507	-0.526	0.014	1 > 2 = 5 > 3 = 4
Girls BMI	1.479	-0.140	-0.312	-0.382	-0.024	1 > 5 > 2 > 3 = 4
Differences between gender	Boys = Girls	Boys > Girls	Girls > Boys	Girls > Boys	Boys = Girls	

Appendix 9.8.

Results from the LPA and LTA conducted on Grade 5 and Secondary 3 students

Model	LL	#fp	Scaling	AIC	CAIC	BIC	ABIC	Entropy	aLMR	BLRT
Grade 5, LPA										
1 Profile	-78146.296	12	1.1335	156316.592	156414.126	156402.126	156363.992	Na	Na	Na
2 Profile	-73094.833	25	1.3632	146239.666	146442.862	146417.862	146338.416	0.734	≤ 0.001	≤ 0.001
3 Profile	-71298.729	38	1.3605	142673.457	142982.315	142944.315	142823.557	0.807	≤ 0.001	≤ 0.001
4 Profile	-69955.505	51	1.4129	140013.010	140427.530	140376.530	140214.460	0.793	≤ 0.001	≤ 0.001
5 Profile	-69090.118	64	1.3934	138308.236	138828.417	138764.417	138561.036	0.805	≤ 0.001	≤ 0.001
6 Profile	-68699.516	77	1.4508	137553.031	138178.874	138101.874	137857.180	0.780	≤ 0.001	≤ 0.001
7 Profile	-68442.256	90	1.5283	137064.512	137796.016	137706.016	137420.011	0.770	0.231	≤ 0.001
8 Profile	-68038.887	103	1.4230	136283.775	137120.941	137017.941	136690.624	0.784	0.150	0.242
Secondary 3, LPA										
1 Profile	-81050.863	12	1.4186	162125.726	162223.698	162211.698	162173.564	Na	Na	Na
2 Profile	-75724.615	25	1.5858	151499.230	151703.337	151678.337	151598.891	0.737	≤ 0.001	≤ 0.001
3 Profile	-73246.384	38	1.3397	146568.768	146879.012	146841.012	146720.254	0.755	≤ 0.001	≤ 0.001
4 Profile	-72306.752	51	1.3755	144715.504	145131.883	145080.883	144918.813	0.803	≤ 0.001	≤ 0.001
5 Profile	-71416.890	64	1.3419	142961.779	143484.294	143420.294	143216.912	0.781	≤ 0.001	≤ 0.001
6 Profile	-70703.644	77	1.3247	141561.289	142189.939	142112.939	141868.245	0.799	≤ 0.001	≤ 0.001
7 Profile	-70080.612	90	1.3543	140341.224	141076.011	140986.011	140700.005	0.779	≤ 0.001	≤ 0.001
8 Profile	-69747.363	103	1.3779	139700.727	140541.649	140438.649	140111.331	0.770	≤ 0.001	≤ 0.001
Model	LL	#fp	Scaling	AIC	CAIC	BIC	ABIC	Entropy	LRT	df
Final 5-Profile LTA Including Correction for Nesting										
Grade 5	-69090.118	64	4.1890	138308.236	138828.417	138764.417	138561.036	0.805		
Secondary 3	-71416.890	64	4.3991	142961.779	143484.294	143420.294	143216.912	0.781		
Configural	-135012.420	144	2.8517	270312.840	271487.637	271343.637	270886.027	0.802		
Structural (M)	-136277.277	114	2.7581	272782.554	273712.601	273598.601	273236.327	0.769	788.717*	30

Note. *: p ≤ .01; LL: Model LogLikelihood; #fp: Number of free parameters; Scaling = scaling factor associated with MLR loglikelihood estimates; AIC: Akaïke Information Criteria; CAIC: Constant AIC; BIC: Bayesian Information Criteria; ABIC: Sample-Size adjusted BIC; aLMR: Adjusted Lo-Mendell-Rubin likelihood ratio test; BLRT: Bootstrap Likelihood ratio test; LRT: Likelihood Ratio Test; df: Degrees of freedom associated with the LRT; M: Means.

Appendix 9.9.

Results from the MRM conducted on Grade 5 students

Model	LL	#fp	Scaling	AIC	CAIC	BIC	ABIC	Entropy	aLMR	
Boys										
1 Profile	-17404.321	20	3.5792	34848.643	34998.007	34978.007	34914.455	Na	Na	
2 Profile	-16684.448	35	2.9021	33438.897	33700.284	33665.284	33554.067	0.859	0.014	
3 Profile	-16109.734	50	2.2521	32319.468	32692.879	32642.879	32483.997	0.705	0.017	
4 Profile	-15894.541	65	2.0129	31919.081	32404.515	32339.515	32132.969	0.682	0.101	
5 Profile	-15742.872	80	1.9593	31645.744	32243.201	32163.201	31908.991	0.728	0.483	
6 Profile	-15645.363	95	1.7853	31480.726	32190.207	32095.207	31793.331	0.710	0.313	
7 Profile	-15584.039	110	1.6912	31388.077	32209.581	32099.581	31750.041	0.671	0.588	
8 Profile	-15524.898	125	1.6294	31299.795	32233.322	32108.322	31711.118	0.671	0.447	
Girls										
1 Profile	-18127.375	20	3.7546	36294.750	36444.152	36424.152	36360.599	Na	Na	
2 Profile	-17148.408	35	2.5658	34366.816	34628.269	34593.269	34482.052	0.624	≤ 0.001	
3 Profile	-16481.320	50	2.3318	33062.640	33436.145	33386.145	33227.263	0.757	0.007	
4 Profile	-16334.425	65	2.1875	32798.850	33284.406	33219.406	33012.860	0.674	0.504	
5 Profile	-16220.750	80	1.9033	32601.500	33199.108	33119.108	32864.898	0.678	0.211	
6 Profile	-16138.482	95	1.7497	32466.965	33176.625	33081.625	32779.749	0.693	0.240	
7 Profile	-16094.422	110	1.6988	32408.844	33230.556	33120.556	32771.016	0.677	0.240	
8 Profile	-16055.877	125	1.5819	32361.754	33295.516	33170.516	32773.312	0.704	≤ 0.001	
Model	LL	#fp	Scaling	AIC	CAIC	BIC	ABIC	Entropy	LRT	df
Final 3-Profile MRM										
Boys Only	-16109.734	50	2.2521	32319.468	32692.879	32642.879	32483.997	0.705		
Girls Only	-16481.320	50	2.3318	33062.640	33436.145	33386.145	33227.263	0.757		
Configural	-39238.006	95	2.7509	78666.011	79441.430	79346.430	79044.535	0.837		
Regression (R)	-39242.173	83	2.9722	78650.346	79327.817	79244.817	78981.056	0.837	6.830	12
Structural (R,M)	-39324.808	68	3.3232	78785.616	79340.653	79272.653	79056.559	0.835	119.674*	15
Dispersion (R,M,V)	-39370.172	53	3.7651	78846.344	79278.946	79225.946	79057.520	0.834	51.497*	15
Distribution (R,M,V,P)	-39372.446	51	3.7293	78846.893	79263.170	79212.170	79050.100	0.834	0.972	2

Note. *: p ≤ .01; LL: Model LogLikelihood; #fp: Number of free parameters; Scaling = scaling factor associated with MLR loglikelihood estimates; AIC: Akaïke Information Criteria; CAIC: Constant AIC; BIC: Bayesian Information Criteria; ABIC: Sample-Size adjusted BIC; aLMR: Adjusted Lo-Mendell-Rubin likelihood ratio test; BLRT: Bootstrap Likelihood ratio test; LRT: Likelihood Ratio Test; df: Degrees of freedom associated with the LRT; R: Regressions; M: Means; V: Variances; P: Probabilities.

Appendix 9.10.

Extended Presentation of GMM

Linear GMM

GMM are built from latent curve models (see Chapter 7; Bollen & Curran, 2006; McArdle & Epstein, 1987; Meredith & Tisak, 1990) and relax the assumption that all individuals from the sample are drawn from a single population. GMM thus represent longitudinal heterogeneity by the identification of subgroups (i.e., latent profiles) following distinct trajectories (e.g., Morin, Maïano et al. 2011). To start at the most basic level, let's assume a linear growth model for outcome y_{it} where i is the index for individual and t is the index for time. To this model, add c, a categorical latent variable with k levels (k = 1, 2, …, K) that is estimated from the data, with each individual i having a probability of membership in each of the k levels.

$$y_{it} = \sum_{k=1}^{K} p_k[\alpha_{iyk} + \beta_{iyk}\lambda_t + \varepsilon_{yitk}] \quad (1)$$

$$\alpha_{iyk} = \mu_{\alpha yk} + \zeta_{\alpha yik} \quad (2)$$

$$\beta_{iyk} = \mu_{\beta yk} + \zeta_{\beta yik} \quad (3)$$

The k subscript indicates that most parameters are allowed to differ across profiles and that each profile can thus be defined by its own latent curve model with independent covariance matrices and mean vectors. In this equation, α_{iyk} and β_{iyk} respectively represent the random intercept and random linear slope of the trajectory for individual i in profile k; $\mu_{\alpha yk}$ and $\mu_{\beta yk}$ represent the average intercept and linear slope in profile k and $\zeta_{\alpha yik}$ and $\zeta_{\beta yik}$ are reflect the variability of the estimated intercepts and slopes across cases within profiles. ε_{yitk} represents the time- individual- and class-specific residual. These errors are assumed to have a mean of 0, to be uncorrelated over time, across cases or with the other model parameters, and are generally allowed to vary across time. The mixing proportion parameter p_k defines the probability that an individual belongs to class k with all $p_k \geq 0$ and $\sum_{k=1}^{K} p_k = 1$. The variance parameters ($\zeta_{\alpha yik}, \zeta_{\beta yik}$) have a mean of zero and a variance-covariance matrix represented by Φ_{yk} :

$$\Phi_{yk} = \begin{bmatrix} \psi_{\alpha\alpha yk} & \\ \psi_{\alpha\beta yk} & \psi_{\beta\beta yk} \end{bmatrix} \quad (4)$$

In these models, Time is represented by λ_t , the factor loading matrix relating the time-specific indicators to the linear slope factor. Time is typically coded to reflect the passage of time and is thus a function of the intervals between measurement points. In the current study, the seven measurements points are equally spaced, and it appears reasonable to set the intercept at Time 1 [$E(\alpha_{iyk}) = \mu_{y1k}$]. Thus, for a linear GMM, time would be coded λ_1 = 0, λ_2 = 1, λ_3 = 2, λ_4 = 3, λ_5 = 4 λ_6 = 5, and λ_7 = 6. Providing a complete coverage of all issues related to time codes is clearly beyond the scope of the current study. However, we would advocate potential users of GMM to consult Biesanz, Deeb-Sossa, Papadakis, Bollen, and Curran (2004) and Metha and West (2000) for more details on time codes and their impact on parameters estimates. Finally, these models allow the inclusion of predictors of class membership. The predictors may also predict the intercept, slopes, time-specific indicators and distal outcomes, and these relationships may be freely estimated in each latent trajectory class.

