

## Estimation of a hierarchical Exploratory Structural Equation Model (ESEM) using ESEM-within-CFA

Alexandre J.S. Morin, Substantive Methodological Synergy Research Laboratory, Department of Psychology, Concordia University, Canada  
& Tihomir Asparouhov, Muthén & Muthén, Los Angeles, CA

Cite as: Morin, A.J.S., & Asparouhov, T. (2018). *Estimation of a hierarchical Exploratory Structural Equation Model (ESEM) using ESEM-within-CFA*. Montreal, QC: Substantive Methodological Synergy Research Laboratory.

Morin, Arens and Marsh (2016, also see Marsh, Morin, Parker, & Kaur, 2014; Morin, Marsh & Nagengast, 2013), based on earlier work by Marsh, Nagengast & Morin (2013) and Joreskog (1969; also see Muthén & Muthén, 2009, slides 133–146) introduced ESEM-Within-CFA (EWC) as a workaround some limitations of exploratory Structural Equation Models (ESEM). Essentially, EWC starts with an ESEM model, and re-express it in the CFA framework using the starts values generated from the initial ESEM model. More precisely, using the exact starts values generated from the initial ESEM model, the EWC model is estimated by adding  $m^2$  constraints for identification purposes. To do so, selected parameter estimates are fixed to the values obtained from the ESEM solution. Typically:

- (1) The  $m$  factor variances are fixed to 1 (in a single-group ESEM, for the first group in a multiple group solution, or the first time point for a longitudinal solution).
- (2) A referent indicator is selected for each factor that has a large (target) loading for the factor it is designed to measure and small (non-target) cross-loadings. Then, for purposes of identification, these small cross-loadings are fixed (@) to their estimated values from the ESEM solution.
- (3) For all other parameter estimates, the pattern of fixed and free estimates should be the same as in the selected ESEM solution.
- (4) It should be noted that the mean structure from the EWC solution can be identified as in a standard CFA model (while using the ESEM start values when possible).

For most applications proposed by Morin and colleagues, this EWC approach works fine. However, we recently realized that this approach was suboptimal for the estimation of higher-order ESEM models reported in Morin, Arens and Marsh (2016). More precisely, to fully re-express the ESEM model, the first-order factor variances have to be set to a value of 1 [i.e., the constraints described under (1) above]. However, in the higher-order EWC models reported in Morin, Arens and Marsh (2016), the constraints described in (1) resulted in into a EWC solution in which the residual variance of the first-order factors were fixed to 1, rather than their variance. In addition, having the factor loading of the first indicator on the higher-order factor fixed to 1 created further interference with the model estimation, taking it even further from the initial ESEM model. More precisely, if we consider one of the first-order factor (F1) and the Higher-order factor (HF):

$$\text{Var}(F1) = \text{residual variance (F1)} + \text{loading}_{HF}^2 * \text{Var}(HF)$$

So that if the constraints described above are imposed:

$$\text{Var}(F1) = 1 + 1 * \text{Var}(HF)$$

To circumvent this issue, we proposed an alternative (and simpler) EWC solution in which the variance of the first-order factors can be freely estimated:

- (1) A referent indicator is selected for each factor that has a large (target) loading for the factor it is designed to measure and small (non-target) cross-loadings. Then, for purposes of identification, these large main loadings and small cross-loadings are fixed (@) to their estimated values from the ESEM solution. All factor variances are freely estimated. Note here that it is important to use the referent indicator approach. Fixing some other arbitrary  $m^2$  loadings may compromise the efficiency of the estimation.

- (2) For all other parameter estimates, the pattern of fixed and free estimates should be the same as in the selected ESEM solution.
- (3) It should be noted that the mean structure from the EWC solution can be identified as in a standard CFA model (while using the ESEM start values when possible).

For most other contexts, the typical EWC approach would work. For instance, Morin, Arens and Marsh (2016) show how to re-express a partial mediation ESEM model with EWC in order to obtain bias-corrected bootstrap confidence intervals. In this example, the residual variance of the outcome ESEM factors has to be fixed to 1 in the EWC model to ensure its equivalence with the ESEM model in which they are also fixed to 1. Likewise, Morin, Marsh and Nagengast (2013) illustrated how EWC could be used to estimate latent change scores in the context of a longitudinal ESEM solution. In this context, the residual variance of the Time 2 factors should not be fixed to 1 in the EWC solution. However, this constraint is not required for this model to match the ESEM solution in which, due to measurement invariance, the variance of the Time 2 factors was allowed to be freely estimated. Still, we urge readers to carefully consider their specific EWC application in order to find the approach that best work with their data.

The revised approach proposed here would lead to new sample inputs for the examples provided by Morin, Arens and Marsh (2016) on pages 8 (middle) and 23 of their online supplemental materials, as well as different fit indices associated with their Higher-Order EWC solutions:

Table 1.

*Adjusted Goodness of Fit Statistics and Information Criteria for the Re-Estimated Higher-Order ESEM models*

Model	$\chi^2$	df	CFI	TLI	RMSEA	RMSEA 90% CI	AIC	CAIC	BIC	SBIC
Simulated data	100.432*	33	.996	.991	.036	.028; .044	40021	403385	403328	40146
SDQ-I	5813.763*	2071	.949	.929	.030	.029; .031	360300	366425	365494	362536
SDQ-I without method control	6219.130*	2113	.944	.924	.032	.031; .032	360835	366684	365795	362971

*Note.* df = Degrees of freedom; CFI = comparative fit index; TLI = Tucker-Lewis index; RMSEA = root mean square error of approximation; CI = confidence interval; AIC = Akaike information criterion; CAIC = Constant AIC; BIC = Bayesian information criterion; ABIC = Sample size adjusted BIC. ESEM models were conducted with target oblique rotation. \* All  $\chi^2$  values are significant ( $p < .01$ ).

**Title: Hierarchical ESEM using ESEM-Within-CFA (Simulated Data)**

! The previous ESEM model is re-expressed using CFA. No rotation is necessary.  
 ! The model section uses the exact values of the non-standardized loadings and cross loadings  
 ! estimated from the previous model as starts values (using \*). First-order factor variances are also freely  
 ! estimated, whereas the variance of the higher-order factor is fixed to 1 for identification purposes.  
 ! For the first-order factors, one item per factor has all loadings and cross loadings  
 ! constrained to be exactly equal to their ESEM values (using @).  
 ! These 3 factors define a higher-order factor HF, with all higher-order loadings freely estimated..