Quadratic GMM

From this linear model, it is relatively easy to extrapolate the estimation of curvilinear (quadratic) GMM which simply involve the addition of one quadratic slope parameter to the model:

$$y_{it} = \sum_{k=1}^{K} p_k[\alpha_{iyk} + \beta_{1iyk}\lambda_t + \beta_{2iyk}\lambda^2{}_t + \varepsilon_{yitk}] \quad (5)$$

$$\beta_{1iyk} = \mu_{\beta 1yk} + \zeta_{\beta 1yik} \quad (6)$$

$$\beta_{2iyk} = \mu_{\beta 2yk} + \zeta_{\beta 2yik} \quad (7)$$

$$\Phi_{yk} = \begin{bmatrix} \psi_{\alpha\alpha yk} & & \\ \psi_{\alpha\beta 1yk} & \psi_{\beta 1\beta 1yk} & \\ \psi_{\alpha\beta 2yk} & \psi_{\beta 1\beta 2yk} & \psi_{\beta 2\beta 2yk} \end{bmatrix} \quad (8)$$

In this model (e.g., Diallo, Morin, & Parker, 2014), α_{iyk} remains defined as in equation 2, λ_t would remained coded as in the previous linear GMM, and β_{1iyk} and β_{2iyk} respectively represent the random linear slope and random quadratic slope of the of the trajectory for individual *i* in profile *k*.

Latent Basis GMM

In typical polynomial (linear, quadratic, etc.) specifications of GMM, time codes λ_t are usually fixed and constrained to equality over groups, although only two of them need to be fixed to 0 and 1 respectively for identification purposes, while the remaining codes can be freely estimated in the context of a latent basis model (Ram & Grim, 2009). Such latent basis models would globally be expressed as in equations 1 to 4 for the linear GMM, but *t*-2 time codes would be freely estimated in λ_t. This model further provides the possibility to freely estimate these *t*-2 times codes in all profiles so that λ_t becomes λ_{tk}, allowing for the extraction of trajectories differing completely in shape across profiles (see Morin et al., 2013 for an illustration). More precisely, rather than fixing time codes to reflect the passage of time and to add polynomial functions to model non-linear trends, a latent basis model freely estimates the time codes to reflect the optimal trajectory. For identification purposes, two time points need to be fixed to 0 and 1 respectively so that $\mu_{\beta yk}$ reflects the total amount of change occurring between these two points. Freely estimated loadings then represent the proportion of the total change ($\mu_{\beta yk}$) that occurred at each specific time point and significance tests associated with these loadings that are routinely reported in any statistical package indicate whether this proportion of change was significant. Here, time was coded so that the intercepts of the trajectories were estimated at Time 1 [$E(\alpha_{iyk}) = \mu_{y1k}$; $\lambda_1 = 0$]. The last time point was coded 1 (λ_7 = 1). The remaining time points ($\lambda_{2k}, \lambda_{3k}, \lambda_{4k}, \lambda_{5k}$, and λ_{6k}) were freely estimated in all classes.

Piecewise GMM

Another flexible way to model non-linear trajectories when there is an expected transition point over the course of the study (school transition, job change, start of an intervention program, etc.) is through the use of piecewise GMM. Piecewise GMM are naturally suited to intervention studies were turning points can be specified as the beginning, or end, of the treatment. In these piecewise models, nonlinearity is through the inclusion of two interrelated linear slopes reflecting growth before and after the transition (e.g., Diallo & Morin, 2014). Globally, piecewise models are specified as:

$$y_{it} = \sum_{k=1}^{K} p_k[\alpha_{iyk} + \beta_{1iyk}\lambda_{1t} + \beta_{2iyk}\lambda_{2t} + \varepsilon_{yitk}] \quad (9)$$

In this model, α_{iyk} remains defined as in equation 2, Φ_{yk} remains defined as in equation 8, β_{1iyk} and β_{2iyk} respectively are linear slopes reflecting the growth occurring before and after the transition point, and are expressed as in Equations 6 and 7. The main difference between this model and the previous one is the reliance on two distinct sets of time scores λ_{1t} and λ_{2t} reflecting the passage of time before, and after the transition point. In the current study, the seven measurements points are equally spaced, and it appears reasonable to set the intercept at Time 1 [$E(\alpha_{iyk})$ = μ_{y1k}] and to set the transition point when the transition to secondary school occurs. Thus, for a piecewise linear GMM, the first set of time scores λ_{1t} would be {0, 1, 2, 2, 2, 2, 2} for time $\lambda_{1t=1}$ to $\lambda_{1t=7}$, and reflect linear growth between the first three time points (after which the equal loadings allow the remaining growth information to be absorbed by the second linear slope factor). Then, the second set of time scores λ_{2t} would be {0, 0, 0, 1, 2, 3, 4} for time $\lambda_{2t=1}$ to $\lambda_{2t=7}$, reflecting linear growth between the last four time points (before which the 0 loadings allow the preceding growth to be absorbed by the first linear slope factor).

It should be noted that there is no need for $\mu_{\beta 2yk}$ to be significantly different from $\mu_{\beta 1yk}$ or for $\zeta_{\beta 2yik}$ to be significantly different from $\zeta_{\beta 1yik}$. Indeed, it may be far more interesting to verify whether the predictors or outcomes of β_{1iyk} differ from those of β_{2iyk}. Similarly, although turning points are typically determined a priori, it is also possible to empirically locate the turning point and even allowing it to differ across subjects (e.g., to study the latency of treatment effects, e.g., Cudeck & Harring, 2007; Cudeck & Klebe, 2002; Kholi, Harring, & Hancock, 2013; Kwok, Luo, & West, 2010).

Additional non-linear GMM specifications

Latent curve models, and by extension GMMs, are quite flexible at modeling various functional forms. We have elected here to focus on the most common (linear and quadratic), and to illustrate two that we considerer to be quite flexible and thus highly useful across many contexts (latent basis and piecewise). However, many additional functional forms can be estimated. Although a complete coverage of these forms would be well beyond the scope of the present chapter, we suggest the following references to interested readers: Blozis, 2007; Browne and DuToit, 1991; Grimm, Ram, and Hamagami, 2011; Grimm et al., 2010; Ram and Grimm, 2007, 2009.

Restricted parameterisations of GMM and implicit invariance assumptions

As noted, the *k* subscript associated with most model parameters indicates that most parameters are allowed to differ across profiles so that each profile can thus be defined by its own latent curve model. However, fully variant GMM are seldom estimated. This may in part be related to the fact that more complex models run more frequently into estimation and convergence problems. But this is also likely to be related to the popularity of simpler more restricted parameterizations (see Morin, Maïano, et al., 2011 for an extensive discussion). Nagin's (1999) group-based latent class growth analysis (LCGA) is arguably the most widely known of these restricted parameterizations. In LCGA, the variances of the growth factors (e.g., $\alpha_{iyk}, \beta_{1iyk}, \beta_{2iyk}$) are constrained to be zero, thus taking out the latent variance-covariance matrix from the model (Φ_{yk} = 0). In this sense, LCGA is essentially a restricted form of GMM in which all members of a profile are assumed to follow the same trajectory. Typically, LCGA also assumes thee time-specific residuals to be equal across profiles ($\varepsilon_{yitk} = \varepsilon_{yit}$).Another typically used restricted parameterisations of GMM is related to the defaults of the Mplus software (Muthén & Muthén, 2014), which specify $\mu_{\alpha yk}$, $\mu_{\beta 1yk}$ and $\mu_{\beta 2yk}$ to be freely estimated in all profiles but constrain the latent variance-covariance parameters as well as the time-specific residuals to be equal across the profiles ($\Phi_{yk} = \Phi_y$ and $\varepsilon_{yitk} = \varepsilon_{yit}$).

Although these restrictions are common, simulation studies have shown that similar restrictions could result in the over-extraction of latent classes and biased parameter estimates in the context of mixture models more generally (e.g., Bauer & Curran, 2004; Enders & Tofighi, 2008; Lubke & Muthén, 2007; Lubke & Neale, 2006, 2008; Magidson & Vermunt, 2004). In discussing the likely impact and meaning of these different restrictions, Morin, Maïano et al. (2011) presented them as untested implicit invariance assumptions that are unlikely to hold in real life and generally fails to be supported when empirically tested. Using a real data set, they further showed that relying on such restricted parameterizations was likely to result in drastically changed substantive conclusions. Unfortunately, arguments supporting the adequacy of these restricted parameterizations are seldom provided in applied research, and tests of these assumptions (which are easy to conduct using the information criteria and LRTs) are almost never implemented. This is worrisome, as these restrictions may substantively change the interpretations of the results. Thus, whenever possible, we suggest that GMM models be estimated with fully independent within-profile models parameters: $\mu_{\alpha yk}$, $\mu_{\beta 1yk}$, $\mu_{\beta 2yk}$, $\zeta_{\alpha yik}$, $\zeta_{\beta 1yik}$, $\zeta_{\beta 2yik}$, Φ_{yk}, ε_{yitk}, and even λ_{tk} in latent basis models.

This is the approach taken in the present chapter. However, as we already noted, more complex models tend to frequently converge on improper solutions, to converge on local maximum, or not to converge at all. These problems, when they cannot be solved by using the strategies proposed in Appendix 9.2 or in the chapter (see the section on piecewise GMM), suggest that the model may have been overparameterized in terms of requesting too many latent profiles, or allowing too many parameters to differ across profiles so that more parsimonious models may be superior (Bauer & Curran, 2003; Chen et al., 2001; Henson et al., 2007). Should GMM users face such problems, we suggest that the following sequence of constraints should be implemented: (1) ε_{yitk} = ε_{yit}; (2) $\psi_{\alpha\beta 1yk}$, $\psi_{\alpha\beta 2yk}$, $\psi_{\beta 1\beta 2yk}$ = $\psi_{\alpha\beta 1y}$, $\psi_{\alpha\beta 2y}$, $\psi_{\beta 1\beta 2y}$; (3) Φ_{yk} = Φ_y; (4) Φ_{yk} = 0. However, this sequence should not be followed blindly and should be adapted to the specific research question that is pursued, and to the specific research context. For instance, in some context research the ability to investigate ε_{yitk} is even more critical than the ability to investigate Φ_{yk} (for examples, see Morin et al., 2012, 2013). Similarly, although we do not recommend the use of LCGA in general, there are some specific research contexts where the sample size makes it impossible to use alternative parameterisations and where LCGA provides the only way to obtain meaningful results.