Model:

f1 BY x1\*.74674; f1 BY x2\*.80372; f1 BY x3@.80739; f1 BY x4\*.83759;  
 f1 BY y1\*-.05015; f1 BY y2\*.20610; f1 BY y3\*-.09183; f1 BY y4@.03835;  
 f1 BY z1\*.22881; f1 BY z2\*.02457; f1 BY z3\*.01376; f1 BY z4@-.13088;  
 f2 BY y1\*.79513; f2 BY y2\*.80701; f2 BY y3\*.95053; f2 BY y4@.90008;  
 f2 BY x1\*-.12434; f2 BY x2\*.15514; f2 BY x3@-.07168; f2 BY x4\*.08193;  
 f2 BY z1\*.06430; f2 BY z2\*.31927; f2 BY z3\*-.14645; f2 BY z4@-.00922;  
 f3 BY z1\*.071349; f3 BY z2\*.66022; f3 BY z3\*.96202; f3 BY z4@.95145;  
 f3 BY x1\*-.0.11211; f3 BY x2\*-.0.13255; f3 BY x3@.0.15235; f3 BY x4\*-.0.02669;  
 f3 BY y1\*.0.16258; f3 BY y2\*-.0.02858; f3 BY y3\*.0.07700; f3 BY y4@-.0.01649;  
 f1-f3\*1;  
 HF BY F1\*1 F2 F3;  
 HF@1;

**Title: Hierarchical ESEM Model of the SDQ-I Using ESEM-Within-CFA (Real Data)**  
**! [...] Analysis and Model sections only**

Model:

esteem BY sdq\_1\*-0.03101; esteem BY sdq\_2\*0.01217; esteem BY sdq\_3\*-0.09151;  
esteem BY sdq\_4@0.02081; esteem BY sdq\_5\*0.03906; esteem BY sdq\_6\*-0.04812;  
esteem BY sdq\_7\*-0.06562; esteem BY sdq\_8\*0.07463; esteem BY sdq\_9\*0.03260;  
esteem BY sdq\_10\*-0.01524; esteem BY sdq\_11\*-0.05772; esteem BY sdq\_12\*0.12921;  
esteem BY sdq\_13\*-0.03180; esteem BY sdq\_14\*-0.07285; esteem BY sdq\_15\*-0.11861;  
esteem BY sdq\_16@-0.02130; esteem BY sdq\_17\*0.02080; esteem BY sdq\_18\*-0.03832;  
esteem BY sdq\_19\*0.00802; esteem BY sdq\_20\*-0.00514; esteem BY sdq\_21@-0.12155;  
esteem BY sdq\_22@-0.08714; esteem BY sdq\_23\*0.09885; esteem BY sdq\_24\*0.01615;  
esteem BY sdq\_25\*-0.03904; esteem BY sdq\_26\*0.01240; esteem BY sdq\_27@-0.03013;  
esteem BY sdq\_28\*-0.00719; esteem BY sdq\_29\*0.22094; esteem BY sdq\_30\*0.18339;  
esteem BY sdq\_31\*0.11053; esteem BY sdq\_32\*0.07525; esteem BY sdq\_33\*0.08772;  
esteem BY sdq\_34\*-0.02310; esteem BY sdq\_35\*-0.05219; esteem BY sdq\_36\*0.14200;  
esteem BY sdq\_37\*0.35244; esteem BY sdq\_38\*-0.03335; esteem BY sdq\_39@0.10041;  
esteem BY sdq\_40\*-0.00166; esteem BY sdq\_41\*-0.01108; esteem BY sdq\_42\*-0.00535;  
esteem BY sdq\_43\*0.04976; esteem BY sdq\_44\*0.08265; esteem BY sdq\_45\*0.43268;  
esteem BY sdq\_46\*0.17244; esteem BY sdq\_47\*0.19713; esteem BY sdq\_48\*0.00646;  
esteem BY sdq\_49\*0.05793; esteem BY sdq\_50@-0.05201; esteem BY sdq\_51@0.03668;  
esteem BY sdq\_52\*0.02084; esteem BY sdq\_53\*0.50177; esteem BY sdq\_54\*0.07961;  
esteem BY sdq\_55\*0.09049; esteem BY sdq\_56@0.01561; esteem BY sdq\_57@0.01464;  
esteem BY sdq\_58\*-0.00679; esteem BY sdq\_59\*0.03239; esteem BY sdq\_60\*0.15877;  
esteem BY sdq\_61\*0.44200; esteem BY sdq\_62\*0.34821; esteem BY sdq\_63\*0.14271;  
esteem BY sdq\_64\*-0.03137; esteem BY sdq\_65\*0.10076; esteem BY sdq\_66\*0.01167;  
esteem BY sdq\_67\*0.57164; esteem BY sdq\_68\*-0.01571; esteem BY sdq\_69\*0.21330;  
esteem BY sdq\_70\*0.42051; esteem BY sdq\_71\*0.09761; esteem BY sdq\_72@0.64738;  
esteem BY sdq\_73\*0.17188; esteem BY sdq\_74\*0.64337; esteem BY sdq\_75\*0.09383;  
esteem BY sdq\_76\*0.59995;  
peer BY sdq\_1\*0.01483; peer BY sdq\_2\*0.00711; peer BY sdq\_3\*0.02157;  
peer BY sdq\_4@0.02496; peer BY sdq\_5\*0.00559; peer BY sdq\_6\*-0.04748;  
peer BY sdq\_7\*0.60133; peer BY sdq\_8\*-0.01487; peer BY sdq\_9\*0.00059;  
peer BY sdq\_10\*-0.01116; peer BY sdq\_11\*0.01508; peer BY sdq\_12\*0.03398;  
peer BY sdq\_13\*0.05970; peer BY sdq\_14\*0.76648; peer BY sdq\_15\*0.03099;  
peer BY sdq\_16@0.04426; peer BY sdq\_17\*0.04696; peer BY sdq\_18\*0.07575;  
peer BY sdq\_19\*-0.02550; peer BY sdq\_20\*-0.02445; peer BY sdq\_21@0.89504;  
peer BY sdq\_22@0.00709; peer BY sdq\_23\*0.02470; peer BY sdq\_24\*0.05091;  
peer BY sdq\_25\*0.01956; peer BY sdq\_26\*0.00629; peer BY sdq\_27@0.04108;  
peer BY sdq\_28\*0.50873; peer BY sdq\_29\*0.06983; peer BY sdq\_30\*-0.05489;  
peer BY sdq\_31\*0.00798; peer BY sdq\_32\*0.01557; peer BY sdq\_33\*0.00357;  
peer BY sdq\_34\*0.10410; peer BY sdq\_35\*0.01648; peer BY sdq\_36\*0.37885;  
peer BY sdq\_37\*0.08214; peer BY sdq\_38\*0.33493; peer BY sdq\_39@-0.00893;  
peer BY sdq\_40\*-0.01058; peer BY sdq\_41\*0.03624; peer BY sdq\_42\*0.08547;  
peer BY sdq\_43\*0.07033; peer BY sdq\_44\*0.59996; peer BY sdq\_45\*-0.07896;  
peer BY sdq\_46\*-0.00915; peer BY sdq\_47\*-0.00916; peer BY sdq\_48\*-0.01087;  
peer BY sdq\_49\*0.05702; peer BY sdq\_50@-0.03239; peer BY sdq\_51@-0.04187;  
peer BY sdq\_52\*0.64059; peer BY sdq\_53\*0.08961; peer BY sdq\_54\*0.06749;  
peer BY sdq\_55\*0.03405; peer BY sdq\_56@-0.03591; peer BY sdq\_57@0.00226;  
peer BY sdq\_58\*-0.03337; peer BY sdq\_59\*0.01515; peer BY sdq\_60\*0.57835;  
peer BY sdq\_61\*0.02987; peer BY sdq\_62\*0.00648; peer BY sdq\_63\*0.00454;  
peer BY sdq\_64\*0.02051; peer BY sdq\_65\*-0.00968; peer BY sdq\_66\*0.02470;  
peer BY sdq\_67\*0.09766; peer BY sdq\_68\*0.03386; peer BY sdq\_69\*0.64351;  
peer BY sdq\_70\*0.24271; peer BY sdq\_71\*0.02979; peer BY sdq\_72@0.00827;  
peer BY sdq\_73\*0.03703; peer BY sdq\_74\*0.04893; peer BY sdq\_75\*-0.01164;  
peer BY sdq\_76\*0.00385;