References used in this Appendix (but not in the main chapter)

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Appendix 9.11.

Results from the GMM

Model	LL	#fp	Scaling	AIC	CAIC	BIC	ABIC	Entropy	aLMR	BLRT
Latent Basis Models of Cardiovascular Fitness										
1 Profile	-23492.749	17	1.6172	47019.499	47159.017	47142.017	47087.993	Na	Na	Na
2 Profile	-19884.348	35	1.9214	39838.696	40125.939	40090.939	39979.714	0.445	≤ 0.001	≤ 0.001
3 Profile	-18476.234	53	2.3299	37058.468	37493.435	37440.435	37272.009	0.442	0.030	≤ 0.001
4 Profile	-17362.897	71	1.9411	34867.795	35450.487	35379.487	35153.860	0.522	0.324	≤ 0.001
5 Profile	-16666.651	89	1.6119	33511.302	34241.719	34152.719	33869.890	0.512	0.162	≤ 0.001
Unconstrained Piecewise Models of Physical Strength										
1 Profile	-21076.156	16	1.4800	42184.311	42315.622	42299.622	42248.776	Na	Na	Na
2 Profile	-18848.542	33	1.2236	37763.084	38033.913	38000.913	37896.044	0.550	≤ 0.001	≤ 0.001
3 Profile	-18346.297	50	1.2111	36792.593	37202.940	37152.940	36994.047	0.496	≤ 0.001	≤ 0.001
4 Profile	-17954.425	67	1.3227	36042.851	36592.716	36525.716	36312.799	0.603	≤ 0.001	≤ 0.001
5 Profile	-17739.077	84	1.4106	35646.153	36335.536	36251.536	35984.596	0.507	0.046	≤ 0.001
Constrained Piecewise Models of Physical Strength										
1 Profile	-21076.156	16	1.4800	42184.311	42315.622	42299.622	42248.776	Na	Na	Na
2 Profile	-18850.562	33	1.2035	37767.124	38037.953	38004.953	37900.083	0.549	Na	≤ 0.001
3 Profile	-18445.386	50	1.4659	36990.773	37401.119	37351.119	37192.227	0.704	Na	≤ 0.001
4 Profile	-18182.396	67	1.2976	36498.793	37048.657	36981.657	36768.741	0.599	Na	≤ 0.001
5 Profile	-17987.048	84	1.2718	36142.095	36831.478	36747.478	36480.538	0.521	Na	≤ 0.001

Note. *: $p \leq .01$; LL: Model LogLikelihood; #fp: Number of free parameters; Scaling = scaling factor associated with MLR loglikelihood estimates; AIC: Akaïke Information Criteria; CAIC: Constant AIC; BIC: Bayesian Information Criteria; ABIC: Sample-Size adjusted BIC; aLMR: Adjusted Lo-Mendell-Rubin likelihood ratio test; BLRT: Bootstrap Likelihood ratio test; LRT: Likelihood Ratio Test; df: Degrees of freedom associated with the LRT.

Appendix 9.12.

Parameters estimates from the final Latent Basis GMM

Parameter	Profile 1 (U-Shaped) Estimate (*t*)	Profile 2 (Increasing) Estimate (*t*)	Profile 3 (Low) Estimate (*t*)
Intercept mean	0.062 (2.000)*	-0.015 (-0.567)	-0.050 (-1.913)
Slope mean	0.023 (2.035)*	0.028 (2.101)*	0.001 (0.144)
Intercept variability ($SD = \sqrt{\sigma}$)	0.937 (17.681)**	0.939 (22.520)**	0.794 (12.404)**
Slope variability ($SD = \sqrt{\sigma}$)	0.152 (5.825)**	0.677 (17.053)**	0.402 (4.336)**
Intercept-slope correlation	-0.273 (-11.340)**	-0.302 (-14.804)**	-0.139 (-3.274)**
Loading Grade 4 (λ_1)	0.000 (NA)	0.000 (NA)	0.000 (NA)
Loading Grade 5 (λ_{2k})	-0.239 (-7.093)**	-0.014 (-1.251)	0.120 (3.008)**
Loading Grade 6 (λ_{3k})	-0.508 (-5.416)**	0.215 (5.602)**	0.802 (7.731)**
Loading Grade 7 (λ_{4k})	-0.314 (-2.525)*	0.626 (12.955)**	1.402 (10.119)**
Loading Grade 8 (λ_{5k})	0.161 (1.711)	0.885 (23.948)**	1.486 (13.620)**
Loading Grade 9 (λ_{6k})	0.800 (19.942)**	1.021 (113.909)**	1.156 (38.834)**
Loading Grade 10 (λ_7)	1.000 (NA)	1.000 (NA)	1.000 (NA)
SD(ε_{yi1})	0.332 (17.002)**	0.237 (16.361)**	0.247 (9.100)**
SD(ε_{yi2})	0.239 (11.741)**	0.122 (6.425)**	0.077 (1.577)
SD(ε_{yi3})	0.148 (7.015)**	0.161 (10.137)**	0.167 (12.294)**
SD(ε_{yi4})	0.221 (7.444)**	0.173 (15.583)**	0.161 (9.430)**
SD(ε_{yi5})	0.161 (8.133)**	0.130 (10.050)**	0.071 (4.486)**
SD(ε_{yi6})	0.130 (6.619)**	0.105 (6.989)**	0.161 (8.838)**
SD(ε_{yi7})	0.192 (7.095)**	0.138 (10.632)**	0.197 (10.032)**

Note. * $p \leq .05$; ** $p \leq .01$; t = Estimate / standard error of the estimate (t value are computed from original variance estimate and not from their square roots); NA = Not applicable; $SD(\varepsilon_{yit})$ = Standard deviations of the time-specific residuals; We present the square roots of the estimates of variability (trajectory factors, time-specific residuals) so that these results can be interpreted in the same units as the constructs used in these models (here, standardized factor scores).

Appendix 9.13.

Parameters estimates from the final Piecewise GMM

Parameter	Profile 1 (Low-Decreasing) Estimate (*t*)	Profile 2 (Increasing) Estimate (*t*)	Profile 3 (High-Stable) Estimate (*t*)	Profile 4 (Mid-Stable) Estimate (*t*)
Intercept mean	-0.323 (-9.926)**	0.044 (0.700)	0.140 (8.909)**	-0.038 (-0.744)
First slope mean	-0.028 (-4.738)**	0.067 (2.603)**	-0.002 (-0.795)	-0.006 (-1.883)
Second slope mean	-0.029 (-5.040)**	0.000 (0.065)	-0.001 (-0.344)	-0.003 (-0.567)
Intercept variability (*SD* = √σ)	1.063 (25.878)**	0.872 (9.713)**	0.801 (32.812)**	0.609 (4.769)**
First slope variability (*SD* = √σ)	0.210 (22.018)**	0.303 (5.441)**	0.138 (10.093)**	0.045 (4.968)**
Second slope variability (*SD* = √σ)	0.195 (17.246)**	0.084 (8.043)**	0.095 (23.060)**	0.045 (4.202)**
Intercept-first slope correlation	-0.060 (-2.471)*	-0.371 (-5.491)**	-0.070 (-2.610)**	0.824 (18.022)**
Intercept-second slope correlation	-0.268 (-11.568)**	-0.192 (-3.320)**	-0.301 (-16.973)**	0.793 (0.067)**
First-second slopes correlations	0.036 (1.237)	-0.396 (-5.462)**	0.017 (0.594)	0.941 (0.043)**
SD(ε_{yi1})	0.000 (0.000)	0.000 (0.000)	0.084 (4.005)**	0.000 (0.000)
SD(ε_{yi2})	0.200 (21.041)**	0.300 (4.510)**	0.141 (12.376)**	0.032 (2.420)*
SD(ε_{yi3})	0.000 (0.000)	0.371 (3.567)**	0.100 (4.931)**	0.000 (0.000)
SD(ε_{yi4})	0.381 (20.853)**	0.148 (2.827)**	0.179 (18.466)**	0.045 (2.491)*
SD(ε_{yi5})	0.345 (22.227)**	0.152 (4.862)**	0.161 (18.167)**	0.063 (3.051)**
SD(ε_{yi6})	0.195 (8.743)**	0.130 (8.560)**	0.114 (16.735)**	0.100 (3.370)**
SD(ε_{yi7})	0.329 (11.157)**	0.000 (0.000)	0.134 (9.282)**	0.197 (4.180)**

Note. *t* = Estimate / standard error of the estimate (*t* value are computed from original variance estimate and not from their square roots); NA = Not applicable; *SD*(ε_{yit}) = Standard deviations of the time-specific residuals; We present the square roots of the estimates of variability (trajectory factors, time-specific residuals) so that these results can be interpreted in the same units as the constructs used in these models (here, standardized factor scores); * $p \leq .05$; ** $p \leq .01$.

Appendix 9.14. Basic Mplus Input Set Up for Mixture Models

In Mplus input files, all section of text preceded by and exclamation point are annotations. Here, we will present the first three sections of the Mplus input file (DATA, VARIABLE, ANALYSIS), as well as the last (OUTPUT) to avoid repeating them in the remaining sections.

The first part of the Mplus input file allows the user to identify the data set that is to be used in the analysis. If the data set is in the same folder as the input file, only the name of the data set needs to be indicated. If the data set is in another folder, then the full path needs to be specified.

```
DATA:
FILE IS DataSing.dat;
```

The next section is the VARIABLE section. The NAMES functions precedes a list of all variables included in the data set, in their order of appearance. Then, the USEVARIABLES functions defines the specific variables to be used in the analysis. The MISSING function defines the code that is used in the data set to identify missing variables (we recommend using the same missing data code for all variables). The IDVARIABLE function defines the unique identifier for participants. The CLUSTER function defines the unique identifier for the clustering (level 2) variable to be controlled in the analysis (here, the school) and should only be included when the user wants to control for clustering. The CLASSES function defines the number of latent profiles required in the analyses (here 5) and thus defines a latent categorical variable (here labeled “c”) with five distinct levels. Finally, the USEOBS function is used to estimate the model using only a subset of the participants. Here, we request that the estimation be limited to participants for whom the variable SEX (representing gender) has a value of 2 (corresponding to females). This function is simply taken out when the model is to be estimated on the full sample.

```
VARIABLE:
NAMES = Index ID Sex P4Code P5Code P6Code S1Code S2Code S3Code S4Code ZP4Sit ZP4Flex ZP4snr
ZP4shut ZP4sbj ZP4Run ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run ZP6Sit ZP6Flex ZP6snr ZP6shut
ZP6sbj ZP6Run ZS1Sit ZS1Flex ZS1snr ZS1shut ZS1sbj ZS1Run ZS2Sit ZS2Flex ZS2snr ZS2shut ZS2sbj
ZS2Run ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run ZS4Sit ZS4Flex ZS4snr ZS4shut ZS4sbj ZS4Run
ZBMI_P4 ZBMI_P5 ZBMI_P6 ZBMI_S1 ZBMI_S2 ZBMI_S3 ZBMI_S4;
USEVARIABLES = ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run;
MISSING = all (-99999);
IDVARIABLE = ID;
CLUSTER = P5Code;
CLASSES = c (5);
USEOBS Sex EQ 2;
```

The next section covers the type of analyses to be conducted. Here, we request the estimation of a mixture model (TYPE = MIXTURE) including a correction for the nesting of students within schools (TYPE = COMPLEX) and using the robust maximum likelihood estimator (ESTIMATOR = MLR). The function STARTS = 5000 200 requests 5000 sets of random start values, and that the best 200 of these starts be kept for final stage optimization. The function STITERATIONS = 100 requests that all random starts be allowed a total of 100 iterations.

```
Analysis:
TYPE = MIXTURE COMPLEX;
ESTIMATOR = MLR;
STARTS = 5000 200; STITERATIONS = 100;
```

The final section of the input covers specific sections of the output to be requested. Here we request standardized model parameters (STDYX), sample statistics (SAMPSTAT), confidence intervals (CINTERVAL), the starts values corresponding to the solution (SVALUES), the residuals (RESIDUAL), the arrays of parameter specifications and starting values (TECH1), the profile-specific sample characteristics (TECH7), the LMR and aLMR (TECH11), and the BLRT (TECH14).

```
OUTPUT:
STDYX SAMPSTAT CINTERVAL SVALUES RESIDUAL TECH1 TECH7 TECH11 TECH14;
```

Appendix 9.15. Estimation of a 5-Profile LPA solution

In mixture models, the MODEL section includes an %OVERALL% section describing the global relations estimated among the constructs, and profile specific statements (here %c#1% to %c#5%, where c corresponds to the labeled used to define the categorical latent variable in the CLASSES section of the VARIABLE: section, and the number 1 to k refers to the specific value of this variable (the specific profile). Here, no relations are estimated between the variables so nothing appears in the %OVERALL% section. The profile specific sections request that the means (indicated by the name of the variable between brackets []) and variances (indicated simply by the names of the variables) of the indicators be freely estimated in all profiles. To estimate profiles with variances that are equal across profiles, the statements (e.g., “ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run”) referring to the variances of the indicators simply need to be taken out.

```
MODEL:
%OVERALL%
%c#1%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run];
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run ;
%c#2%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run];
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run ;
%c#3%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run];
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run ;
%c#4%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run];
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run ;
%c#5%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run];
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run ;
```

Appendix 9.16. Estimation of a 5-Profile Factor Mixture Solution

Here we present an input for the estimation of the global factor mixture model described in Morin and Marsh (2014) and described in Appendix 9.4. as providing a way to control for global levels shared among the indicator in order to estimate clearer latent profiles. The only difference with the previous model is the introduction of a common factor model in the %OVERALL% section of the input (as this factor model is specified as invariant across profiles, nothing needs to be added to the profile-specific statements). Here, the common factor is labeled G, and defined by the various indicators (the command BY defines factor loadings). All loadings on this factor are freely estimated (the * associated with the first indicators is to override the Mplus default of constraining the loading of the first factor to be 1 for identification purposes, which requires its variance to be fixed to 1 (the @ is used to fix a parameter to a specific value). Because the intercepts of the indicators of this factor will be freely estimated across profiles, the factor means needs to be fixed to 0 for identification purposes.

```
MODEL:
%OVERALL%
G BY ZP5Sit* ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run ;
G@1;
[G@0];
%c#1%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run];
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run ;
%c#2%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run];
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run ;
%c#3%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run];
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run ;
%c#4%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run];
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run ;
%c#5%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run];
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run ;
```

Appendix 9.17.

Estimation of a 5-Profile LPA in Multiple Observed Groups (Configural Invariance)

In the VARIABLE section of the input, the multiple observed groups across which the model will be estimated needs to be defined. In mixture models, this is done using the KNOWCLASS function, which uses a label (here we use cg) to define this new grouping variable, and the levels of this new grouping variables are defined as: (a) including participants with a value of 1 (male) on the variable SEX, and (b) including participants with a value of 2 (female) on the variable SEX. The mixture model will now considered that there are two latent grouping variables, C estimated as part of the model estimation (the profiles) and having *k* levels (here we are still working with a solution including 5 profiles) and CG reflecting the observed subgroups (gender) which has 2 levels. Participants are allowed to be cross classified on these two grouping variables.

```
KNOWCLASS = cg (Sex = 1 Sex = 2);
CLASSES = cg (2) c (5);
```

The %OVERALL% section of the model section, are used to indicate that the class sizes are freely estimated in all observed samples (males and females) using the ON function (reflecting regressions) indicating that profile membership is conditional on gender. Only *k*-1 statements are required (i.e., 4 for a 5-profile model). Then, profile-specific statements now need to be defined using a combination of both the known classes CG and the estimated classes C. Labels in parentheses identify parameters that are estimated to be equal across groups. Here, even though all parameters are labeled, none of these labels are share between groups, so that the means and variances are freely estimated in all combinations of profiles and gender. Lists of constraints (e.g., m1-m3) apply to the parameters in order of appearance (e.g., m1 applies to ZP5Sit, m2 to ZP5Flex, m3 to ZP5snr and so on).

```
%OVERALL%
c#1 on cg#1; c#2 on cg#1; c#3 on cg#1; c#4 on cg#1;
%cg#1.c#1%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m1-m6);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v1-v6);
%cg#1.c#2%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m7-m12);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v7-v12);
%cg#1.c#3%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m13-m18);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v13-v18);
%cg#1.c#4%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m19-m24);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v19-v24);
%cg#1.c#5%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m25-m30);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v25-v30);

%cg#2.c#1%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (mm1-mm6);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (vv1-vv6);
%cg#2.c#2%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (mm7-mm12);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (vv7-vv12);
%cg#2.c#3%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (mm13-mm18);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (vv13-vv18);
%cg#2.c#4%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (mm19-mm24);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (vv19-vv24);
%cg#2.c#5%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (mm25-mm30);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (vv25-vv30);
```

Appendix 9.18.

Estimation of a 5-Profile LPA in Multiple Observed Groups (Structural Invariance)

The only difference between this model and the previous one is that the means are constrained to be equal across gender within each profile using identical labels in parentheses.

```
%OVERALL%
c#1 on cg#1; c#2 on cg#1; c#3 on cg#1; c#4 on cg#1;
%cg#1.c#1%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m1-m6);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v1-v6);
%cg#1.c#2%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m7-m12);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v7-v12);
%cg#1.c#3%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m13-m18);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v13-v18);
%cg#1.c#4%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m19-m24);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v19-v24);
%cg#1.c#5%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m25-m30);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v25-v30);

%cg#2.c#1%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m1-m6);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (vv1-vv6);
%cg#2.c#2%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m7-m12);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (vv7-vv12);
%cg#2.c#3%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m13-m18);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (vv13-vv18);
%cg#2.c#4%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m19-m24);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (vv19-vv24);
%cg#2.c#5%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m25-m30);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (vv25-vv30);
```

Appendix 9.19.

Estimation of a 5-Profile LPA in Multiple Observed Groups (Dispersion Invariance)

The only difference between this model and the previous one is that the variances are also constrained to be equal across gender within each profile using identical labels in parentheses.

```
%OVERALL%
c#1 on cg#1; c#2 on cg#1; c#3 on cg#1; c#4 on cg#1;
%cg#1.c#1%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m1-m6);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v1-v6);
%cg#1.c#2%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m7-m12);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v7-v12);
%cg#1.c#3%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m13-m18);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v13-v18);
%cg#1.c#4%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m19-m24);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v19-v24);
%cg#1.c#5%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m25-m30);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v25-v30);

%cg#2.c#1%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m1-m6);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v1-v6);
%cg#2.c#2%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m7-m12);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v7-v12);
%cg#2.c#3%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m13-m18);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v13-v18);
%cg#2.c#4%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m19-m24);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v19-v24);
%cg#2.c#5%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m25-m30);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v25-v30);
```

Appendix 9.20.

Estimation of a 5-Profile LPA in Multiple Observed Groups (Distribution Invariance)

Given that the dispersion invariance of the model was not supported, this model was built from the model of structural invariance. However, to build it from the model of dispersion invariance, one only needs to reinstate the invariance constraints on the variance parameters. The only difference between this model and the model of structural invariance one is that nothing appears in the %OVERALL% section of the input to reflect the fact that the sizes of the profiles are no longer conditional on gender.

```
%OVERALL%
%cg#1.c#1%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m1-m6);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v1-v6);
%cg#1.c#2%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m7-m12);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v7-v12);
%cg#1.c#3%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m13-m18);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v13-v18);
%cg#1.c#4%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m19-m24);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v19-v24);
%cg#1.c#5%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m25-m30);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v25-v30);

%cg#2.c#1%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m1-m6);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (vv1-vv6);
%cg#2.c#2%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m7-m12);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (vv7-vv12);
%cg#2.c#3%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m13-m18);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (vv13-vv18);
%cg#2.c#4%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m19-m24);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (vv19-vv24);
%cg#2.c#5%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m25-m30);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (vv25-vv30);
```

Appendix 9.21.

Estimation of a 5-Profile LPA in Multiple Observed Groups, Including Predictors with Effects Freely Estimated Across Gender.