appear BY sdq\_1\*0.71159; appear BY sdq\_2\*0.09873; appear BY sdq\_3\*0.13287;  
appear BY sdq\_4@0.06279; appear BY sdq\_5\*0.00368; appear BY sdq\_6\*-0.00785;  
appear BY sdq\_7\*-0.04277; appear BY sdq\_8\*0.70614; appear BY sdq\_9\*0.11704;  
appear BY sdq\_10\*-0.01854; appear BY sdq\_11\*0.11321; appear BY sdq\_12\*-0.03001;  
appear BY sdq\_13\*0.02418; appear BY sdq\_14\*-0.00685; appear BY sdq\_15\*1.00287;  
appear BY sdq\_16@0.07795; appear BY sdq\_17\*-0.12160; appear BY sdq\_18\*0.04556;  
appear BY sdq\_19\*0.01240; appear BY sdq\_20\*0.01631; appear BY sdq\_21@-0.04602;  
appear BY sdq\_22@1.02221; appear BY sdq\_23\*-0.00912; appear BY sdq\_24\*-0.10848;  
appear BY sdq\_25\*0.07942; appear BY sdq\_26\*0.03758; appear BY sdq\_27@0.07097;  
appear BY sdq\_28\*-0.04027; appear BY sdq\_29\*0.05582; appear BY sdq\_30\*0.58827;  
appear BY sdq\_31\*-0.01727; appear BY sdq\_32\*0.11016; appear BY sdq\_33\*0.04432;  
appear BY sdq\_34\*-0.00522; appear BY sdq\_35\*0.02454; appear BY sdq\_36\*0.21725;  
appear BY sdq\_37\*0.18567; appear BY sdq\_38\*0.48975; appear BY sdq\_39@0.02326;  
appear BY sdq\_40\*0.00452; appear BY sdq\_41\*0.01634; appear BY sdq\_42\*-0.01316;  
appear BY sdq\_43\*-0.02090; appear BY sdq\_44\*0.09312; appear BY sdq\_45\*0.23782;  
appear BY sdq\_46\*0.46624; appear BY sdq\_47\*-0.00479; appear BY sdq\_48\*-0.00881;  
appear BY sdq\_49\*-0.01399; appear BY sdq\_50@0.00336; appear BY sdq\_51@0.00420;  
appear BY sdq\_52\*0.05729; appear BY sdq\_53\*-0.01538; appear BY sdq\_54\*0.54715;  
appear BY sdq\_55\*0.01001; appear BY sdq\_56@0.05502; appear BY sdq\_57@0.02400;  
appear BY sdq\_58\*0.00958; appear BY sdq\_59\*0.03606; appear BY sdq\_60\*0.07554;  
appear BY sdq\_61\*-0.01190; appear BY sdq\_62\*0.40915; appear BY sdq\_63\*0.01776;  
appear BY sdq\_64\*0.08782; appear BY sdq\_65\*-0.04775; appear BY sdq\_66\*-0.04915;  
appear BY sdq\_67\*-0.04068; appear BY sdq\_68\*-0.03990; appear BY sdq\_69\*0.04566;  
appear BY sdq\_70\*-0.00037; appear BY sdq\_71\*-0.02092; appear BY sdq\_72@0.21957;  
appear BY sdq\_73\*-0.04005; appear BY sdq\_74\*0.00244; appear BY sdq\_75\*-0.01976;  
appear BY sdq\_76\*-0.01995;  
phy BY sdq\_1\*0.02194; phy BY sdq\_2\*0.01452; phy BY sdq\_3\*0.78947; phy BY sdq\_4@-0.02107;  
phy BY sdq\_5\*0.01981; phy BY sdq\_6\*0.01287; phy BY sdq\_7\*0.03353; phy BY sdq\_8\*0.06273;  
phy BY sdq\_9\*0.00662; phy BY sdq\_10\*0.59057; phy BY sdq\_11\*-0.02425;  
phy BY sdq\_12\*-0.06819; phy BY sdq\_13\*-0.01218; phy BY sdq\_14\*0.00452;  
phy BY sdq\_15\*-0.03428; phy BY sdq\_16@0.03458; phy BY sdq\_17\*0.84509;  
phy BY sdq\_18\*0.01133; phy BY sdq\_19\*0.01286; phy BY sdq\_20\*-0.01088;  
phy BY sdq\_21@0.03890; phy BY sdq\_22@-0.05882; phy BY sdq\_23\*-0.00818;  
phy BY sdq\_24\*0.96238; phy BY sdq\_25\*0.00073; phy BY sdq\_26\*-0.00136;  
phy BY sdq\_27@-0.02262; phy BY sdq\_28\*0.04810; phy BY sdq\_29\*0.06779;  
phy BY sdq\_30\*-0.01366; phy BY sdq\_31\*0.04801; phy BY sdq\_32\*0.44694;  
phy BY sdq\_33\*0.00927; phy BY sdq\_34\*-0.00148; phy BY sdq\_35\*0.03654;  
phy BY sdq\_36\*-0.02582; phy BY sdq\_37\*-0.05735; phy BY sdq\_38\*0.02221;  
phy BY sdq\_39@0.05285; phy BY sdq\_40\*0.96105; phy BY sdq\_41\*0.05432;  
phy BY sdq\_42\*0.02684; phy BY sdq\_43\*0.04910; phy BY sdq\_44\*-0.03567;  
phy BY sdq\_45\*0.05679; phy BY sdq\_46\*0.31726; phy BY sdq\_47\*0.02376;  
phy BY sdq\_48\*0.89070; phy BY sdq\_49\*0.00098; phy BY sdq\_50@-0.01488;  
phy BY sdq\_51@0.03171; phy BY sdq\_52\*0.00920; phy BY sdq\_53\*0.06230;  
phy BY sdq\_54\*0.07194; phy BY sdq\_55\*0.02209; phy BY sdq\_56@1.07046;  
phy BY sdq\_57@0.01120; phy BY sdq\_58\*-0.02499; phy BY sdq\_59\*0.01162;  
phy BY sdq\_60\*0.03135; phy BY sdq\_61\*-0.03602; phy BY sdq\_62\*-0.02723;  
phy BY sdq\_63\*0.01613; phy BY sdq\_64\*0.72703; phy BY sdq\_65\*-0.00122;  
phy BY sdq\_66\*0.04326; phy BY sdq\_67\*0.04468; phy BY sdq\_68\*0.06114;  
phy BY sdq\_69\*0.01422; phy BY sdq\_70\*-0.04907; phy BY sdq\_71\*0.03776;  
phy BY sdq\_72@0.02647; phy BY sdq\_73\*0.06399; phy BY sdq\_74\*0.01812;  
phy BY sdq\_75\*0.03129; phy BY sdq\_76\*0.04231;  
parent BY sdq\_1\*-0.02668; parent BY sdq\_2\*0.02864; parent BY sdq\_3\*-0.00981;  
parent BY sdq\_4@0.02773; parent BY sdq\_5\*0.57959; parent BY sdq\_6\*0.07208;  
parent BY sdq\_7\*0.03752; parent BY sdq\_8\*0.00950; parent BY sdq\_9\*0.11439;  
parent BY sdq\_10\*0.05511; parent BY sdq\_11\*0.03639; parent BY sdq\_12\*0.39861;

parent BY sdq\_13\*-0.01614; parent BY sdq\_14\*-0.04690; parent BY sdq\_15\*0.00667;  
parent BY sdq\_16@0.08799; parent BY sdq\_17\*-0.03809; parent BY sdq\_18\*0.02067;  
parent BY sdq\_19\*0.38859; parent BY sdq\_20\*0.03514; parent BY sdq\_21@0.01897;  
parent BY sdq\_22@0.02610; parent BY sdq\_23\*0.07107; parent BY sdq\_24\*-0.04255;  
parent BY sdq\_25\*0.06513; parent BY sdq\_26\*0.38376; parent BY sdq\_27@0.01378;  
parent BY sdq\_28\*0.07242; parent BY sdq\_29\*0.02260; parent BY sdq\_30\*0.06201;  
parent BY sdq\_31\*0.05306; parent BY sdq\_32\*-0.00869; parent BY sdq\_33\*0.03554;  
parent BY sdq\_34\*0.73127; parent BY sdq\_35\*0.01952; parent BY sdq\_36\*-0.01827;  
parent BY sdq\_37\*0.11768; parent BY sdq\_38\*-0.02072; parent BY sdq\_39@0.01320;  
parent BY sdq\_40\*-0.01890; parent BY sdq\_41\*0.06423; parent BY sdq\_42\*0.76171;  
parent BY sdq\_43\*0.03504; parent BY sdq\_44\*0.01979; parent BY sdq\_45\*0.11303;  
parent BY sdq\_46\*0.04475; parent BY sdq\_47\*0.06342; parent BY sdq\_48\*0.07274;  
parent BY sdq\_49\*0.00470; parent BY sdq\_50@0.86884; parent BY sdq\_51@-0.02296;  
parent BY sdq\_52\*-0.01037; parent BY sdq\_53\*0.13723; parent BY sdq\_54\*-0.01702;  
parent BY sdq\_55\*0.03126; parent BY sdq\_56@-0.01330; parent BY sdq\_57@0.06805;  
parent BY sdq\_58\*0.79620; parent BY sdq\_59\*0.00180; parent BY sdq\_60\*0.00609;  
parent BY sdq\_61\*0.20753; parent BY sdq\_62\*0.02179; parent BY sdq\_63\*0.01376;  
parent BY sdq\_64\*0.05815; parent BY sdq\_65\*0.04031; parent BY sdq\_66\*0.85028;  
parent BY sdq\_67\*0.01947; parent BY sdq\_68\*0.01100; parent BY sdq\_69\*-0.01439;  
parent BY sdq\_70\*0.00436; parent BY sdq\_71\*0.02078; parent BY sdq\_72@-0.05797;  
parent BY sdq\_73\*-0.02819; parent BY sdq\_74\*-0.05795; parent BY sdq\_75\*0.03648;  
parent BY sdq\_76\*0.01392;  
schocom BY sdq\_1\*-0.03574; schocom BY sdq\_2\*0.31972; schocom BY sdq\_3\*0.06532;  
schocom BY sdq\_4@0.19823; schocom BY sdq\_5\*-0.04472; schocom BY sdq\_6\*-0.09553;  
schocom BY sdq\_7\*-0.19154; schocom BY sdq\_8\*-0.17366; schocom BY sdq\_9\*0.17193;  
schocom BY sdq\_10\*-0.10055; schocom BY sdq\_11\*-0.09512; schocom BY sdq\_12\*-0.00304;  
schocom BY sdq\_13\*0.05550; schocom BY sdq\_14\*-0.01751; schocom BY sdq\_15\*0.00149;  
schocom BY sdq\_16@0.41965; schocom BY sdq\_17\*-0.17914; schocom BY sdq\_18\*0.12824;  
schocom BY sdq\_19\*-0.07013; schocom BY sdq\_20\*-0.03820; schocom BY sdq\_21@-0.08778;  
schocom BY sdq\_22@0.04052; schocom BY sdq\_23\*-0.08808; schocom BY sdq\_24\*-0.17587;  
schocom BY sdq\_25\*-0.07910; schocom BY sdq\_26\*-0.06565; schocom BY sdq\_27@0.10586;  
schocom BY sdq\_28\*-0.09862; schocom BY sdq\_29\*0.06475; schocom BY sdq\_30\*-0.17817;  
schocom BY sdq\_31\*0.34433; schocom BY sdq\_32\*0.14926; schocom BY sdq\_33\*0.04007;  
schocom BY sdq\_34\*0.09566; schocom BY sdq\_35\*-0.03328; schocom BY sdq\_36\*0.00935;  
schocom BY sdq\_37\*-0.12996; schocom BY sdq\_38\*0.14800; schocom BY sdq\_39@0.26675;  
schocom BY sdq\_40\*-0.04308; schocom BY sdq\_41\*-0.03316; schocom BY sdq\_42\*0.15598;  
schocom BY sdq\_43\*0.09851; schocom BY sdq\_44\*0.05164; schocom BY sdq\_45\*-0.22275;  
schocom BY sdq\_46\*0.06105; schocom BY sdq\_47\*0.01216; schocom BY sdq\_48\*0.17752;  
schocom BY sdq\_49\*0.10702; schocom BY sdq\_50@0.01514; schocom BY sdq\_51@-0.07166;  
schocom BY sdq\_52\*0.11216; schocom BY sdq\_53\*-0.06347; schocom BY sdq\_54\*0.30532;  
schocom BY sdq\_55\*0.19671; schocom BY sdq\_56@0.06650; schocom BY sdq\_57@-0.03430;  
schocom BY sdq\_58\*0.00228; schocom BY sdq\_59\*0.07978; schocom BY sdq\_60\*0.18743;  
schocom BY sdq\_61\*-0.00385; schocom BY sdq\_62\*0.02041; schocom BY sdq\_63\*0.40308;  
schocom BY sdq\_64\*0.12476; schocom BY sdq\_65\*-0.08413; schocom BY sdq\_66\*0.10837;  
schocom BY sdq\_67\*0.11549; schocom BY sdq\_68\*-0.02362; schocom BY sdq\_69\*0.02854;  
schocom BY sdq\_70\*0.00562; schocom BY sdq\_71\*0.24421; schocom BY sdq\_72@-0.01005;  
schocom BY sdq\_73\*0.18943; schocom BY sdq\_74\*0.08310; schocom BY sdq\_75\*0.02012;  
schocom BY sdq\_76\*0.06152;  
schoaff BY sdq\_1\*0.01267; schoaff BY sdq\_2\*0.30037; schoaff BY sdq\_3\*-0.06672;  
schoaff BY sdq\_4@-0.02909; schoaff BY sdq\_5\*0.02703; schoaff BY sdq\_6\*0.01571;  
schoaff BY sdq\_7\*0.09787; schoaff BY sdq\_8\*0.06019; schoaff BY sdq\_9\*0.57874;  
schoaff BY sdq\_10\*0.09970; schoaff BY sdq\_11\*0.17712; schoaff BY sdq\_12\*0.02106;  
schoaff BY sdq\_13\*-0.05071; schoaff BY sdq\_14\*0.04516; schoaff BY sdq\_15\*0.02970;  
schoaff BY sdq\_16@0.35375; schoaff BY sdq\_17\*0.12652; schoaff BY sdq\_18\*-0.05048;  
schoaff BY sdq\_19\*0.00282; schoaff BY sdq\_20\*0.16064; schoaff BY sdq\_21@0.02939;