This models uses the SVALUES associated with the previous model of dispersion invariance (specified as * followed by the value of the estimated parameters), and simply include covariates effects on profile membership (c#1-c#4 ON ZBMI_P4;, reflecting the idea that profile membership is conditional on BMI). To allow these effects to be freely estimated across gender, they need to be constrained to 0 in the %OVERALL% section, and freely estimated in both gender groups in a new section of the input specifically referring to CG. See all sections in bold.

```
%OVERALL%
[ cg#1*-0.00217 ]; [ c#1*-0.49034 ]; [ c#2*0.64971 ]; [ c#3*0.38954 ]; [ c#4*-0.53810 ];
c#1-c#4 ON ZBMI_P4@0;
%CG#1.C#1%
[ zp5sit*-1.03305 ] (m1); [ zp5flex*-1.08704 ] (m2);
[ zp5snr*-0.42895 ] (m3); [ zp5shut*1.35821 ] (m4);
[ zp5sbj*-1.35181 ] (m5); [ zp5run*1.29563 ] (m6);
zp5sit*0.93220 (v1); zp5flex*0.14704 (v2);
zp5snr*0.79632 (v3); zp5shut*1.09086 (v4);
zp5sbj*0.74147 (v5); zp5run*0.68367 (v6);
%CG#1.C#2%
[ zp5sit*-0.26254 ] (m7); [ zp5flex*-0.45790 ] (m8);
[ zp5snr*-0.64127 ] (m9); [ zp5shut*0.26265 ] (m10);
[ zp5sbj*-0.34736 ] (m11); [ zp5run*0.27622 ] (m12);
zp5sit*0.59074 (v7); zp5flex*0.36366 (v8);
zp5snr*0.21725 (v9); zp5shut*0.35089 (v10);
zp5sbj*0.24847 (v11); zp5run*0.57875 (v12);
%CG#1.C#3%
[ zp5sit*0.58893 ] (m13); [ zp5flex*0.19735 ] (m14);
[ zp5snr*-0.39566 ] (m15); [ zp5shut*-0.74653 ] (m16);
[ zp5sbj*0.72653 ] (m17); [ zp5run*-0.67554 ] (m18);
zp5sit*0.68416 (v13); zp5flex*0.42929 (v14);
zp5snr*0.22035 (v15); zp5shut*0.43196 (v16);
zp5sbj*0.58758 (v17); zp5run*0.46840 (v18);
%CG#1.C#4%
[ zp5sit*0.72479 ] (m19); [ zp5flex*1.56628 ] (m20);
[ zp5snr*1.47831 ] (m21); [ zp5shut*-0.82727 ] (m22);
[ zp5sbj*0.98243 ] (m23); [ zp5run*-0.85448 ] (m24);
zp5sit*0.54772 (v19); zp5flex*0.37416 (v20);
zp5snr*0.26672 (v21); zp5shut*0.60201 (v22);
zp5sbj*0.73433 (v23); zp5run*0.36566 (v24);
%CG#1.C#5%
[ zp5sit*-0.16071 ] (m25); [ zp5flex*0.38795 ] (m26);
[ zp5snr*1.20871 ] (m27); [ zp5shut*0.24698 ] (m28);
[ zp5sbj*-0.15526 ] (m29); [ zp5run*0.17023 ] (m30);
zp5sit*0.69737 (v25); zp5flex*1.03973 (v26);
zp5snr*0.27897 (v27); zp5shut*0.43848 (v28);
zp5sbj*0.41388 (v29); zp5run*0.55283 (v30);
%CG#2.C#1%
[ zp5sit*-1.03305 ] (m1); [ zp5flex*-1.08704 ] (m2);
[ zp5snr*-0.42895 ] (m3); [ zp5shut*1.35821 ] (m4);
```

```
[ zp5sbj*-1.35181 ] (m5); [ zp5run*1.29563 ] (m6);
zp5sit*1.03814 (vv1); zp5flex*0.09550 (vv2);
zp5snr*0.86685 (vv3); zp5shut*1.14245 (vv4);
zp5sbj*0.83652 (vv5); zp5run*1.01864 (vv6);
%CG#2.C#2%
[ zp5sit*-0.26254 ] (m7); [ zp5flex*-0.45790 ] (m8);
[ zp5snr*-0.64127 ] (m9); [ zp5shut*0.26265 ] (m10);
[ zp5sbj*-0.34736 ] (m11); [ zp5run*0.27622 ] (m12);
zp5sit*0.78006 (vv7); zp5flex*0.39196 (vv8);
zp5snr*0.20980 (vv9); zp5shut*0.49815 (vv10);
zp5sbj*0.42562 (vv11); zp5run*0.60355 (vv12);
%CG#2.C#3%
[ zp5sit*0.58893 ] (m13); [ zp5flex*0.19735 ] (m14);
[ zp5snr*-0.39566 ] (m15); [ zp5shut*-0.74653 ] (m16);
[ zp5sbj*0.72653 ] (m17); [ zp5run*-0.67554 ] (m18);
zp5sit*0.56603 (vv13); zp5flex*0.49250 (vv14);
zp5snr*0.24091 (vv15); zp5shut*0.51077 (vv16);
zp5sbj*0.60299 (vv17); zp5run*0.57497 (vv18);
%CG#2.C#4%
[ zp5sit*0.72479 ] (m19); [ zp5flex*1.56628 ] (m20);
[ zp5snr*1.47831 ] (m21); [ zp5shut*-0.82727 ] (m22);
[ zp5sbj*0.98243 ] (m23); [ zp5run*-0.85448 ] (m24);
zp5sit*0.78422 (vv19); zp5flex*0.72949 (vv20);
zp5snr*0.29214 (vv21); zp5shut*0.53613 (vv22);
zp5sbj*0.63470 (vv23); zp5run*0.62658 (vv24);
%CG#2.C#5%
[ zp5sit*-0.16071 ] (m25); [ zp5flex*0.38795 ] (m26);
[ zp5snr*1.20871 ] (m27); [ zp5shut*0.24698 ] (m28);
[ zp5sbj*-0.15526 ] (m29); [ zp5run*0.17023 ] (m30);
zp5sit*0.78790 (vv25); zp5flex*1.14398 (vv26);
zp5snr*0.31522 (vv27); zp5shut*0.58147 (vv28);
zp5sbj*0.49954 (vv29); zp5run*0.68549 (vv30);
MODEL cg:
%cg#1%
c#1-c#4 ON  ZBMI_P4;
%cg#2%
c#1-c#4 ON  ZBMI_P4;
```

Appendix 9.22.

Estimation of a 5-Profile LPA in Multiple Observed Groups, Including Predictors with Effects Constrained to Invariance Across Gender (Deterministic Invariance).

This model is almost identical to the previous one. In order for the effects of the predictors to be constrained to invariance across genders, they simply need to be specified as freely estimated in the %OVERALL% section (c#1-c#4 ON ZBMI_P4;), while taking out the gender specific sections.

```
%OVERALL%
[ cg#1*-0.00217 ]; [ c#1*-0.49034 ]; [ c#2*0.64971 ]; [ c#3*0.38954 ]; [ c#4*-0.53810 ];
c#1-c#4 ON ZBMI_P4;
%CG#1.C#1%
[ zp5sit*-1.03305 ] (m1); [ zp5flex*-1.08704 ] (m2);
[ zp5snr*-0.42895 ] (m3); [ zp5shut*1.35821 ] (m4);
[ zp5sbj*-1.35181 ] (m5); [ zp5run*1.29563 ] (m6);
zp5sit*0.93220 (v1); zp5flex*0.14704 (v2);
zp5snr*0.79632 (v3); zp5shut*1.09086 (v4);
zp5sbj*0.74147 (v5); zp5run*0.68367 (v6);
%CG#1.C#2%
[ zp5sit*-0.26254 ] (m7); [ zp5flex*-0.45790 ] (m8);
[ zp5snr*-0.64127 ] (m9); [ zp5shut*0.26265 ] (m10);
[ zp5sbj*-0.34736 ] (m11); [ zp5run*0.27622 ] (m12);
zp5sit*0.59074 (v7); zp5flex*0.36366 (v8);
zp5snr*0.21725 (v9); zp5shut*0.35089 (v10);
zp5sbj*0.24847 (v11); zp5run*0.57875 (v12);
%CG#1.C#3%
[ zp5sit*0.58893 ] (m13); [ zp5flex*0.19735 ] (m14);
[ zp5snr*-0.39566 ] (m15); [ zp5shut*-0.74653 ] (m16);
[ zp5sbj*0.72653 ] (m17); [ zp5run*-0.67554 ] (m18);
zp5sit*0.68416 (v13); zp5flex*0.42929 (v14);
zp5snr*0.22035 (v15); zp5shut*0.43196 (v16);
zp5sbj*0.58758 (v17); zp5run*0.46840 (v18);
%CG#1.C#4%
[ zp5sit*0.72479 ] (m19); [ zp5flex*1.56628 ] (m20);
[ zp5snr*1.47831 ] (m21); [ zp5shut*-0.82727 ] (m22);
[ zp5sbj*0.98243 ] (m23); [ zp5run*-0.85448 ] (m24);
zp5sit*0.54772 (v19); zp5flex*0.37416 (v20);
zp5snr*0.26672 (v21); zp5shut*0.60201 (v22);
zp5sbj*0.73433 (v23); zp5run*0.36566 (v24);
%CG#1.C#5%
[ zp5sit*-0.16071 ] (m25); [ zp5flex*0.38795 ] (m26);
[ zp5snr*1.20871 ] (m27); [ zp5shut*0.24698 ] (m28);
[ zp5sbj*-0.15526 ] (m29); [ zp5run*0.17023 ] (m30);
zp5sit*0.69737 (v25); zp5flex*1.03973 (v26);
zp5snr*0.27897 (v27); zp5shut*0.43848 (v28);
zp5sbj*0.41388 (v29); zp5run*0.55283 (v30);
%CG#2.C#1%
[ zp5sit*-1.03305 ] (m1); [ zp5flex*-1.08704 ] (m2);
[ zp5snr*-0.42895 ] (m3); [ zp5shut*1.35821 ] (m4);
[ zp5sbj*-1.35181 ] (m5); [ zp5run*1.29563 ] (m6);
zp5sit*1.03814 (vv1); zp5flex*0.09550 (vv2);
zp5snr*0.86685 (vv3); zp5shut*1.14245 (vv4);
zp5sbj*0.83652 (vv5); zp5run*1.01864 (vv6);
```

```
%CG#2.C#2%
[ zp5sit*-0.26254 ] (m7); [ zp5flex*-0.45790 ] (m8);
[ zp5snr*-0.64127 ] (m9); [ zp5shut*0.26265 ] (m10);
[ zp5sbj*-0.34736 ] (m11); [ zp5run*0.27622 ] (m12);
zp5sit*0.78006 (vv7); zp5flex*0.39196 (vv8);
zp5snr*0.20980 (vv9); zp5shut*0.49815 (vv10);
zp5sbj*0.42562 (vv11); zp5run*0.60355 (vv12);
%CG#2.C#3%
[ zp5sit*0.58893 ] (m13); [ zp5flex*0.19735 ] (m14);
[ zp5snr*-0.39566 ] (m15); [ zp5shut*-0.74653 ] (m16);
[ zp5sbj*0.72653 ] (m17); [ zp5run*-0.67554 ] (m18);
zp5sit*0.56603 (vv13); zp5flex*0.49250 (vv14);
zp5snr*0.24091 (vv15); zp5shut*0.51077 (vv16);
zp5sbj*0.60299 (vv17); zp5run*0.57497 (vv18);
%CG#2.C#4%
[ zp5sit*0.72479 ] (m19); [ zp5flex*1.56628 ] (m20);
[ zp5snr*1.47831 ] (m21); [ zp5shut*-0.82727 ] (m22);
[ zp5sbj*0.98243 ] (m23); [ zp5run*-0.85448 ] (m24);
zp5sit*0.78422 (vv19); zp5flex*0.72949 (vv20);
zp5snr*0.29214 (vv21); zp5shut*0.53613 (vv22);
zp5sbj*0.63470 (vv23); zp5run*0.62658 (vv24);
%CG#2.C#5%
[ zp5sit*-0.16071 ] (m25); [ zp5flex*0.38795 ] (m26);
[ zp5snr*1.20871 ] (m27); [ zp5shut*0.24698 ] (m28);
[ zp5sbj*-0.15526 ] (m29); [ zp5run*0.17023 ] (m30);
zp5sit*0.78790 (vv25); zp5flex*1.14398 (vv26);
zp5snr*0.31522 (vv27); zp5shut*0.58147 (vv28);
zp5sbj*0.49954 (vv29); zp5run*0.68549 (vv30);
```

Appendix 9.23.