schoaff BY sdq\_22@0.03393; schoaff BY sdq\_23\*0.20753; schoaff BY sdq\_24\*0.11611; schoaff BY sdq\_25\*0.19140; schoaff BY sdq\_26\*-0.02982; schoaff BY sdq\_27@-0.00763; schoaff BY sdq\_28\*0.08785; schoaff BY sdq\_29\*0.09841; schoaff BY sdq\_30\*0.01285; schoaff BY sdq\_31\*0.20521; schoaff BY sdq\_32\*-0.15592; schoaff BY sdq\_33\*-0.07493; schoaff BY sdq\_34\*0.06088; schoaff BY sdq\_35\*0.12109; schoaff BY sdq\_36\*0.02556; schoaff BY sdq\_37\*0.06750; schoaff BY sdq\_38\*-0.04896; schoaff BY sdq\_39@0.72990; schoaff BY sdq\_40\*-0.01937; schoaff BY sdq\_41\*0.17576; schoaff BY sdq\_42\*-0.02253; schoaff BY sdq\_43\*-0.05860; schoaff BY sdq\_44\*-0.04273; schoaff BY sdq\_45\*0.09265; schoaff BY sdq\_46\*0.03906; schoaff BY sdq\_47\*0.00893; schoaff BY sdq\_48\*-0.12509; schoaff BY sdq\_49\*-0.09122; schoaff BY sdq\_50@-0.03307; schoaff BY sdq\_51@0.08933; schoaff BY sdq\_52\*-0.09556; schoaff BY sdq\_53\*0.05800; schoaff BY sdq\_54\*-0.17392; schoaff BY sdq\_55\*0.66120; schoaff BY sdq\_56@-0.04085; schoaff BY sdq\_57@0.20644; schoaff BY sdq\_58\*0.00145; schoaff BY sdq\_59\*-0.04844; schoaff BY sdq\_60\*-0.18589; schoaff BY sdq\_61\*0.02385; schoaff BY sdq\_62\*-0.06655; schoaff BY sdq\_63\*0.37975; schoaff BY sdq\_64\*-0.07081; schoaff BY sdq\_65\*0.12569; schoaff BY sdq\_66\*-0.07108; schoaff BY sdq\_67\*-0.02117; schoaff BY sdq\_68\*0.12000; schoaff BY sdq\_69\*-0.04491; schoaff BY sdq\_70\*0.03310; schoaff BY sdq\_71\*0.72868; schoaff BY sdq\_72@0.00713; schoaff BY sdq\_73\*-0.01735; schoaff BY sdq\_74\*-0.04215; schoaff BY sdq\_75\*-0.06356; schoaff BY sdq\_76\*0.03296;

germcom BY sdq\_1\*0.02464; germcom BY sdq\_2\*0.31979; germcom BY sdq\_3\*0.11665; germcom BY sdq\_4@0.88819; germcom BY sdq\_5\*0.00246; germcom BY sdq\_6\*0.06922; germcom BY sdq\_7\*0.00814; germcom BY sdq\_8\*-0.00219; germcom BY sdq\_9\*-0.01496; germcom BY sdq\_10\*-0.02036; germcom BY sdq\_11\*0.34848; germcom BY sdq\_12\*0.13294; germcom BY sdq\_13\*-0.12882; germcom BY sdq\_14\*0.06945; germcom BY sdq\_15\*0.02824; germcom BY sdq\_16@0.42804; germcom BY sdq\_17\*-0.03356; germcom BY sdq\_18\*0.84978; germcom BY sdq\_19\*-0.01960; germcom BY sdq\_20\*0.02158; germcom BY sdq\_21@0.11222; germcom BY sdq\_22@-0.02171; germcom BY sdq\_23\*0.10716; germcom BY sdq\_24\*-0.05364; germcom BY sdq\_25\*0.14376; germcom BY sdq\_26\*0.00068; germcom BY sdq\_27@0.00548; germcom BY sdq\_28\*0.01925; germcom BY sdq\_29\*-0.03785; germcom BY sdq\_30\*0.00515; germcom BY sdq\_31\*0.23068; germcom BY sdq\_32\*-0.15803; germcom BY sdq\_33\*0.76849; germcom BY sdq\_34\*-0.00141; germcom BY sdq\_35\*0.01786; germcom BY sdq\_36\*0.01015; germcom BY sdq\_37\*0.03312; germcom BY sdq\_38\*-0.01284; germcom BY sdq\_39@-0.12649; germcom BY sdq\_40\*0.04597; germcom BY sdq\_41\*0.15598; germcom BY sdq\_42\*-0.07653; germcom BY sdq\_43\*-0.04550; germcom BY sdq\_44\*-0.02531; germcom BY sdq\_45\*-0.07370; germcom BY sdq\_46\*0.06314; germcom BY sdq\_47\*0.24289; germcom BY sdq\_48\*0.04927; germcom BY sdq\_49\*0.67107; germcom BY sdq\_50@-0.05134; germcom BY sdq\_51@0.09341; germcom BY sdq\_52\*-0.12721; germcom BY sdq\_53\*-0.00815; germcom BY sdq\_54\*-0.12946; germcom BY sdq\_55\*-0.13366; germcom BY sdq\_56@0.06107; germcom BY sdq\_57@0.18819; germcom BY sdq\_58\*-0.04104; germcom BY sdq\_59\*-0.04233; germcom BY sdq\_60\*-0.01171; germcom BY sdq\_61\*0.20377; germcom BY sdq\_62\*0.04860; germcom BY sdq\_63\*0.13074; germcom BY sdq\_64\*-0.16007; germcom BY sdq\_65\*0.28651; germcom BY sdq\_66\*-0.01054; germcom BY sdq\_67\*0.00605; germcom BY sdq\_68\*0.01225; germcom BY sdq\_69\*-0.02624; germcom BY sdq\_70\*-0.00307; germcom BY sdq\_71\*-0.16968; germcom BY sdq\_72@-0.01015; germcom BY sdq\_73\*0.53200; germcom BY sdq\_74\*0.04742; germcom BY sdq\_75\*-0.03063; germcom BY sdq\_76\*-0.03434;

germaff BY sdq\_1\*-0.03627; germaff BY sdq\_2\*-0.17220; germaff BY sdq\_3\*-0.03133; germaff BY sdq\_4@0.01653; germaff BY sdq\_5\*0.03043; germaff BY sdq\_6\*-0.14578; germaff BY sdq\_7\*-0.02423; germaff BY sdq\_8\*-0.01526; germaff BY sdq\_9\*0.12656; germaff BY sdq\_10\*0.03245; germaff BY sdq\_11\*0.77733; germaff BY sdq\_12\*-0.08203; germaff BY sdq\_13\*0.03138; germaff BY sdq\_14\*-0.00193; germaff BY sdq\_15\*0.05813; germaff BY sdq\_16@-0.23242; germaff BY sdq\_17\*-0.07994; germaff BY sdq\_18\*0.24452; germaff BY sdq\_19\*0.02966; germaff BY sdq\_20\*-0.02903; germaff BY sdq\_21@-0.10068; germaff BY sdq\_22@0.04173; germaff BY sdq\_23\*0.14279; germaff BY sdq\_24\*0.00064; germaff BY sdq\_25\*0.82360; germaff BY sdq\_26\*0.02056; germaff BY sdq\_27@-0.00937; germaff BY sdq\_28\*-0.01456; germaff BY sdq\_29\*0.14622; germaff BY sdq\_30\*-0.02322;