Estimation of a 5-Profile LPA in Multiple Observed Groups, Including Distal Outcomes with Relations Freely Estimated Across Gender.

This model also uses the SVALUES associated with the model of distributional invariance. Here, we simply request the free estimation of the distal outcome means in all profiles x genders ([ZBMI_P6]). We also use labels in parentheses to identify these new parameters, which will then be used in a new MODEL CONSTRAINT section to request tests of the significance of mean differences between profiles and genders.

```
%OVERALL%
[ cg#1*-0.00217 ]; [ c#1*-0.49034 ]; [ c#2*0.64971 ]; [ c#3*0.38954 ]; [ c#4*-0.53810 ];
%CG#1.C#1%
[ zp5sit*-1.03305 ] (m1); [ zp5flex*-1.08704 ] (m2);
[ zp5snr*-0.42895 ] (m3); [ zp5shut*1.35821 ] (m4);
[ zp5sbj*-1.35181 ] (m5); [ zp5run*1.29563 ] (m6);
zp5sit*0.93220 (v1); zp5flex*0.14704 (v2);
zp5snr*0.79632 (v3); zp5shut*1.09086 (v4);
zp5sbj*0.74147 (v5); zp5run*0.68367 (v6);
[ZBMI_P6] (oa1);
%CG#1.C#2%
[ zp5sit*-0.26254 ] (m7); [ zp5flex*-0.45790 ] (m8);
[ zp5snr*-0.64127 ] (m9); [ zp5shut*0.26265 ] (m10);
[ zp5sbj*-0.34736 ] (m11); [ zp5run*0.27622 ] (m12);
zp5sit*0.59074 (v7); zp5flex*0.36366 (v8);
zp5snr*0.21725 (v9); zp5shut*0.35089 (v10);
zp5sbj*0.24847 (v11); zp5run*0.57875 (v12);
[ZBMI_P6] (oa2);
%CG#1.C#3%
[ zp5sit*0.58893 ] (m13); [ zp5flex*0.19735 ] (m14);
[ zp5snr*-0.39566 ] (m15); [ zp5shut*-0.74653 ] (m16);
[ zp5sbj*0.72653 ] (m17); [ zp5run*-0.67554 ] (m18);
zp5sit*0.68416 (v13); zp5flex*0.42929 (v14);
zp5snr*0.22035 (v15); zp5shut*0.43196 (v16);
zp5sbj*0.58758 (v17); zp5run*0.46840 (v18);
[ZBMI_P6] (oa3);
%CG#1.C#4%
[ zp5sit*0.72479 ] (m19); [ zp5flex*1.56628 ] (m20);
[ zp5snr*1.47831 ] (m21); [ zp5shut*-0.82727 ] (m22);
[ zp5sbj*0.98243 ] (m23); [ zp5run*-0.85448 ] (m24);
zp5sit*0.54772 (v19); zp5flex*0.37416 (v20);
zp5snr*0.26672 (v21); zp5shut*0.60201 (v22);
zp5sbj*0.73433 (v23); zp5run*0.36566 (v24);
[ZBMI_P6] (oa4);
%CG#1.C#5%
[ zp5sit*-0.16071 ] (m25); [ zp5flex*0.38795 ] (m26);
[ zp5snr*1.20871 ] (m27); [ zp5shut*0.24698 ] (m28);
[ zp5sbj*-0.15526 ] (m29); [ zp5run*0.17023 ] (m30);
zp5sit*0.69737 (v25); zp5flex*1.03973 (v26);
zp5snr*0.27897 (v27); zp5shut*0.43848 (v28);
zp5sbj*0.41388 (v29); zp5run*0.55283 (v30);
[ZBMI_P6] (oa5);
```

```
%CG#2.C#1%
[ zp5sit*-1.03305 ] (m1); [ zp5flex*-1.08704 ] (m2);
[ zp5snr*-0.42895 ] (m3); [ zp5shut*1.35821 ] (m4);
[ zp5sbj*-1.35181 ] (m5); [ zp5run*1.29563 ] (m6);
zp5sit*1.03814 (vv1); zp5flex*0.09550 (vv2);
zp5snr*0.86685 (vv3); zp5shut*1.14245 (vv4);
zp5sbj*0.83652 (vv5); zp5run*1.01864 (vv6);
[ZBMI_P6] (ob1);
%CG#2.C#2%
[ zp5sit*-0.26254 ] (m7); [ zp5flex*-0.45790 ] (m8);
[ zp5snr*-0.64127 ] (m9); [ zp5shut*0.26265 ] (m10);
[ zp5sbj*-0.34736 ] (m11); [ zp5run*0.27622 ] (m12);
zp5sit*0.78006 (vv7); zp5flex*0.39196 (vv8);
zp5snr*0.20980 (vv9); zp5shut*0.49815 (vv10);
zp5sbj*0.42562 (vv11); zp5run*0.60355 (vv12);
[ZBMI_P6] (ob2);
%CG#2.C#3%
[ zp5sit*0.58893 ] (m13); [ zp5flex*0.19735 ] (m14);
[ zp5snr*-0.39566 ] (m15); [ zp5shut*-0.74653 ] (m16);
[ zp5sbj*0.72653 ] (m17); [ zp5run*-0.67554 ] (m18);
zp5sit*0.56603 (vv13); zp5flex*0.49250 (vv14);
zp5snr*0.24091 (vv15); zp5shut*0.51077 (vv16);
zp5sbj*0.60299 (vv17); zp5run*0.57497 (vv18);
[ZBMI_P6] (ob3);
%CG#2.C#4%
[ zp5sit*0.72479 ] (m19); [ zp5flex*1.56628 ] (m20);
[ zp5snr*1.47831 ] (m21); [ zp5shut*-0.82727 ] (m22);
[ zp5sbj*0.98243 ] (m23); [ zp5run*-0.85448 ] (m24);
zp5sit*0.78422 (vv19); zp5flex*0.72949 (vv20);
zp5snr*0.29214 (vv21); zp5shut*0.53613 (vv22);
zp5sbj*0.63470 (vv23); zp5run*0.62658 (vv24);
[ZBMI_P6] (ob4);
%CG#2.C#5%
[ zp5sit*-0.16071 ] (m25); [ zp5flex*0.38795 ] (m26);
[ zp5snr*1.20871 ] (m27); [ zp5shut*0.24698 ] (m28);
[ zp5sbj*-0.15526 ] (m29); [ zp5run*0.17023 ] (m30);
zp5sit*0.78790 (vv25); zp5flex*1.14398 (vv26);
zp5snr*0.31522 (vv27); zp5shut*0.58147 (vv28);
zp5sbj*0.49954 (vv29); zp5run*0.68549 (vv30);
[ZBMI_P6] (ob5);
MODEL CONSTRAINT:
  ! New parameters are created using this function and reflect pairwise mean differences between
  ! profiles. So the first of those (y12) reflect the differences between the means of profiles 1 and 2.
  ! This will be included in the outputs as new parameters reflecting the significance of
  ! the differences between the means, without those parameters having an impact on the model.
NEW (y12);
y12 = oa1-oa2;
NEW (y13);
y13 = oa1-oa3;
NEW (y14);
y14 = oa1-oa4;
NEW (y15);
y15 = oa1-oa5;
NEW (y23);
y23 = oa2-oa3;
```

```
NEW (y24);
y24 = oa2-oa4;
NEW (y25);
y25 = oa2-oa5;
NEW (y34);
y34 = oa3-oa4;
NEW (y35);
y35 = oa3-oa5;
NEW (y45);
y45 = oa4-oa5;
NEW (z12);
z12 = ob1-ob2;
NEW (z13);
z13 = ob1-ob3;
NEW (z14);
z14 = ob1-ob4;
NEW (z15);
z15 = ob1-ob5;
NEW (z23);
z23 = ob2-ob3;
NEW (z24);
z24 = ob2-ob4;
NEW (z25);
z25 = ob2-ob5;
NEW (z34);
z34 = ob3-ob4;
NEW (z35);
z35 = ob3-ob5;
NEW (z45);
z45 = ob4-ob5;
NEW (w11);
w11 = oa1-ob1;
NEW (w22);
w22= oa2-ob2;
NEW (w33);
w33= oa3-ob3;
NEW (w44);
w44= oa4-ob4;
NEW (w55);
w55= oa5-ob5;
```

Appendix 9.24.

Estimation of a 5-Profile LPA in Multiple Observed Groups, Including Distal Outcomes with Relations Constrained to Invariance Across Gender (Predictive Invariance).