germaff BY sdq\_31\*-0.01167; germaff BY sdq\_32\*0.11916; germaff BY sdq\_33\*0.21202; germaff BY sdq\_34\*0.00199; germaff BY sdq\_35\*0.00157; germaff BY sdq\_36\*-0.04778; germaff BY sdq\_37\*-0.13279; germaff BY sdq\_38\*0.02415; germaff BY sdq\_39@0.23207; germaff BY sdq\_40\*-0.04781; germaff BY sdq\_41\*0.83719; germaff BY sdq\_42\*0.03849; germaff BY sdq\_43\*0.03811; germaff BY sdq\_44\*0.03801; germaff BY sdq\_45\*-0.00583; germaff BY sdq\_46\*-0.02773; germaff BY sdq\_47\*0.01603; germaff BY sdq\_48\*0.03247; germaff BY sdq\_49\*0.37662; germaff BY sdq\_50@0.05242; germaff BY sdq\_51@-0.08716; germaff BY sdq\_52\*0.09654; germaff BY sdq\_53\*-0.00154; germaff BY sdq\_54\*0.04087; germaff BY sdq\_55\*0.28296; germaff BY sdq\_56@-0.02630; germaff BY sdq\_57@0.89432; germaff BY sdq\_58\*-0.01001; germaff BY sdq\_59\*0.05214; germaff BY sdq\_60\*0.01003; germaff BY sdq\_61\*-0.16349; germaff BY sdq\_62\*0.02547; germaff BY sdq\_63\*-0.03054; germaff BY sdq\_64\*0.04280; germaff BY sdq\_65\*0.78931; germaff BY sdq\_66\*-0.05294; germaff BY sdq\_67\*0.01370; germaff BY sdq\_68\*0.01890; germaff BY sdq\_69\*0.00657; germaff BY sdq\_70\*-0.00344; germaff BY sdq\_71\*0.27667; germaff BY sdq\_72@-0.03087; germaff BY sdq\_73\*0.37518; germaff BY sdq\_74\*0.03031; germaff BY sdq\_75\*0.00124; germaff BY sdq\_76\*0.03225;

mathcom BY sdq\_1\*-0.01355; mathcom BY sdq\_2\*0.17807; mathcom BY sdq\_3\*-0.04975; mathcom BY sdq\_4@-0.12521; mathcom BY sdq\_5\*0.00470; mathcom BY sdq\_6\*0.21715; mathcom BY sdq\_7\*0.06967; mathcom BY sdq\_8\*0.04412; mathcom BY sdq\_9\*-0.08619; mathcom BY sdq\_10\*0.00770; mathcom BY sdq\_11\*-0.02471; mathcom BY sdq\_12\*0.13379; mathcom BY sdq\_13\*0.94431; mathcom BY sdq\_14\*0.05180; mathcom BY sdq\_15\*0.02195; mathcom BY sdq\_16@0.17164; mathcom BY sdq\_17\*-0.01023; mathcom BY sdq\_18\*-0.00722; mathcom BY sdq\_19\*0.00664; mathcom BY sdq\_20\*0.12631; mathcom BY sdq\_21@0.08614; mathcom BY sdq\_22@0.03202; mathcom BY sdq\_23\*-0.02866; mathcom BY sdq\_24\*-0.01810; mathcom BY sdq\_25\*0.06313; mathcom BY sdq\_26\*0.02995; mathcom BY sdq\_27@0.97555; mathcom BY sdq\_28\*0.00675; mathcom BY sdq\_29\*-0.03438; mathcom BY sdq\_30\*0.08342; mathcom BY sdq\_31\*0.26082; mathcom BY sdq\_32\*0.04376; mathcom BY sdq\_33\*-0.00976; mathcom BY sdq\_34\*-0.05170; mathcom BY sdq\_35\*0.15805; mathcom BY sdq\_36\*-0.03068; mathcom BY sdq\_37\*0.05779; mathcom BY sdq\_38\*-0.09554; mathcom BY sdq\_39@-0.03657; mathcom BY sdq\_40\*-0.01202; mathcom BY sdq\_41\*0.01682; mathcom BY sdq\_42\*-0.02365; mathcom BY sdq\_43\*0.82870; mathcom BY sdq\_44\*-0.05670; mathcom BY sdq\_45\*0.06601; mathcom BY sdq\_46\*-0.01572; mathcom BY sdq\_47\*0.16245; mathcom BY sdq\_48\*0.02372; mathcom BY sdq\_49\*0.02387; mathcom BY sdq\_50@0.03635; mathcom BY sdq\_51@0.10636; mathcom BY sdq\_52\*0.06893; mathcom BY sdq\_53\*0.00662; mathcom BY sdq\_54\*0.03602; mathcom BY sdq\_55\*-0.06226; mathcom BY sdq\_56@-0.06294; mathcom BY sdq\_57@0.01686; mathcom BY sdq\_58\*-0.01569; mathcom BY sdq\_59\*0.94183; mathcom BY sdq\_60\*0.00518; mathcom BY sdq\_61\*0.02473; mathcom BY sdq\_62\*-0.11578; mathcom BY sdq\_63\*0.25556; mathcom BY sdq\_64\*0.11057; mathcom BY sdq\_65\*0.04248; mathcom BY sdq\_66\*-0.13649; mathcom BY sdq\_67\*0.08974; mathcom BY sdq\_68\*0.20559; mathcom BY sdq\_69\*-0.08096; mathcom BY sdq\_70\*-0.13314; mathcom BY sdq\_71\*0.09010; mathcom BY sdq\_72@-0.09731; mathcom BY sdq\_73\*0.08917; mathcom BY sdq\_74\*0.08093; mathcom BY sdq\_75\*0.84844; mathcom BY sdq\_76\*0.05491;

mathaff BY sdq\_1\*0.02917; mathaff BY sdq\_2\*-0.04590; mathaff BY sdq\_3\*0.03817; mathaff BY sdq\_4@0.11083; mathaff BY sdq\_5\*-0.00099; mathaff BY sdq\_6\*1.10437; mathaff BY sdq\_7\*-0.07729; mathaff BY sdq\_8\*0.02207; mathaff BY sdq\_9\*0.18071; mathaff BY sdq\_10\*-0.03058; mathaff BY sdq\_11\*-0.02118; mathaff BY sdq\_12\*-0.05929; mathaff BY sdq\_13\*0.17265; mathaff BY sdq\_14\*-0.05697; mathaff BY sdq\_15\*-0.03669; mathaff BY sdq\_16@-0.02219; mathaff BY sdq\_17\*-0.03774; mathaff BY sdq\_18\*0.02009; mathaff BY sdq\_19\*-0.01505; mathaff BY sdq\_20\*1.09659; mathaff BY sdq\_21@-0.11460; mathaff BY sdq\_22@-0.04446; mathaff BY sdq\_23\*0.20464; mathaff BY sdq\_24\*-0.03226; mathaff BY sdq\_25\*-0.03306; mathaff BY sdq\_26\*-0.03018; mathaff BY sdq\_27@0.03160; mathaff BY sdq\_28\*0.00992; mathaff BY sdq\_29\*0.03104; mathaff BY sdq\_30\*-0.00932; mathaff BY sdq\_31\*-0.04989; mathaff BY sdq\_32\*0.02745; mathaff BY sdq\_33\*0.07847; mathaff BY sdq\_34\*0.00078; mathaff BY sdq\_35\*1.06929; mathaff BY sdq\_36\*0.02782; mathaff BY sdq\_37\*-0.05197; mathaff BY sdq\_38\*0.05628; mathaff BY sdq\_39@0.17280;



```

mathaff BY sdq_40*-0.00423; mathaff BY sdq_41*-0.03239; mathaff BY sdq_42*-0.00995;
mathaff BY sdq_43*0.25704; mathaff BY sdq_44*0.06873; mathaff BY sdq_45*-0.03605;
mathaff BY sdq_46*-0.00531; mathaff BY sdq_47*-0.01081; mathaff BY sdq_48*0.06323;
mathaff BY sdq_49*-0.00535; mathaff BY sdq_50@0.00584; mathaff BY sdq_51@1.26654;
mathaff BY sdq_52*-0.00616; mathaff BY sdq_53*-0.00407; mathaff BY sdq_54*-0.07068;
mathaff BY sdq_55*0.23184; mathaff BY sdq_56@0.00812; mathaff BY sdq_57@-0.00597;
mathaff BY sdq_58*-0.00621; mathaff BY sdq_59*0.20240; mathaff BY sdq_60*0.04015;
mathaff BY sdq_61*-0.02459; mathaff BY sdq_62*0.09024; mathaff BY sdq_63*-0.05669;
mathaff BY sdq_64*0.03170; mathaff BY sdq_65*-0.03723; mathaff BY sdq_66*0.10015;
mathaff BY sdq_67*-0.00482; mathaff BY sdq_68*0.95882; mathaff BY sdq_69*0.06475;
mathaff BY sdq_70*0.09384; mathaff BY sdq_71*0.13859; mathaff BY sdq_72@0.06366;
mathaff BY sdq_73*-0.09910; mathaff BY sdq_74*-0.05801; mathaff BY sdq_75*0.18125;
mathaff BY sdq_76*-0.04185;
esteem*1 peer*1 appear*1 phy*1 parent*1;
schocom*1 schoaff*1 Germcom*1 Germaff*1 MathAff*1 MathCom*1 ;
! Higher-order factor
general by esteem*1 peer appear phy parent
schocom schoaff Germcom Germaff MathAff MathCom ;
general@1;
! Method Factor (negative items)
MF BY SDQ_30* SDQ_17 SDQ_12 SDQ_21 SDQ_47 SDQ_23 SDQ_33 SDQ_65
SDQ_75 SDQ_6 SDQ_37 SDQ_61; MF@1; [MF@0];
MF WITH general@0 esteem@0 peer@0 appear@0 phy@0 parent@0 schocom@0
schoaff@0 Germcom@0 Germaff@0 MathAff@0 MathCom@0;
! correlated uniquenesses between parallel worded items
SDQ_11 with SDQ_51 SDQ_71; SDQ_51 with SDQ_71;
SDQ_25 with SDQ_35 SDQ_39; SDQ_35 with SDQ_39;
SDQ_41 with SDQ_68 SDQ_9; SDQ_68 with SDQ_9;
SDQ_57 with SDQ_20 SDQ_55; SDQ_20 with SDQ_55;
SDQ_23 with SDQ_6 SDQ_65; SDQ_6 with SDQ_65;
SDQ_4 with SDQ_27 SDQ_16; SDQ_27 with SDQ_16;
SDQ_18 with SDQ_59 SDQ_2; SDQ_59 with SDQ_2;
SDQ_49 with SDQ_13 SDQ_63; SDQ_13 with SDQ_63;
SDQ_73 with SDQ_43 SDQ_31; SDQ_43 with SDQ_31;
SDQ_47 with SDQ_75 SDQ_33; SDQ_75 with SDQ_33;

```

### References:

- Jöreskog, K. G. (1969). A general approach to confirmatory maximum likelihood factor analysis. *Psychometrika*, 34, 183–202.
- Morin, A. J. S., Arens, A., & Marsh, H. (2016). A bifactor exploratory structural equation modeling framework for the identification of distinct sources of construct-relevant psychometric multidimensionality. *Structural Equation Modeling*, 23, 116-139
- Marsh, H.W., Morin, A.J.S., Parker, P.D., & Kaur, G. (2014). Exploratory structural equation modelling: An integration of the best features of exploratory and confirmatory factor analyses. *Annual Review of Clinical Psychology*, 10, 85-110.
- Marsh, H.W., Nagengast, B., & Morin, A.J.S. (2013). Measurement invariance of big-five factors Construct-Relevant Multidimensionality over the life span: ESEM tests of gender, age, plasticity, maturity, and La Dolce Vita effects. *Developmental Psychology*, 49, 1194-1218.
- Morin, A. J. S., Marsh, H. W., & Nagengast, B. (2013). Exploratory structural equation modeling. In Hancock, G. R., & Mueller, R. O. (Eds.). *Structural equation modeling: A second course* (2nd ed., pp. 395-436). Charlotte, NC: Information Age Publishing, Inc.
- Muthén, L. K., & Muthén, B. O. (2009). Mplus short courses: Topic 1: Exploratory factor analysis, confirmatory factor analysis, and structural equation modeling for continuous outcomes. Los Angeles CA: Muthén & Muthén. Retrieved from [http://www.statmodel.com/course\\_materials.shtml](http://www.statmodel.com/course_materials.shtml)