This model is almost identical to the previous one except that the parameter labels are used to constrain the outcome means to be invariant across genders. As a result, less lines of code are required in the MODEL COSNTRAINT section.

```
%OVERALL%
[ cg#1*-0.00217 ]; [ c#1*-0.49034 ]; [ c#2*0.64971 ]; [ c#3*0.38954 ]; [ c#4*-0.53810 ];
%CG#1.C#1%
[ zp5sit*-1.03305 ] (m1); [ zp5flex*-1.08704 ] (m2);
[ zp5snr*-0.42895 ] (m3); [ zp5shut*1.35821 ] (m4);
[ zp5sbj*-1.35181 ] (m5); [ zp5run*1.29563 ] (m6);
zp5sit*0.93220 (v1); zp5flex*0.14704 (v2);
zp5snr*0.79632 (v3); zp5shut*1.09086 (v4);
zp5sbj*0.74147 (v5); zp5run*0.68367 (v6);
[ZBMI_P6] (oa1);
%CG#1.C#2%
[ zp5sit*-0.26254 ] (m7); [ zp5flex*-0.45790 ] (m8);
[ zp5snr*-0.64127 ] (m9); [ zp5shut*0.26265 ] (m10);
[ zp5sbj*-0.34736 ] (m11); [ zp5run*0.27622 ] (m12);
zp5sit*0.59074 (v7); zp5flex*0.36366 (v8);
zp5snr*0.21725 (v9); zp5shut*0.35089 (v10);
zp5sbj*0.24847 (v11); zp5run*0.57875 (v12);
[ZBMI_P6] (oa2);
%CG#1.C#3%
[ zp5sit*0.58893 ] (m13); [ zp5flex*0.19735 ] (m14);
[ zp5snr*-0.39566 ] (m15); [ zp5shut*-0.74653 ] (m16);
[ zp5sbj*0.72653 ] (m17); [ zp5run*-0.67554 ] (m18);
zp5sit*0.68416 (v13); zp5flex*0.42929 (v14);
zp5snr*0.22035 (v15); zp5shut*0.43196 (v16);
zp5sbj*0.58758 (v17); zp5run*0.46840 (v18);
[ZBMI_P6] (oa3);
%CG#1.C#4%
[ zp5sit*0.72479 ] (m19); [ zp5flex*1.56628 ] (m20);
[ zp5snr*1.47831 ] (m21); [ zp5shut*-0.82727 ] (m22);
[ zp5sbj*0.98243 ] (m23); [ zp5run*-0.85448 ] (m24);
zp5sit*0.54772 (v19); zp5flex*0.37416 (v20);
zp5snr*0.26672 (v21); zp5shut*0.60201 (v22);
zp5sbj*0.73433 (v23); zp5run*0.36566 (v24);
[ZBMI_P6] (oa4);
%CG#1.C#5%
[ zp5sit*-0.16071 ] (m25); [ zp5flex*0.38795 ] (m26);
[ zp5snr*1.20871 ] (m27); [ zp5shut*0.24698 ] (m28);
[ zp5sbj*-0.15526 ] (m29); [ zp5run*0.17023 ] (m30);
zp5sit*0.69737 (v25); zp5flex*1.03973 (v26);
zp5snr*0.27897 (v27); zp5shut*0.43848 (v28);
zp5sbj*0.41388 (v29); zp5run*0.55283 (v30);
[ZBMI_P6] (oa5);
%CG#2.C#1%
[ zp5sit*-1.03305 ] (m1); [ zp5flex*-1.08704 ] (m2);
[ zp5snr*-0.42895 ] (m3); [ zp5shut*1.35821 ] (m4);
[ zp5sbj*-1.35181 ] (m5); [ zp5run*1.29563 ] (m6);
zp5sit*1.03814 (vv1); zp5flex*0.09550 (vv2);
zp5snr*0.86685 (vv3); zp5shut*1.14245 (vv4);
zp5sbj*0.83652 (vv5); zp5run*1.01864 (vv6);
[ZBMI_P6] (oa1);
%CG#2.C#2%
```

```
[ zp5sit*-0.26254 ] (m7); [ zp5flex*-0.45790 ] (m8);
[ zp5snr*-0.64127 ] (m9); [ zp5shut*0.26265 ] (m10);
[ zp5sbj*-0.34736 ] (m11); [ zp5run*0.27622 ] (m12);
zp5sit*0.78006 (vv7); zp5flex*0.39196 (vv8);
zp5snr*0.20980 (vv9); zp5shut*0.49815 (vv10);
zp5sbj*0.42562 (vv11); zp5run*0.60355 (vv12);
[ZBMI_P6] (oa2);
%CG#2.C#3%
[ zp5sit*0.58893 ] (m13); [ zp5flex*0.19735 ] (m14);
[ zp5snr*-0.39566 ] (m15); [ zp5shut*-0.74653 ] (m16);
[ zp5sbj*0.72653 ] (m17); [ zp5run*-0.67554 ] (m18);
zp5sit*0.56603 (vv13); zp5flex*0.49250 (vv14);
zp5snr*0.24091 (vv15); zp5shut*0.51077 (vv16);
zp5sbj*0.60299 (vv17); zp5run*0.57497 (vv18);
[ZBMI_P6] (oa3);
%CG#2.C#4%
[ zp5sit*0.72479 ] (m19); [ zp5flex*1.56628 ] (m20);
[ zp5snr*1.47831 ] (m21); [ zp5shut*-0.82727 ] (m22);
[ zp5sbj*0.98243 ] (m23); [ zp5run*-0.85448 ] (m24);
zp5sit*0.78422 (vv19); zp5flex*0.72949 (vv20);
zp5snr*0.29214 (vv21); zp5shut*0.53613 (vv22);
zp5sbj*0.63470 (vv23); zp5run*0.62658 (vv24);
[ZBMI_P6] (oa4);
%CG#2.C#5%
[ zp5sit*-0.16071 ] (m25); [ zp5flex*0.38795 ] (m26);
[ zp5snr*1.20871 ] (m27); [ zp5shut*0.24698 ] (m28);
[ zp5sbj*-0.15526 ] (m29); [ zp5run*0.17023 ] (m30);
zp5sit*0.78790 (vv25); zp5flex*1.14398 (vv26);
zp5snr*0.31522 (vv27); zp5shut*0.58147 (vv28);
zp5sbj*0.49954 (vv29); zp5run*0.68549 (vv30);
[ZBMI_P6] (oa5);
MODEL CONSTRAINT:
NEW (y12);
y12 = oa1-oa2;
NEW (y13);
y13 = oa1-oa3;
NEW (y14);
y14 = oa1-oa4;
NEW (y15);
y15 = oa1-oa5;
NEW (y23);
y23 = oa2-oa3;
NEW (y24);
y24 = oa2-oa4;
NEW (y25);
y25 = oa2-oa5;
NEW (y34);
y34 = oa3-oa4;
NEW (y35);
y35 = oa3-oa5;
NEW (y45);
y45 = oa4-oa5;
```

Appendix 9.25. Estimation of a 5-Profile LTA (Configural Invariance).

The estimation of a latent transition model is highly similar to the estimation of a multiple-group LPA with the exception that the latent categorical variable representing the observed group defined using the KNOWCLASS function (CG in the previous examples) is replaced by another unknown latent categorical variable representing profiles groups estimated at the second time point.

```
CLASSES = c1 (5) c2 (5);
```

Because of the similarity of the input set ups, we will not comment the sequence of invariance tests in the next sections. In the basic LTA model, the %OVERALL% section states that membership into the profiles at the second time point (C2) is conditional on membership in the profiles estimated at the first time points (C1). This is necessary to estimate the individual transition probabilities over time. Then two sections of the inputs are used to defined the profiles estimated at the first (MODEL C1:) and second (MODEL C2:) time points, where the profiles are defined by distinct variables reflecting the mixture indicators measured at either the first (e.g., ZP5Sit) or second (e.g., ZS3Sit) time point.

```
%OVERALL%
c2 on c1;
MODEL C1:
%c1#1%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m1-m6);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v1-V6);
%c1#2%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m11-m16);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v11-V16);
%c1#3%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m21-m26);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v21-v26);
%c1#4%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m31-m36);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v31-V36);
%c1#5%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m41-m46);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v41-V46);
MODEL C2:
%c2#1%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (mm1-mm6);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (vv1-vv6);
%c2#2%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (mm11-mm16);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (vv11-vv16);
%c2#3%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (mm21-mm26);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (vv21-vv26);
%c2#4%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (mm31-mm36);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (vv31-vv36);
%c2#5%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (mm41-mm46);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (vv41-vv46);
```

Appendix 9.26. Estimation of a 5-Profile LTA (Structural Invariance).

Here, parameter labels are simply used to constrain the means to be invariant over time within similar profiles.

```
%OVERALL%
c2 on c1;
MODEL C1:
%c1#1%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m1-m6);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v1-V6);
%c1#2%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m11-m16);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v11-V16);
%c1#3%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m21-m26);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v21-v26);
%c1#4%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m31-m36);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v31-V36);
%c1#5%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m41-m46);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v41-V46);
MODEL C2:
%c2#1%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (m1-m6);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (vv1-vv6);
%c2#2%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (m11-m16);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (vv11-vv16);
%c2#3%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (m21-m26);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (vv21-vv26);
%c2#4%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (m31-m36);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (vv31-vv36);
%c2#5%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (m41-m46);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (vv41-vv46);
```

Appendix 9.27. Estimation of a 5-Profile LTA (Dispersion Invariance).

Here, we provide an input that assumes structural invariance.

```
%OVERALL%
c2 on c1;
MODEL C1:
%c1#1%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m1-m6);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v1-V6);
%c1#2%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m11-m16);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v11-V16);
%c1#3%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m21-m26);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v21-v26);
%c1#4%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m31-m36);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v31-V36);
%c1#5%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m41-m46);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v41-V46);
MODEL C2:
%c2#1%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (m1-m6);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (v1-v6);
%c2#2%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (m11-m16);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (v11-v16);
%c2#3%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (m21-m26);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (v21-v26);
%c2#4%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (m31-m36);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (v31-v36);
%c2#5%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (m41-m46);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (v41-v46);
```

Appendix 9.28. Estimation of a 5-Profile LTA (Distributional Invariance).

Here, we provide an input that assumes structural and dispersion invariance. To request distributional invariance, labels are used to request that the sizes of the profiles be invariant over time.

```
%OVERALL%
c2 on c1;
[ c1#1] (p1);
[ c1#2] (p2);
[ c1#3] (p3);
[ c1#4] (p4);
[ c2#1] (p1);
[ c2#2] (p2);
[ c2#3] (p3);
[ c2#4] (p4);
MODEL C1:
%c1#1%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m1-m6);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v1-V6);
%c1#2%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m11-m16);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v11-V16);
%c1#3%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m21-m26);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v21-v26);
%c1#4%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m31-m36);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v31-V36);
%c1#5%
[ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run] (m41-m46);
ZP5Sit ZP5Flex ZP5snr ZP5shut ZP5sbj ZP5Run (v41-V46);
MODEL C2:
%c2#1%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (m1-m6);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (v1-v6);
%c2#2%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (m11-m16);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (v11-v16);
%c2#3%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (m21-m26);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (v21-v26);
%c2#4%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (m31-m36);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (v31-v36);
%c2#5%
[ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run] (m41-m46);
ZS3Sit ZS3Flex ZS3snr ZS3shut ZS3sbj ZS3Run (v41-v46);
```

Appendix 9.29. Estimation of a 3-Profile Mixture Regression.

In contrast with the previous LPA solution, a mixture regression model specifies a regression model in the %OVERALL% section of the input indicating here that Grade 6 BMI (ZBMI_P6) is regressed (ON) a series of predictors (ZBMI_P5 SP5 CP5 ZP5snr). Then, the profile-specific sections of the input request that these regression coefficients be freely estimated in all profiles, together with the means and variances of all predictors and outcomes.

```
%OVERALL%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr;
%c#1%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr;
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5;
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5];
%c#2%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr;
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5;
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5];
%c#3%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr;
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5;
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5];
```

Appendix 9.30.

Estimation of a Multiple Group Mixture Regression (Configural Invariance).

This set-up is highly similar to the one used for the multiple groups LPA models. Here again, the KNOWCLASS option is used to defined the observed groups in the VARIABLE section:

```
KNOWCLASS = cg (Sex = 1 Sex = 2);
CLASSES = cg (2) c (3);
```

Then the main section of the input is specified to request the estimation of a model of configural invariance.

```
%OVERALL%
c#1 on cg#1;
c#2 on cg#1;
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr ;
%cg#1.c#1%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r1-r4);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m1-m5);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v1-v5);
%cg#1.c#2%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r11-r14);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m11-m15);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v11-v15);
%cg#1.c#3%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r21-r24);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m21-m25);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v21-v25);
%cg#2.c#1%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (rr1-rr4);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (mm1-mm5);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (vv1-vv5);
%cg#2.c#2%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (rr11-rr14);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (mm11-mm15);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (vv11-vv15);
%cg#2.c#3%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (rr21-rr24);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (mm21-mm25);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (vv21-vv25);
```

Appendix 9.31.

Estimation of a Multiple Group Mixture Regression (Regression Invariance).

Here, parameter labels are used to specify the invariance of the regression coefficients across genders.

```
%OVERALL%
c#1 on cg#1;
c#2 on cg#1;
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr ;
%cg#1.c#1%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r1-r4);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m1-m5);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v1-v5);
%cg#1.c#2%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r11-r14);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m11-m15);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v11-v15);
%cg#1.c#3%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r21-r24);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m21-m25);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v21-v25);
%cg#2.c#1%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r1-r4);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (mm1-mm5);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (vv1-vv5);
%cg#2.c#2%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r11-r14);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (mm11-mm15);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (vv11-vv15);
%cg#2.c#3%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r21-r24);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (mm21-mm25);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (vv21-vv25);
```

Appendix 9.32.

Estimation of a Multiple Group Mixture Regression (Structural Invariance).

Here, parameter labels are used to specify the invariance of the means across genders.

```
%OVERALL%
c#1 on cg#1;
c#2 on cg#1;
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr ;
%cg#1.c#1%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r1-r4);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m1-m5);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v1-v5);
%cg#1.c#2%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r11-r14);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m11-m15);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v11-v15);
%cg#1.c#3%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r21-r24);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m21-m25);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v21-v25);
%cg#2.c#1%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r1-r4);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m1-m5);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (vv1-vv5);
%cg#2.c#2%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r11-r14);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m11-m15);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (vv11-vv15);
%cg#2.c#3%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r21-r24);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m21-m25);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (vv21-vv25);
```

Appendix 9.33.

Estimation of a Multiple Group Mixture Regression (Dispersion Invariance).

Here, parameter labels are used to specify the invariance of the variances across genders.

```
%OVERALL%
c#1 on cg#1;
c#2 on cg#1;
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr ;
%cg#1.c#1%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r1-r4);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m1-m5);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v1-v5);
%cg#1.c#2%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r11-r14);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m11-m15);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v11-v15);
%cg#1.c#3%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r21-r24);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m21-m25);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v21-v25);
%cg#2.c#1%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r1-r4);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m1-m5);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v1-v5);
%cg#2.c#2%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r11-r14);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m11-m15);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v11-v15);
%cg#2.c#3%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r21-r24);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m21-m25);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v21-v25);
```

Appendix 9.34.

Estimation of a Multiple Group Mixture Regression (Distribution Invariance).

Here, the statements making profile membership conditional on gender are simply taken out from the %OVERALL% statement.

```
%OVERALL%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr ;
%cg#1.c#1%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r1-r4);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m1-m5);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v1-v5);
%cg#1.c#2%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r11-r14);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m11-m15);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v11-v15);
%cg#1.c#3%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r21-r24);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m21-m25);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v21-v25);
%cg#2.c#1%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r1-r4);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m1-m5);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v1-v5);
%cg#2.c#2%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r11-r14);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m11-m15);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v11-v15);
%cg#2.c#3%
ZBMI_P6 ON ZBMI_P5 SP5 CP5 ZP5snr (r21-r24);
ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5 (m21-m25);
[ZP5snr ZBMI_P5 ZBMI_P6 SP5 CP5] (v21-v25);
```

Appendix 9.35. Estimation of a 3-Profile Latent Basis GMM

In the estimation of latent curve model, the “I S |” function serves as a shortcut to define longitudinal intercepts and slope parameters and are generally followed by a specification of the time-varying indicators and their loadings on the slope factor (the loadings on the intercept factor are fixed to 1). In a latent basis model, two loadings (typically the first and last) need to be respectively fixed to 0 and 1 (@0 and @1) while the others are freely estimated. Here, we also request that these be freely estimated in all profiles by repeating this function in the profile-specific sections. In this input, we also request that the means of the intercepts and slope factors ([I S];), their variances (I S;) and covariances (I WITH S;) and all time specific residuals (CP4 CP5 CP6 CS1 CS2 CS3 CS4;) be freely estimated in all profiles.

```
%OVERALL%
I S | CP4@0 CP5*  CP6* CS1* CS2* CS3* CS4@1;
%c#1%
I S | CP4@0 CP5*  CP6* CS1* CS2* CS3* CS4@1;
I S;
[I S];
I WITH S;
CP4 CP5  CP6 CS1 CS2 CS3 CS4;
%c#2%
I S | CP4@0 CP5*  CP6* CS1* CS2* CS3* CS4@1;
I S;
[I S];
I WITH S;
CP4 CP5  CP6 CS1 CS2 CS3 CS4;
%c#3%
I S | CP4@0 CP5*  CP6* CS1* CS2* CS3* CS4@1;
I S;
[I S];
I WITH S;
CP4 CP5  CP6 CS1 CS2 CS3 CS4;
```

An interesting function available in Mplus allows the user to obtain plots of the trajectories.

```
PLOT:
TYPE IS PLOT3;
SERIES = CP4-CS4(*);
```

Appendix 9.36. Estimation of a 4-Profile Piecewise GMM

This input is similar to the previous one, with the main differences that two slope factors (S1 and S2) are requested and defined using the pattern of time codes described in Appendix 9.10.

```
%OVERALL%
I S1 | SP4@0 SP5@1  SP6@2 SS1@2 SS2@2 SS3@2 SS4@2;
I S2 | SP4@0 SP5@0  SP6@0 SS1@1 SS2@2 SS3@3 SS4@4;
%c#1%
I S1 | SP4@0 SP5@1  SP6@2 SS1@2 SS2@2 SS3@2 SS4@2;
I S2 | SP4@0 SP5@0  SP6@0 SS1@1 SS2@2 SS3@3 SS4@4;
I S1 S2;
[I S1 S2];
I WITH S1 S2;
S1 WITH S2;
SP4 SP5  SP6 SS1 SS2 SS3 SS4;
%c#2%
I S1 | SP4@0 SP5@1  SP6@2 SS1@2 SS2@2 SS3@2 SS4@2;
I S2 | SP4@0 SP5@0  SP6@0 SS1@1 SS2@2 SS3@3 SS4@4;
I S1 S2;
[I S1 S2];
I WITH S1 S2;
S1 WITH S2;
SP4 SP5  SP6 SS1 SS2 SS3 SS4;
%c#3%
I S1 | SP4@0 SP5@1  SP6@2 SS1@2 SS2@2 SS3@2 SS4@2;
I S2 | SP4@0 SP5@0  SP6@0 SS1@1 SS2@2 SS3@3 SS4@4;
I S1 S2;
[I S1 S2];
I WITH S1 S2;
S1 WITH S2;
SP4 SP5  SP6 SS1 SS2 SS3 SS4;
%c#4%
I S1 | SP4@0 SP5@1  SP6@2 SS1@2 SS2@2 SS3@2 SS4@2;
I S2 | SP4@0 SP5@0  SP6@0 SS1@1 SS2@2 SS3@3 SS4@4;
I S1 S2;
[I S1 S2];
I WITH S1 S2;
S1 WITH S2;
SP4 SP5  SP6 SS1 SS2 SS3 SS4;
```

Appendix 9.37. Estimation of a 4-Profile Piecewise GMM (Constrained Estimation)

This input is similar to the previous one, with the main differences that the time specific residuals are now labeled and constrained to take a non-zero value.

```
%OVERALL%
I S1 | SP4@0 SP5@1  SP6@2 SS1@2 SS2@2 SS3@2 SS4@2;
I S2 | SP4@0 SP5@0  SP6@0 SS1@1 SS2@2 SS3@3 SS4@4;
%c#1%
I S1 | SP4@0 SP5@1  SP6@2 SS1@2 SS2@2 SS3@2 SS4@2;
I S2 | SP4@0 SP5@0  SP6@0 SS1@1 SS2@2 SS3@3 SS4@4;
I S1 S2;
[I S1 S2];
I WITH S1 S2;
S1 WITH S2;
    sp4* (r1);
    sp5* (r2);
    sp6* (r3);
    ss1* (r4);
    ss2* (r5);
    ss3* (r6);
    ss4* (r7);
%c#2%
I S1 | SP4@0 SP5@1  SP6@2 SS1@2 SS2@2 SS3@2 SS4@2;
I S2 | SP4@0 SP5@0  SP6@0 SS1@1 SS2@2 SS3@3 SS4@4;
I S1 S2;
[I S1 S2];
I WITH S1 S2;
S1 WITH S2;
    sp4* (r11);
    sp5* (r12);
    sp6* (r13);
    ss1* (r14);
    ss2* (r15);
    ss3* (r16);
    ss4* (r17);
%c#3%
I S1 | SP4@0 SP5@1  SP6@2 SS1@2 SS2@2 SS3@2 SS4@2;
I S2 | SP4@0 SP5@0  SP6@0 SS1@1 SS2@2 SS3@3 SS4@4;
I S1 S2;
[I S1 S2];
I WITH S1 S2;
S1 WITH S2;
    sp4* (r21);
    sp5* (r22);
    sp6* (r23);
    ss1* (r24);
    ss2* (r25);
    ss3* (r26);
    ss4* (r27);
%c#4%
I S1 | SP4@0 SP5@1  SP6@2 SS1@2 SS2@2 SS3@2 SS4@2;
I S2 | SP4@0 SP5@0  SP6@0 SS1@1 SS2@2 SS3@3 SS4@4;
I S1 S2;
[I S1 S2];
```

```
I WITH S1 S2;
S1 WITH S2;
    sp4* (r31);
    sp5* (r32);
    sp6* (r33);
    ss1* (r34);
    ss2* (r35);
    ss3* (r36);
    ss4* (r37);
MODEL CONSTRAINT:
r1 > 0;
r2 > 0;
r3 > 0;
r4 > 0;
r5 > 0;
r6 > 0;
r7 > 0;
r11 > 0;
r12 > 0;
r13 > 0;
r14 > 0;
r15 > 0;
r16 > 0;
r17 > 0;
r21 > 0;
r22 > 0;
r23 > 0;
r24 > 0;
r25 > 0;
r26 > 0;
r27 > 0;
r31 > 0;
r32 > 0;
r33 > 0;
r34 > 0;
r35 > 0;
r36 > 0;
r37 > 0;
```